EXAM IN MODELING AND LEARNING FOR DYNAMICAL SYSTEMS (TSRT92)

ROOM: ASGÅRD

TIME: Monday, August 21, 2023, 08.00–12.00

COURSE: TSRT92 Modeling and Learning for Dynamical Systems

CODE: DAT1

DEPARTMENT: ISY

NUMBER OF EXERCISES: 4

NUMBER OF PAGES (including cover page): 5

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APPROVED TOOLS:

1. *L. Ljung, T. Glad & A. Hansson* "Modeling and Identification of Dynamical Systems" Previous editions (including Swedish edition) of the book are also allowed.
2. *T. Glad & L. Ljung*: "Reglerteknik. Grundläggande teori"
3. *A. Hansson*: "Optimal Experiment Design"
4. Mathematical tables and formulas (e.g.*L. Råde & B. Westergren*: "Mathematics handbook",*C. Nordling & J.Österman*: "Physics handbook", *S. Söderkvist*: "Formler & tabeller")
5. Calculator and Matlab

Notes in the above books are allowed.

SOLUTIONS: Linked from the course home page after the exam.

The exam can be inspected and checked out 2023-09-12, 12.30-13:00 in room 2A:573, B-building, entrance 25, A-corridor.

PRELIMINARY GRADING:	grade 3	15 points
	grade 4	23 points
	grade 5	30 points

All solutions should be well motivated. Writing should be neat and clean.

Good Luck!

COMPUTER TIPS:

- To open Matlab:
  - open a terminal (right-click on the background and choose `open terminal`)
  - type

    ```
    module add prog/matlab
    matlab &
    ```

- Print out the model description and the plots requested

- Remember to write your AID number on each printed page you include

- In the identification exercise using the System Identification toolbox:
  - To print the model description: Right-click on the icon of the model you have computed and then click `Present`. The model description appears then on the Matlab main window. Copy it into a file and print it.
  - the SysId plots cannot be directly printed. You have to choose `File` → `Copy figure`, which gives an ordinary Matlab plot you can print.

- Printing in Linux:
  - A file called `file.pdf` can be printed out for instance typing in a terminal

    ```
    lp -d printername file.pdf
    ```

    (replace `printername` with the name of the printer in the room you sit in).
  - It is possible to print using `File` → `Print` in a matlab plot, but one must select the printer name writing `-Pprintername` in the `Device option` (again `printername` is the name of your printer).

1. (a) What kind of information on a linear dynamical system can you obtain from its step response? (2p)

   (b) Put the following higher order ODE

$$\dddot{y} + \dot{y}y^2\ddot{y} + yu + \dot{y}y^3 = 0$$

   in state space form [in swedish: tillståndsform]. (2p)

   (c) What is the stability region of the 1st order implicit Adams method (also known as the backward Euler method)?

$$x_n = x_{n-1} + hf(x_n)$$

   How would you choose h to correctly simulate the behavior of the following ODE?

$$\dot{x} = 10x$$

   (3p)

   (d) Consider the following system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -0.8 & 0 & 0 \\ 0 & -1.3 & 0 \\ 0 & 0 & -20 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 2 \\ 14 \end{bmatrix} u(t) \\ y &= \begin{bmatrix} 2 & 4 & 0.5 \end{bmatrix} x(t) \end{aligned}$$

   Extract from it a two-dimensional system that approximates the original system and disregards its fastest time constant. (3p)

2. The system

$$y(t) = 0.2u(t-1) - 0.3u(t-2) + w(t)$$

is identified minimizing the prediction error from the following model

$$y(t) = b_1 u(t-1) + b_2 u(t-2) + e(t)$$

where $e(t)$ is a white noise of variance 1. Assume that also w is a white noise of variance 1 and that u is independent of w and e. What is the covariance of the parameter estimates one obtains asymptotically in the following cases:

   (a) u is a white noise of variance r

   (4p)

   (b) u has covariance function

   $$R_u(\tau) = a^{|\tau|} \quad (|a| < 1)$$

   (3p)

   (c) In case b), what is the variance of the estimate of b_1? And that of $b_1 + b_2$? (3p)

3. The data for this exercise are in a file named

   `sysid_data_20151027.mat.`

   In this file you will find the sampled signals u and y (the sample time is 0.1). Notice that the data are *not* produced with a closed loop system (hence disregard the `feedback` command in the SysId toolbox)

   (a) Consider first the non-parametric models. What can you deduce from them in this case? (2p)

   (b) Construct one or more appropriate black-box models. For one or more of these models report

      - plot of the fitted model vs. validation data
      - parameter values and uncertainty
      - quality of the fit
      - Bode plots
      - poles and zeros placement

   Discuss and comment your choices and results. (8p)

4. Consider the system

$$E\dot{x} + Fx = Gu$$

where

$$E = \begin{bmatrix} 1 & 0 & 1 \\ 1 & \alpha & 0 \\ 0 & 1 & 1 \end{bmatrix}, \qquad F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \qquad G = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

(a) Show that there is a value α_o such that for all $\alpha \neq \alpha_o$ the system can be written as

$$\dot{x} = Ax + Bu$$

What is α_o? What is the expression for A and B as a function of E, F, and G? (3p)

(b) What is the index of the system? (3p)

(c) For $\alpha = \alpha_o$ (value computed in (a)), show that the system can be written as

$$\frac{d}{dx}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + B_1 u + B_2 \dot{u}$$

$$x_3 = C_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + D_1 u$$

and compute the expressions for the matrices A_1, B_1, B_2, C_1 and D_1. (4p)