# Modeling and Learning for Dynamical Systems

Martin Enqvist



Lecturer and examiner: Martin Enqvist

Teaching assistants:

Gustav Zetterqvist Joel Nilsson

Course room in Lisam (mainly for lab sign-up and hand-ins)

Course webpage: http://www.control.isy.liu.se/student/tsrt92/



#### Who am I?

#### Martin Enqvist:

Education: MSc from Y program at LiU 1996-2000, PhD student in automatic control at LiU 2000-2005 (PhD thesis: "Linear Models of Nonlinear Systems")

Postdoc at a university in Brussels, Belgium during 2006

Back at LiU since 2007

Now: Associate professor in automatic control, research about system identification (e.g. aircraft, vehicles, ships, robots, electronics, sound), collaboration with Saab, ABB and Oticon













The course should give knowledge about methods and principles for constructing mathematical models of dynamical systems, and about how properties of the models can be studied through simulation.

Three themes:

- First principles modeling
- System identification and model learning
- Simulation



#### Organization

12 lectures

12 exercise sessions (7 in computer rooms)

3 labs:

- Modeling and simulation of a measurement robot
- System identification of a weather vane (written report + peer review)
- Nonlinear system identification and machine learning

Register as soon as possible!

Computer exam (textbook allowed)



#### Course Evaluation Last Year

#### Result:

- Overall evaluation: mean 2.05
- Response frequency: 36%
- Request for more focus on practical aspects
- Request for closer connection between lectures and exercise sessions
- Some labs took too long time to finish

#### Changes:

- New examiner
- Revised lecture series
- Compendium about experiment design removed (changed lecture and exercise session)
- Two lab assistants for lab 1 and 3



#### The Use of Models



#### Systems and Models

- A **system** is any kind of physically or conceptually bounded object (e.g. the solar system, a human brain cell, an electrical motor)
- A model is a description of a system

System properties can be investigated directly in experiments. However, this can be

- expensive (e.g. reduced quality in a production plant)
- dangerous (e.g. nuclear power plant)
- impossible (e.g. solar system, new inventions or products)



#### Models of Different Nature

- physical models (e.g. a small-scale ship model)
- mental models (e.g. our understanding of how to ride a bike)
- verbal models (e.g. an increased power will result in a higher temperature)
- mathematical models
  - based on first principles (domain knowledge, often acquired over long time)
  - based on data-driven modeling (system identification based on experimental observations)



#### The Use of Models

Models can be used for

- simulation (numerical experiments producing computed system behaviors given a particular starting condition)
- prediction (numerical forecasts over a bounded horizon given past measurements)
- signal processing
- realistic visualization and virtual reality
- fault detection and diagnosis (by comparing model predictions and measurements)
- control design
- . . .



#### Example: Crash Tests

- Crash tests are expensive
- Simulations are an attractive complement
- A mathematical model and reliable simulation tools are needed





### Example: Aircraft Development

- Simulations can be used to evaluate the aircraft performance before the first prototype has been built
- Simulations increase the safety during development
- Model-based control design
- Pilot training using simulators





#### Example: Weather and Climate Models

- Weather forecasts are based on model-based predictions
- Climate models can be used for long-term predictions and simulation of particular scenarios (*what is the average temperature by 2100?* - the alternative is just to wait)
- At the same time: Extrapolations have to be considered carefully (values might become larger or smaller than predicted)







## The Use of Models within Automatic Control



#### Control without Models

Much of what we humans do can be viewed as automatic control without (mathematical) models



- Simple control problems can be solved fairly well without models.
- Controllers can be tuned iteratively by repeated experiments.



#### Model-based Control Design

Most modern control design methods are model-based.

## **Benefits:** Model-based control design

- saves lives
- saves time
- saves money
- can be used to analyze a system before it exists



Airbus A380



## The Two Sides of Automatic Control

#### An unforgiving side:

Control design using an incorrect model might result in an unstable closed-loop system.

- Controllers must be robust against model errors.
- Models must come with a quality measure.

#### A forgiving side:

A simple approximate model might be enough for control design purposes in many cases.

**Conclusion:** It is OK to use rather simple models for automatic control.



## Example: Approximate Models (I)

Assume that we have two models of a particular system:

$$G_1(s) = \frac{0.4}{s}e^{-0.5s}$$
$$G_2(s) = \frac{2}{5s+1}e^{-0.5s}$$

Input (a step):

$$u(t) = \begin{cases} 0, & t < 0, \\ 1, & t \ge 0 \end{cases}$$

The step responses of the models differ significantly:





## Example: Approximate Models (II)

Assume that the controller is:

$$F(s) = 4.4 \left( 1 + \frac{1}{5.2s} \right)$$

Closed-loop systems based on the two models:

$$G_{c,k}(s) = \frac{G_k(s)F(s)}{1 + G_k(s)F(s)}$$

k=1,2

The step responses of the two closed-loop systems are similar despite the model differences:





## The Cost of Modeling

The modeling of an unknown system can be quite **time-consuming** and is often a significant part of an industrial control-design project.

- In particular, this is true for modeling based on first principles (physical laws, known relations, etc.).
- A convenient alternative: Data-driven modeling via **system identification**



Paper machine 12 at StoraEnso Kvarnsveden has 15 000 control loops.



The Cost of Modeling...

Normally modeling costs account for over 75% of the expenditures in the design of an advanced control project [1].

Identification is undoubtedly the most important aspect of any control project, easily taking in excess of 75% of the total project resources [2].

The plant test and subsequent model identification are the most important steps in an MPC project, and incur the most time, representing up to 50% of the total project time [3].

- M. A. Hussain, Review of neural networks in chemical process control simulation and online implementation, Artificial Intelligence in Engineering, 13(1): 55-68, January 1999.
- [2] J. W. MacArthur and C. Zhan, A practical global multi-stage method for fully automated closed-loop identification of industrial processes, Journal of Process Control, 17(10):770-786, December 2007.
- [3] M.L. Darby and M. Nikolaou, MPC: Current practice and challenges, Control Engineering Practice, 20(4):328-342, April 2012.

## **Example Plots**



#### Example: Hare-Lynx Cycles





Example: Hare-Lynx Cycles...



Number of hares (blue) and lynxes (red) according to simple model



#### State-space Models



#### A common model structure: State-space models:

$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = h(x(t), u(t))$$

where  $\boldsymbol{x}(t)$  are states,  $\boldsymbol{u}(t)$  inputs and  $\boldsymbol{y}(t)$  outputs



#### State-space Models

What is so special with state-space models?

```
\dot{x}(t) = f(x(t), u(t))
```

Assume  $x(t_0)$  and u(t) (continuous) are given.

- f continuously differentiable and u(t) is piece-wise continuous  $\Rightarrow$  a unique solution exists for  $t \geq t_0$
- Reliable numerical methods for finding this solution are available
- For these reasons, it is often beneficial (ideal) to express models on state-space form
- A common (but not always desireable) alternative: Differential-algebraic equations (DAEs):  $F(\dot{z}, z, u) = 0$



#### Linearization

Consider a nonlinear model

$$\dot{x} = f(x, u)$$
$$y = h(x, u)$$

with the stationary solution (equilibrium)  $x_0, u_0, y_0$ :

$$0 = f(x_0, u_0)$$
$$y_0 = h(x_0, u_0)$$

Let

$$\delta_x = x - x_0$$
  

$$\delta_u = u - u_0$$
  

$$\delta_y = y - y_0$$



#### Linearization...

Linearized state-space model:

$$\dot{\delta_x} = A\delta_x + B\delta_u$$
$$\delta_y = C\delta_x + D\delta_u$$

(good approximation for small  $\delta_x$ ,  $\delta_u$ ,  $\delta_y$ )

Here:

$$A = f_x(x_0, u_0) \quad B = f_u(x_0, u_0) C = h_x(x_0, u_0) \quad D = h_u(x_0, u_0)$$

(Jacobians with element i, j equal to  $\frac{\partial f_i}{\partial x_j}$ )



#### Summary

- Systems and models
- Model useage: simulation, prediction, signal processing, visualization, diagnosis, control design, ...
- Models for control design
- State-space models: Relatively easy to analyze and simulate, linearization



#### www.liu.se

