Modeling of Dynamic Systems - Simulation Exercises

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Exercise 1

First solve exercise 11.11 in the exercise book.

Then, simulate the differential equation in 11.11b with an input signal added according to

$$
\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \\
y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t).
\end{aligned}
$$

Simulate a step response in Simulink with the two methods through these settings in `Simulation` – `Model Configuration Parameters...`:

- `Type: Fixed-step`
- `Fixed-step size`: desired step length
- `Solver`: `ode1` for Euler forward, och `ode14x` for Euler backward.
- `Extrapolation order`: 1 for Euler backward..

Investigate whether the step length limits you derived in exercise 11.11b are correct.

Hint: You can use the Simulink model `uppg11_11.mdl` from the course webpage.

Uppgift 2

Exercise 11.12 from the book.

Uppgift 3

Consider exercise 4.9c from the book.

(a) What are the poles of the system? What is the largest step length that can be used in the Euler forward method in order for the simulation to be stable?

(b) We will now focus on the slow dynamics, but we will simulate the original system with both the fast and slow dynamics. Simulate the step response of the system for 5000 seconds with:

  - `ode1` (Euler forward)
  - `ode14x` (Euler backward)

(See Exercise 1 for settings)

Compare the computational time and results when the step length is changed.

Hint: You can use the Simulink model `uppg4_9.mdl` from the course webpage.

(c) Now use a solver with a variable step length (`Type: Variable-step`) and compare with the solvers in (b):

- `ode45` (Explicit variable-step method)
- `ode15s` (Implicit variable-step method for stiff problems)

Are there any advantages (computational time, accuracy) by using a variable-step method for this problem? Compare, in particular, `ode1` with `ode45` and `ode14x` with `ode15s`. Are both the slow and fast dynamics accurately described? Why/why not?

Exercise 4

Exercise 11.15 in the exercise book.

Solutions

Exercise 3

(a) The poles of the system are $\lambda_1 = -0.001$ and $\lambda_2 = -1000$. For Euler forward, $|1 + h\lambda_i| < 1$ for $i = 1, 2$, which implies $0 < h < 0.002$.

(b) For `ode1` (Euler forward), the step length h must be less than 0.002, for instance $h = 0.001$. This requires $5000/0.001 = 5 \cdot 10^6$ steps! For `ode14x` (Euler backward), it suffices to use a step length which gives an acceptable solution for the slow dynamics, like a choice $h = 50$, which requires 100 steps. The computational time on a 1.2 GHz computer is 21.34 and 0.19 seconds, respectively.

(c) With a variable step length, it is possible to capture also the fast dynamics (zoom to the first 0.01 seconds). If this is to be captured with a fixed-step method, the step length must be chosen to even less than the stability requirement $h < 0.002$, which requires a very long simulation time.

The simulation time with the `ode45` and `ode15s` solvers (with default settings) are 18.35 and 0.12 seconds, respectively. Then they have taken $1.5 \cdot 10^6$ and 100 steps respectively.

`ode45` captures the fast dynamics in the start, but is then unable to use longer step sizes because of the stability of the explicit solver. In this case, it takes steps of average length 0.0033. Because of this, the computational time is approximately as long as for `ode1` (depending on the chosen relative error and step size).

Both `ode14x` and `ode15s` are very fast in comparison to the other methods. The difference is that `ode15s` also describes the fast dynamics. If this is to be done with `ode14x`, one must again use a step size which is approximately as short as `ode1`, which requires a lot of computational time. An implicit solver with variable step length (such as `ode15s`) is thus useful for simulation of stiff systems.