Modeling and Learning for Dynamical Systems

Lecture 9

Martin Enqvist

LINKÖPING UNIVERSITY

Subspace Methods for State-Space Models

Introduction

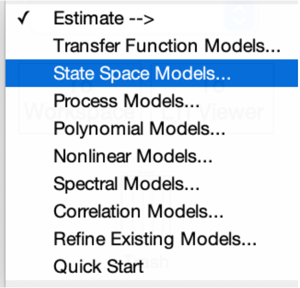
- State-space models can be parameterized directly (e.g., all matrix elements are parameters) and estimated using the prediction-error method (PEM)
- However, initialization of the PEM can be an issue (remember that a change of basis will give a different state-space description of the same input-output relation)
- An alternative is to use a class of methods known as *subspace methods* where a state-space model is estimated in a computationally efficient way without requiring a particular parameterization

Properties

- Computationally efficient method (no iterative search)
- Well-suited for large datasets and MIMO problems
- Limitation: Only for linear state-space models
- Limitation: Closed-loop data is a challenge (although special methods exist for particular cases)

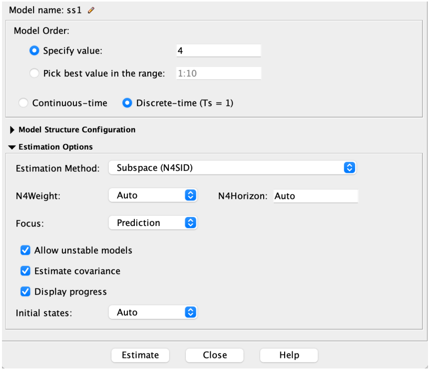
Matlab Example

Consider the dataset we studied during Lecture 7 and estimate a state-space model



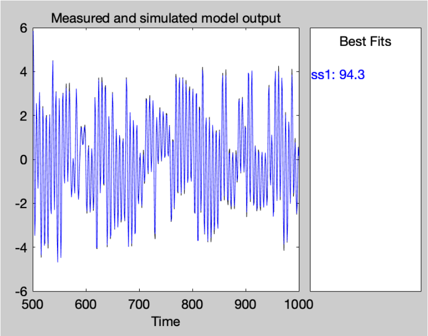
Matlab Example. . .

Select model order, method (subspace or PEM) and N4Horizon (contains r, s_y and s_u) (we have used r and $s = s_y = s_u$)



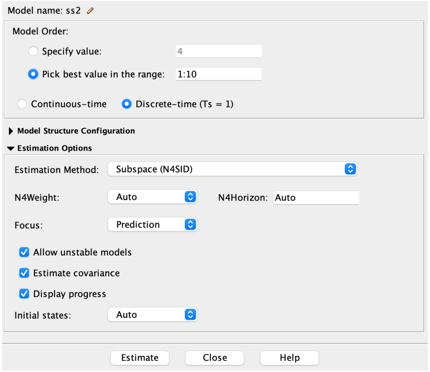
Matlab Example. . .

A fourth-order model gives a high model fit in this case



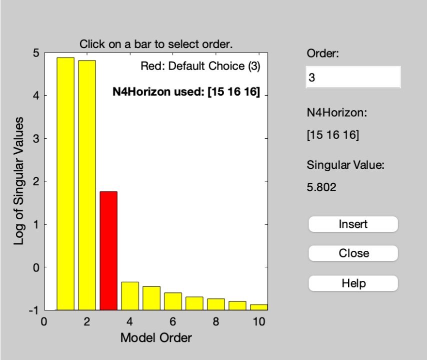
Matlab Example...

We can also try several model orders simultaneously



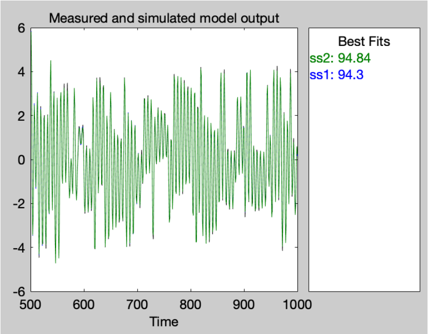
Matlab Example. . .

The singular values indicate that a third-order model might be enough



Matlab Example. . .

The third-order model gives a small improvement of the model fit



Estimation of Grey-Box State-Space Models

Grey-Box State-Space Models

- Grey-box models (which are based on first-principles modeling and prior knowledge) are often convenient to write on state-space form
- It is then straightforward to estimate them using the prediction-error method
- However, there is often a risk for ending in a local minimum and black-box modeling using for example subspace methods can be a way to find suitable starting points

Grey-Box Example

Consider a model of a DC motor with a particular physically motivated parameterization:

$$
\begin{aligned}
\dot{x}(t) &= \begin{pmatrix} 0 & 1 \\ 0 & \theta_1 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix} u(t) \\
y(t) &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x(t) + e(t) \\
x(0) &= \begin{pmatrix} \theta_3 \\ 0 \end{pmatrix}
\end{aligned}
$$

(You will see another parameterization at the exercise sessions.)

Grey-Box Example...

Specify the model structure in a Matlab function (`par` contains the parameters and `T` is the sampling time (not used here)).

```
function [A,B,C,D,K,x0] = myfunc(par,T)
A = [0 1; 0 par(1)];
B = [0;par(2)];
C = eye(2);
D = zeros(2,1);
K = zeros(2,2);
x0 = [par(3);0];
```

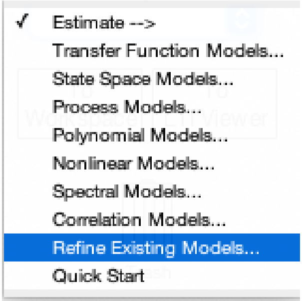
The model is then defined using the command

```
m0=idgrey('myfunc',[-1;0.25;0],'c')
```

(The second argument contains the initial values for the parameters and the third (`'c'`) states that the model is in continuous time)

Grey-Box Example. . .

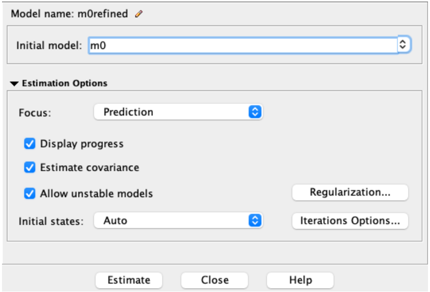
The grey-box model can now be estimated by refining an existing model (m0).



Grey-Box Example. . .

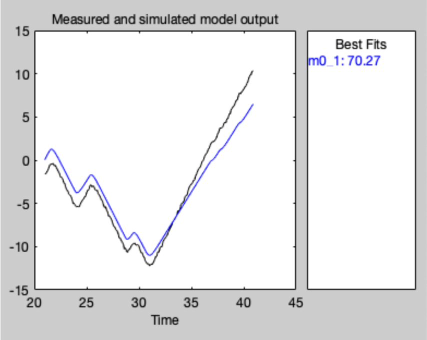
Model estimation gives parameter estimates

$$\hat{\theta} = \begin{pmatrix} -4.642 \\ 1.003 \\ 3.662 \end{pmatrix}$$



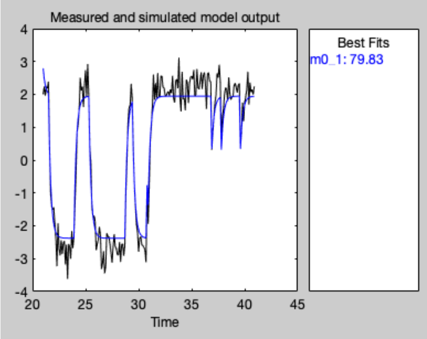
Grey-Box Example. . .

Result (motor angle)



Grey-Box Example. . .

Result (motor velocity)



Summary

- Subspace methods for state-space models are based on linear algebra, singular value decomposition and the least-squares method
- Estimation of grey-box state-space models