## EXAM IN MODELING AND LEARNING FOR DYNAMICAL SYSTEMS (TSRT92)

ROOM: Egypten and Asgård

TIME: Thursday, January 5, 2023, 14.00–18.00

COURSE: TSRT92 Modeling and Learning for Dynamical Systems

CODE: DAT1

DEPARTMENT: ISY

NUMBER OF EXERCISES: 4

NUMBER OF PAGES (including cover page): 5

EXAMINER: Anders Hansson, 070-3004401

COURSE ADMINISTRATOR: Ninna Stensgård 013-282225, ninna.stensgard@liu.se

## APPROVED TOOLS:

1. L. Ljung, T. Glad & A. Hansson "Modeling and Identification of Dynamical Systems" Previous editions (including Swedish edition) of the book are also allowed.

2. T. Glad & L. Ljung: "Reglerteknik. Grundläggande teori"

3. A. Hansson: "Optimal Experiment Design"

4. Mathematical tables and formulas (e.g. *L. Råde & B. Westergren*: "Mathematics handbook", *C. Nordling & J.Österman*: "Physics handbook", *S. Söderkvist*: "Formler & tabeller")

5. Calculator and Matlab

Notes in the above books are allowed.

SOLUTIONS: Linked from the course home page after the exam.

The exam can be inspected and checked out 2023-01-26, 12.30-13:00 in room 2A:573, B-building, entrance 25, A-corridor.

PRELIMINARY GRADING:	grade 3	15  points
	grade 4	23 points
	grade 5	30 points

All solutions should be well motivated. Writing should be neat and clean.

Good Luck!

## COMPUTER TIPS:

- To open Matlab:
  - open a terminal (right-click on the background and choose open terminal)
  - type

module add prog/matlab
matlab &

- Print out the model description and the plots requested
- Remember to write your AID number on each printed page you include
- In the identification exercise using the System Identification toolbox:
  - To print the model description: Right-click on the icon of the model you have computed and then click Present. The model description appears then on the Matlab main window. Copy it into a file and print it.
  - the SysId plots cannot be directly printed. You have to choose File  $\rightarrow$  Copy figure, which gives an ordinary Matlab plot you can print.
- Printing in Linux:
  - A file called file.pdf can be printed out for instance typing in a terminal

lp -d printername file.pdf

(replace **printername** with the name of the printer in the room you sit in).

 It is possible to print using File → Print in a matlab plot, but one must select the printer name writing -Pprintername in the Device option (again printername is the name of your printer).

- 1. (a) Explain what model validation is and why it is important. (2p)
  - (b) Why can it be difficult to investigate if a system is stable or unstable using simulations? (2p)
  - (c) Which are the stationary points for

$$\dot{x}_1 = x_2$$
  
 $\dot{x}_2 = x_3$   
 $\dot{x}_3 = \tan(x_2 + x_3^2) + \sin(x_1 + 10x_2x_3)$ 

(2p)

- (d) Give an intuitive explanation why it is more difficult to simulate a DAE the higher its index is. (2p)
- (e) Linus is going to design a cruise controller for a car, and therefore he will model the dynamics of the car from the gas pedal to the speed of the car. Linus is ambitious, and therefore he carefully models the combustion of the engine, the gear box, the contact forces between the tires and the road, the air resistance and the slope of the road. When he tries to simulate the model he gets disappointed since the simulations run very slowly. What is a likely reason for this problem and how can it be circumvented? (2p)
- (a) Linnea is given the task to derive a model for an unknown system using system identification. Describe *briefly* how she should carry out this task. You should specifically comment on experiment design, choice of model structure and validation. (3p)
  - (b) The model

$$y(t) = b_1 u(t) + b_2 u(t-1)$$

if fitted to measurement data using the least squares method. The input signal is

$$u(t) = 1 + (-1)^t, \quad t = 1, \dots, N$$

In reality the true system is given by

$$y(t) = u(t) + u(t-1) + \varepsilon$$

where  $\varepsilon$  is a constant disturbance. What values will you obtain for the estimates of  $b_1$  och  $b_2$  when  $N \to \infty$ ? (7p) 3. A physical system is described by

$$y(t) = G_0(q)u(t) + w(t)$$

where w is white noise independent of u and where

$$G_0(q) = \frac{q^{-1} + 0.92q^{-2}}{1 - 1.71q^{-1} + 0.79q^{-2}}$$

The system is identified by minimizing the criterion

$$V_N(\theta) = \frac{1}{N} \sum_{t=1}^{N} (y(t) - \hat{y}(t|\theta))^2$$

(a) Suppose you fit an OE-model

$$y(t) = \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1} + a_2 q^{-2}} u(t) + e(t)$$
(1)

for which the input signal u(t) is white noise. What values do you obtain for the estimated parameters  $b_1$ ,  $b_2$ ,  $a_1$  och  $a_2$  as  $N \to \infty$ ? Motivate! (2p)

(b) If you instead fit a static model

$$y(t) = bu(t) + e(t)$$

and use a constant input signal, what value do you obtain for the estimate of b as  $N \to \infty$ ? Motivate! (2p)

(c) Suppose you are going to identify an OE-model according to (1), but that you are uncertain about how to carry out the identification experiment in order to obtain as good model as possible. You have three different data sets to choose from, z1, z2, z3 (load the file ex091012\_4c.mat), which all have the same maximum value for the input signal but different frequency contents.

Estimate an OE-model as in (1) for each data set and motivate then carefully which data set that is most suitable to use for system identification. The resulting model is going to be used for controller design where the closed loop band-width should be 3 rad/s. (6p)

- 4. Consider the following simplified system. In a partially insulated tank there are two inflows and one outflow. The inflows are  $F_1$  [m<sup>3</sup>/s] with temperature  $T_1$  [K] and  $F_2$  [m<sup>3</sup>/s] with temperature  $T_2$  [K]. The outflow is  $F_3$  [m<sup>3</sup>/s]. The specific heat capacity of the liquid is c [J/(kg K)] and the density is  $\rho$  [kg/m<sup>3</sup>]. Because of insufficient insulation there is a flow of energy between the liquid in the tank and the ambient air, which has temperature  $T_0$  [K]. The energy flow depends on the heat transfer coefficient U [W/(m<sup>2</sup> K)] and the cross-section area A [m<sup>2</sup>].
  - (a) Determine a mathematical model that describes how the volume V(t) [m<sup>3</sup>] and the temperature T(t) [K] varies for the liquid in the tank assuming that there is perfect mixing. (7p)
  - (b) Show by means of analysis of dimensions that the expressions for V(t) and T(t) in a) are not wrong. (3p)