

Computer Exercises in System Identification

Part 1

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Objective

This document contains computer exercises in system identification. The field is to a large extent fairly theoretical, and the connection to its many practical applications is not always obvious. This document aims to give an overview of how to find a good model of a dynamic system from data. The objectives are as follows:

- Introduce a workflow for system identification
- Give an overview of how SYSTEM IDENTIFICATION TOOLBOX (SITB) for MATLAB is used
- Indicate what quantities are useful to study for model validation

SITB's Graphical User Interface

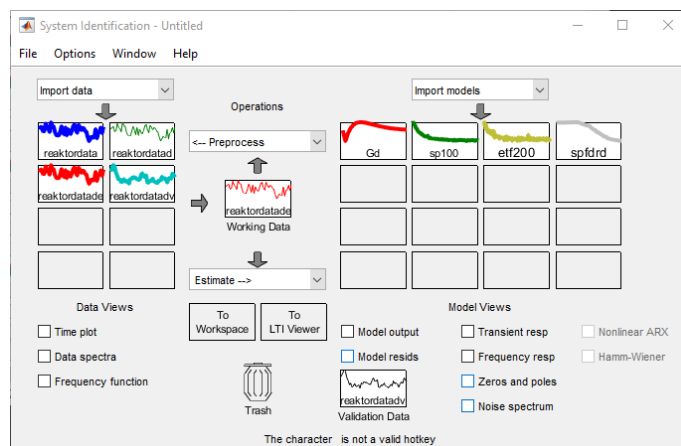


Figure 1: The graphical user interface of the SITB.

Most things related to pre-processing of data, identification, and validation can be done in a graphical user interface (GUI) in SYSTEM IDENTIFICATION TOOLBOX (SITB) in MATLAB, see Figure 1. This GUI can be started by entering

```
>> systemIdentification
```

in MATLAB's command prompt. To understand how the GUI works, a brief description is now given. In the left of Figure 1, data sets used for modeling are illustrated with icons. These data sets can be estimation and validation data sets, as well as any other data sets relevant for modeling. These data sets can be plotted in the time and frequency domain using `Time plot` and `Data spectra` in `Data Views`.

The data set which is currently used for data preprocessing and estimation of models is in the field `Working Data`. This data set can be set and changed by dragging a data set icon to `Working Data`. This data set can be preprocessed (`Preprocess`) or used for model estimation (`Estimate`). The preprocessed data set will appear in the data area with a new name, and to make further preprocessing steps, this new data set must be dragged to `Working Data`. The estimated models are illustrated by icons in the right-hand side of the GUI. Properties of the estimated models can be studied by marking the checkboxes in `Model Views`. When data is needed for these plots (to validate the model), the data set in `Validation Data` can be used. Icons can be discarded in `Trash` if they will not be used anymore. New data sets can be moved to `Working Data`, `Validation Data` or `Trash` by dragging the data set icons there.

Any work that has been done can (and should) be saved, now and then, in order to be able to easily continue working at a later occasion. This can be done by clicking `File` and then `Save session`. This is particularly useful in this exercise set, because many upcoming exercises are based on the these ones!

Note

- MATLAB is opened by writing the following in the terminal window:
 `> module add courses/TSRT92`
 `> matlab`
- The files for the exercises are found on the course webpage.
- Everything that can be done through the GUI (and even more) can also be done in MATLAB code. This is sometimes easier, especially if many different models are to be estimated.
- Enter `help ident` to get a list of SITB commands. `iddemo` is also useful. Right-clicking estimated models in the GUI shows how to compute the estimated model using code.

Importing and preprocessing data

If data are saved in the file `myfile`, they can be loaded by

```
>> load myfile
```

These data sequences can then be imported into the GUI by clicking **Import data** and choosing **Time domain data**. Data can be stored in two different ways: as vectors/matrices or as `iddata` objects. One of these options must be chosen in **Data Format for Signals**. The data set can also be named in this window, and the sample time and start time can be chosen.

By clicking **Preprocess**, the menu in Figure 2 is shown. Clicking any of

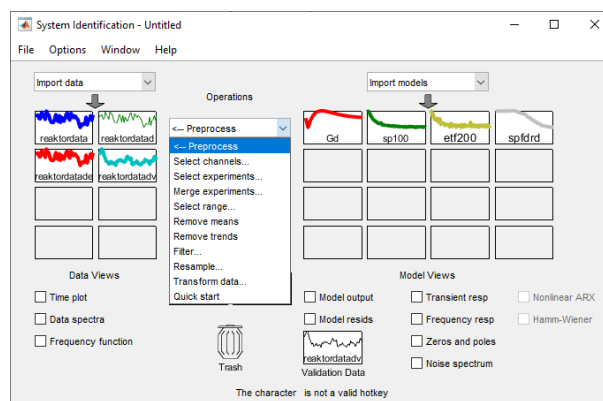


Figure 2: Ways to pre-process data in SITB.

these options performs a preprocessing step on the data set in **Working Data**. Those which are of interest for this document are explained here. For more information, see [2, Chapter 17.3].

- **Select range**. Choose what data can be included in the different data sets.
- **Remove means**. Remove means from input and output signals.
- **Quick start**. Remove means and then splits the data into two equally-sized parts (for estimation and validation). Make sure to check the result!

1 Nonparametric Identification

We will now study nonparametric frequency estimation methods, see [2, Chapter 10]. These can be used in the GUI by clicking **Estimate** and then **Spectral Models...** Three methods are implemented: Blackman-Tukey's method (SPA), SPAFDR, and ETFE. The models can be studied by clicking **Frequency resp** in **Model Views**.

We will also compute impulse responses (CRA). In this exercise, we will do it outside the graphical user interface (preferably by writing an `m` file).

1. Simplistically, a chemical reactor can be described as a tank which contains a liquid mixture undergoing a chemical reaction. The reactor often has a continual inflow of reagents and an outflow of chemical products. The inflow is typically varied in order to control the concentration of certain chemical products, and the mixture is kept homogeneous by constant stirring. A 100 liter Van de Vusse reactor can at a certain linearization point be described using the model

$$G(s) = \frac{-1.1170s + 3.1472}{s^2 + 4.6429s + 5.3821},$$

where the input signal is the inflow (in l/min), and the output signals is the concentration (in mol/l) of the interesting chemical product in the outflow.

Now assume that the model is unknown, but that a number of experiments has been done on the reactor. These are stored in the file `reaktordata.mat` as a number of `iddata` objects (run `load reaktordata`). The file also contains the transfer functions `Gd` and `Fd`, where `Gd` is the model $G(s)$ discretized with sample time $T = 0.1$ min, and `Fd` is the controller which was used for the experiments `z2–z5`. The transfer functions `Gd` and `Fd` can be imported into the SITB by selecting **Import models**.

- (a) Import the data `z1` to SITB and study its time and frequency properties in **Data Views**.
- (b) Estimate a few nonparametric models (**Spectral models...**). Use both SPA and ETFE, and vary the frequency resolution (in [2] it is denoted γ). How does the frequency resolution influence the spectral model?

Also try the more advanced method SPAFDR and vary the number of frequencies. Set the frequency resolution to `Default`.

- (c) Data set `z1` was collected by varying the input signal randomly, and measuring the output of the system. Now, import data set `z2` which was collected from closed-loop operation of the system. The input signal is now computed using the controller `Fd` based on the difference between a random reference signal and the measured system output. Do the nonparametric models appear good? What is the problem?
- (d) In order to further investigate the influence of feedback, two more data sets `z3` and `z4` are available, where the amplitude of the reference signal is either 10 times larger (`z3`) or is zero (`z4`). Using `z2`, `z3` and `z4`, explain how the estimate is influenced by the feedback and the reference signal.
Hint: How much of the input energy comes from the reference signal, and how much comes from the feedback?
- (e) Bonus question: Periodic data. Also try the periodic data set `z5`. The reference signal at the time of the experiment was periodic noise (thus, the exact same noise sequence was repeated 10 times). SITB takes this into account since `z5.Period` has been set to $N/10 = 200$. Compare the ETFE estimates from `z5` and `z2` (which has the same reference signal amplitude, but is non-periodic).

2. When simulating acoustics in a room, thousands of samples of the acoustic behavior of the room is often needed. In this exercise we will compare the acoustics of two rooms (a lecture hall and an office) both by listing and by studying the impulse responses. We will estimate the impulse responses using CRA, and study how the rooms differ.

The acoustic experiment has been carried out as follows: a speaker playing a trumpet fanfare, and a microphone recording the sound, are placed at the same location in a room. The file `akustikdata.mat` (run `load akustikdata`) contains the signals `u`, `y1`, and `y2`, where `u` is the trumpet fanfare and `y1` and `y2` are recordings of the fanfare in the two rooms. All signals consist of $N = 41154$ samples with sample frequency $f_s = 16$ kHz.

Note: This exercise should be solved in code, i.e, outside the GUI. The reason is that certain commands (such as `cra`¹) are unavailable in the GUI.

¹In `Estimate -> Correlation Models`, only the method `impulse` can be used. This

- (a) Study the behavior of the signals, for example by plotting them. Also listen to the sounds if possible. Can you tell what signal comes from what room?

Hint: To play a sound signal u , run the code

```
fs=16000; soundsc(u,fs);
```

- (b) Estimate the first 500 lags of the impulse response for the rooms by using the method `cra`. In order to get a good estimate, you must increase the order of the whitening filter ($L(q)$ in page 410 of [2]). Increase the model order until the estimated impulse response appears to be the same as the last order.

Hint: An impulse response for $M = 500$ lags with whitening filter order $n_a = 100$ is estimated by:

```
T=1/fs; z1=iddata(y1,u,T); M=500; na=100;
g1hat=cra(z1,M,na,0);
figure; stairs(g1hat);
```

- (c) Use the estimated impulse responses to compute the distance to the closest object in the rooms.
- (d) Use your estimated impulse response to simulate the room acoustics. Are you able to recreate a realistic trumpet sound? Try various numbers of lags M and whitening filter orders n_a and see how they influence the sound.

Hint: Simulation of the acoustics can be done through:

```
y1hat=filter(g1hat,1,u);
```

3. For many mechanical systems (e.g., bridges, airplanes, cars) it is common to perform vibration analysis. This often consists of studying the resonance modes of the system by estimating the frequency response from applied force to movements in the system (typically acceleration). Forces are usually generated in one of three ways: an impulse hammer, a shaker, or normal operation. The movements are usually measured using an accelerometer.

This exercise consists of computing spectral models from two data sets. The file `vibrationdata.mat` (run `load vibrationdata`) contains the `iddata` objects `zh` (impulse hammer) and `zs` (shaker). The true system `Gd` is also included; you can use it to verify the estimated models.

method is different from CRA, and often gives a better result; however, it is too slow if many impulse response coefficients must be estimated.

- (a) Estimate spectral models for the two data sets by using SPA, ETFE and SPAFDR. How many resonance peaks can you find?
 - (b) The same maximum force amplitude has been used in the two data sets (which is reasonable, since the mechanical construction cannot withstand arbitrarily large forces). How does this influence the accuracy of the spectral models between the two data sets?
4. You have been assigned to compute a model of an electric motor with a mechanical load, see Figure 3. An approximate physical model for such a motor is given by

$$G(s) = \frac{1}{s^3 + 0.9s^2 + 1.1s + 0.46} \quad (1)$$

In an experiment, 1000 data points have been collected with a sample time of 0.3 seconds. The data set has been saved in `elmotor.mat`, where the input signal (voltage, V) is `u` and the output signal (angular velocity, rad/s) is `y`.

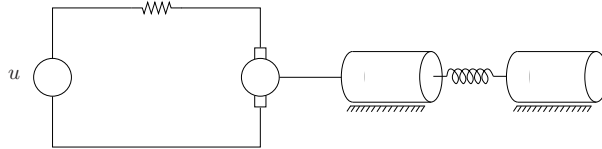


Figure 3: Electric motor with load

- (a) Import the data set into SITB and study its time and frequency properties. Ensure that the data set appears normal, i.e. that no trends are in the data, and that no outlier has been included.
- (b) How does the Bode plot of the motor model defined in (1) look?
- (c) Estimate a few nonparametric models. Use SPA, SPAFDR, and ETFE, and vary the frequency resolution. How does the choice of frequency resolution influence the estimate? Does the Bode plot of the physical model agree with the estimated spectral model?

Solutions with Discussion

The solutions below are not necessarily the only correct solution. Depending on choices during data processing and estimation, your results may be perfectly correct even if they do not exactly agree with the discussion below.

Nonparametric Identification

1. (a) The signals appear OK. Thus, proceed to (b).
- (b) A small frequency resolution yields a bias (systematic errors), but a large frequency resolution yields high variance (“fluttery” errors). A good compromise is a frequency resolution of 200 for SPA and ETFE. For SPAFDR, choose **Frequency Spacing Logarithmic**. The quality of the estimate is then quite good for a wide frequency band. Note that the “flutter” at high frequencies disappears when logarithmic frequency spacing is used instead of linear.
- (c) The estimates are systematically wrong in the whole frequency band. The reason is that the input signal is correlated to the noise because of the feedback. Spectral methods assume that the input signal is uncorrelated to measurement noise.
- (d) We first provide an intuitive explanation, and then give more mathematical detail. Spectral methods assume that the input signal is uncorrelated with the measurement noise. In this case, this does not hold since $u(t) = F(q)(r(t) - y(t)) = F(s)R(s) - F(q)y(t)$, and $y(t)$ depends on the measurement noise. However, think about how big the correlation is. If r is 0, the only signal influencing u will be the measurement noise, so the correlation will be large. If r is large, only a smaller fraction of the energy in u will be due to measurement noise, so the correlation will be smaller (but still nonzero).

Mathematically, for ETFE, the frequency estimate in feedback is given by (when $N \rightarrow \infty$):

$$\hat{G}(i\omega) \rightarrow \frac{G(i\omega)\Phi_r(\omega) - [F(i\omega)]^{-1}\Phi_v(\omega)}{\Phi_r(\omega) + \Phi_v(\omega)}.$$

The bias thus depends on the ratio between $\Phi_r(\omega)$ and $\Phi_v(\omega)$; the signal-to-noise ratio. If r is 10 times larger, Φ_r is 100 times larger,

and the bias should decrease proportionally. If $r = 0$, $\Phi_r = 0$ and thus $\hat{G}(i\omega) \rightarrow [F(i\omega)]^{-1}$. This can be verified by loading `Fd` through `Import models`.

- (e) The bias is reduced when `z5` is used. This is because ETFE averages over the periods, which reduces the noise variance Φ_v by a factor of 10. This also reduces the frequency resolution by the same factor, however.

2. (a) Plot the signals by, for example:

```
fs=16000; T=1/fs; N=length(u); t=T*(1:N)';
figure; plot(t,[u y1 y2]);
```

Already in this plot, we can see that the recorded signals from the rooms contain reverberations (echos). The signal `y1` has the longest reverberations, so it should be from the larger room (the lecture hall). By listening, this difference can be clearly heard:

```
soundsc(u,fs); soundsc(y1,fs); soundsc(y2,fs);
```

- (b) The impulse response for `y1` can be estimated as follows:

```
z1=iddata(y1,u,T);
M = 500; na = 300;
g1hat=cra(z1,M,na,0);
t=T*(0:M)';
figure; stairs(t,[g1hat,g1(1:M+1)])
```

where the true impulse response `g1` (from `akustikdata.mat`) is shown for comparison. The order has to be around $M = 300$ in order to accurately estimate the first 100 lags close to 0. We get similar results for `y2`.

- (c) There is a direct term at time 0, but afterwards the impulse response is close to 0 until 6.5 ms for `y1` and 3 ms for `y2`. Multiplying by the speed of sound 343 m/s yields that the traveled distance is 2.2 m and 1 m respectively. Since the sound travels from the trumpet to the closest object and back to the microphone, the distance to the closest object in the rooms is half that: 1.1 m and 0.5 m respectively.

- (d) The most difficult signal to recreate is `y1`. Run the following code for different `M` and `na`:

```
M=2000; na=300;
g1hat=cra(z1,M,na,0);
y1hat=filter(g1hat,1,u);
soundsc(y1,fs);
soundsc(y1hat,fs);
```

The number of lags `M` must be at least 2000 in order to get the sound “right”. If `na` is reduced, the sound becomes muffled.

3. (a) The system has resonances at frequencies 2.2, 6.5, 12, 21, and 40 rad/s. The highest frequency is drowned out by noise for both the data sets. The first resonance at 2.2 rad/s can also be hard to distinguish with ETFE and SPA (with **Frequency Spacing Linear**). The other resonance peaks can be easily distinguished in the data set `zs` if the frequency resolution is chosen large enough (5000 works well). For the dataset `zh`, the fourth resonance may be difficult to see because the noise level is higher.

For SPAFDR, **Frequency Spacing Logarithmic** and 500 frequencies gives a good result for data set `zs`. The first resonance can be seen easily. This can also be done with SPA with **Frequency Spacing Logarithmic** with frequency resolution 10000. For data set `zh`, it may be hard to find a setting where the fourth resonance is distinguishable.

It is difficult to accurately estimate the damping (the magnitude of the resonance peaks) for this resonant system. This can be verified by importing the true system `Gd` into the GUI.

- (b) Comparing the signals in `Time plot`, it can be seen that the output when the shaker is used is roughly 100 times larger. The spectrum of the signals (`Data spectra`) is also dramatically different. In order to get similar results with an impulse hammer as with a shaker, much more force must be applied.
4. (a) The signals appear OK. Thus, proceed to (b).
- (b) The bode plot of the system can be seen in 4. The system has a resonance peak at around 1 rad/s, and at high frequencies the slope is -3 .

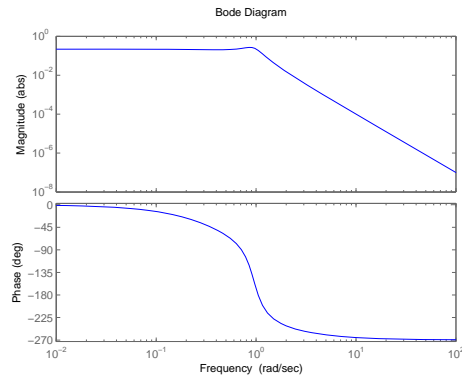


Figure 4: Bode plot for the electric motor with a load.

- (c) When the frequency resolution is increased, the noisiness is increased, but the bias is reduced. A good trade-off for this system is at around 70–80.

References

- [1] T. Glad and L. Ljung. *Reglerteknik. Grundläggande teori*. Studentlitteratur, 2006.
- [2] L. Ljung, T. Glad and A. Hansson. *Modeling and identification of Dynamic Systems*. Studentlitteratur, 2021.