

Modeling and Learning for Dynamical Systems

Lecture 9

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Subspace Methods for State-Space Models

Introduction

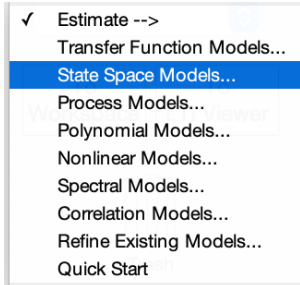
- State-space models can be parameterized directly (e.g., all matrix elements are parameters) and estimated using the prediction-error method (PEM)
- However, initialization of the PEM can be an issue (remember that a change of basis will give a different state-space description of the same input-output relation)
- An alternative is to use a class of methods known as *subspace methods* where a state-space model is estimated in a computationally efficient way without requiring a particular parameterization

Properties

- Computationally efficient method (no iterative search)
- Well-suited for large datasets and MIMO problems
- Limitation: Only for linear state-space models
- Limitation: Closed-loop data is a challenge (although special methods exist for particular cases)


Matlab Example

Consider the dataset we studied during Lecture 7 and estimate a state-space model



Matlab Example. . .

Select model order, method (subspace or PEM) and N4Horizon (contains r , s_y and s_u) (we have used r and $s = s_y = s_u$)

Model name: ss1 

Model Order:


Specify value:


Pick best value in the range:


Continuous-time Discrete-time (Ts = 1)

► Model Structure Configuration

▼ Estimation Options

Estimation Method: 


N4Weight:  N4Horizon:

Focus: 

Allow unstable models

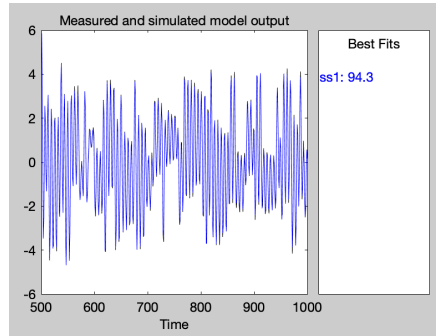
Estimate covariance

Display progress

Initial states: 

Matlab Example...

A fourth-order model gives a high model fit in this case



Matlab Example. . .

We can also try several model orders simultaneously

The screenshot shows the MATLAB Model Configuration dialog box for a state-space model. The model name is 'ss2'. Under 'Model Order', there are three options: 'Specify value' (set to 4), 'Pick best value in the range' (set to 1:10), and 'Continuous-time' (unselected) vs 'Discrete-time (Ts = 1)' (selected). The 'Model Structure Configuration' section is expanded to show 'Estimation Options'. The 'Estimation Method' is 'Subspace (N4SID)'. 'N4Weight' is 'Auto' and 'N4Horizon' is 'Auto'. 'Focus' is 'Prediction'. Three checkboxes are checked: 'Allow unstable models', 'Estimate covariance', and 'Display progress'. 'Initial states' is 'Auto'. At the bottom are 'Estimate', 'Close', and 'Help' buttons.

Model name: ss2

Model Order:

- Specify value: 4
- Pick best value in the range: 1:10
- Continuous-time Discrete-time (Ts = 1)

► Model Structure Configuration

▼ Estimation Options

Estimation Method: Subspace (N4SID)

N4Weight: Auto N4Horizon: Auto

Focus: Prediction

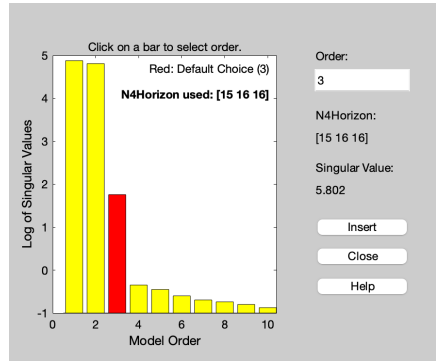
- Allow unstable models
- Estimate covariance
- Display progress

Initial states: Auto

Estimate Close Help

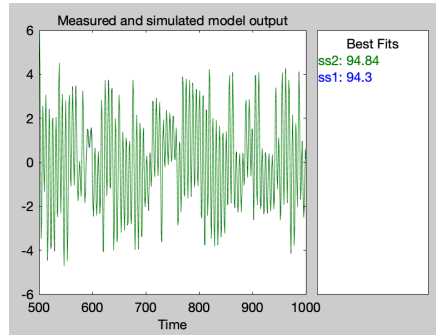
Matlab Example. . .

The singular values indicate that a third-order model might be enough



Matlab Example. . .

The third-order model gives a small improvement of the model fit



Estimation of Grey-Box State-Space Models

Grey-Box State-Space Models

- Grey-box models (which are based on first-principles modeling and prior knowledge) are often convenient to write on state-space form
- It is then straightforward to estimate them using the prediction-error method
- However, there is often a risk for ending in a local minimum and black-box modeling using for example subspace methods can be a way to find suitable starting points

Grey-Box Example

Consider a model of a DC motor with a particular physically motivated parameterization:

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ 0 & \theta_1 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x(t) + e(t)$$

$$x(0) = \begin{pmatrix} \theta_3 \\ 0 \end{pmatrix}$$

(You will see another parameterization at the exercise sessions.)

Grey-Box Example. . .

Specify the model structure in a Matlab function (par contains the parameters and T is the sampling time (not used here)).

```
function [A,B,C,D,K,x0] = myfunc(par,T)
A = [0 1; 0 par(1)];
B = [0;par(2)];
C = eye(2);
D = zeros(2,1);
K = zeros(2,2);
x0 = [par(3);0];
```

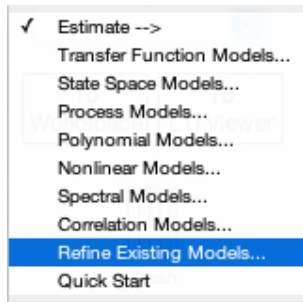
The model is then defined using the command

```
m0=idgrey('myfunc',[-1;0.25;0], 'c')
```

(The second argument contains the initial values for the parameters and the third ('c') states that the model is in continuous time)

Grey-Box Example. . .

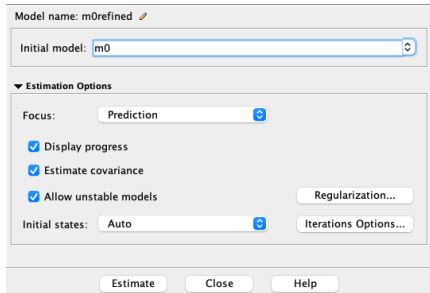
The grey-box model can now be estimated by refining an existing model (m0).



Grey-Box Example...

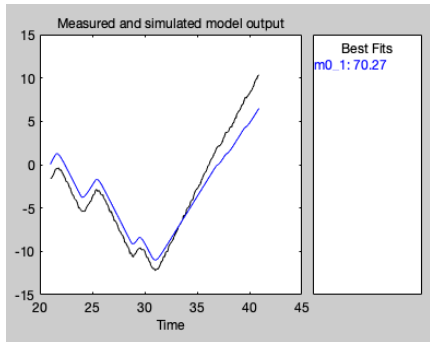
Model estimation gives parameter estimates

$$\hat{\theta} = \begin{pmatrix} -4.642 \\ 1.003 \\ 3.662 \end{pmatrix}$$



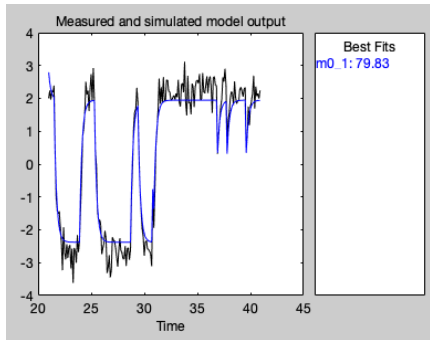
Grey-Box Example...

Result (motor angle)



Grey-Box Example...

Result (motor velocity)



Summary

- Subspace methods for state-space models are based on linear algebra, singular value decomposition and the least-squares method
- Estimation of grey-box state-space models

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