EXAM IN MODELING AND LEARNING FOR DYNAMICAL SYSTEMS (TSRT92)

ROOM: Egypten, Asgård, Glan, Olympen

TIME: Friday, October 28, 2022, 14.00–18.00

COURSE: TSRT92 Modeling and Learning for Dynamical Systems

CODE: DAT1

DEPARTMENT: ISY

NUMBER OF EXERCISES: 4

NUMBER OF PAGES (including cover page): 6

EXAMINER: Anders Hansson, 070-3004401

VISITS: 15.00 and 17.00

COURSE ADMINISTRATOR: Ninna Stensgård 013-282225, ninna.stensgard@liu.se

APPROVED TOOLS:
1. *L. Ljung, T. Glad & A. Hansson* "Modeling and Identification of Dynamical Systems" Previous editions (including Swedish edition) of the book are also allowed.
    2. *T. Glad & L. Ljung*: "Reglerteknik. Grundläggande teori"
    3. *A. Hansson*: "Optimal Experiment Design"
    4. Mathematical tables and formulas (e.g.*L. Råde & B. Westergren*: "Mathematics handbook",*C. Nordling & J.Österman*: "Physics handbook", *S. Söderkvist*: "Formler & tabeller")
    5. Calculator and Matlab

Notes in the above books are allowed.

SOLUTIONS: Linked from the course home page after the exam.

The exam can be inspected and checked out 2022-11-24, 12.30-13:00 in Ljungeln, B-building, entrance 25, A-corridor, room 2A:514.

PRELIMINARY GRADING: grade 3   15 points
                                   grade 4   23 points
                                   grade 5   30 points

All solutions should be well motivated. Writing should be neat and clean.

Good Luck!

COMPUTER TIPS:

- To open Matlab:

  - open a terminal (right-click on the background and choose `open terminal`)
  - type

    ```
    module add prog/matlab
    matlab &
    ```

- Print out the model description and the plots requested

- Remember to write your AID number on each printed page you include

- In the identification exercise using the System Identification toolbox:

  - To print the model description: Right-click on the icon of the model you have computed and then click `Present`. The model description appears then on the Matlab main window. Copy it into a file and print it.
  - the SysId plots cannot be directly printed. You have to choose `File → Copy figure`, which gives an ordinary Matlab plot you can print.

- Printing in Linux:

  - A file called `file.pdf` can be printed out for instance typing in a terminal

    `lp -d printername file.pdf`

    (replace `printername` with the name of the printer in the room you sit in).
  - It is possible to print using `File → Print` in a matlab plot, but one must select the printer name writing `-Pprintername` in the `Device option` (again `printername` is the name of your printer).

1. (a) Explain what are explicit and implicit methods, respectively, for numerically solving differential equations? Discuss pros and cons for these types of methods. (2p)

   (b) Describe how to proceed in order to determine if a system is linear or nonlinear using experiments. (2p)

   (c) You want to simulate the systems S_1 och S_2, interconnected as in the diagram below.
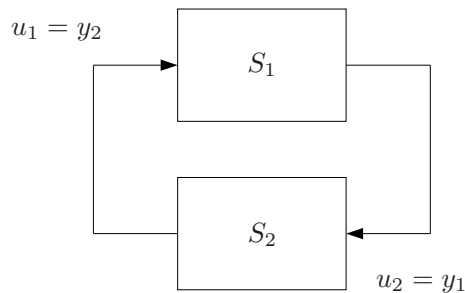
   S_1 is given by

   $$y_1(t) = -0.4y_1(t-1) + 0.2y_1(t-2) + u_1(t)$$

   while S_2 is given either by

   $$\text{(i)} \quad y_2(t) = 0.1y_2(t-1) + u_2(t)$$

   or

   $$\text{(ii)} \quad y_2(t) = u_2(t-1) + u_2(t-2)$$

   For one of the cases you get an error message from the simulation program, while for the other case there are no problems. Explain the reason for this. (2p)

   (d) When modelling a system involving heating and cooling there are infinitely many states, since every single spatial point in the system has its own temperature. What type of simplification can be used to obtain a finite dimensional model? What assumptions have to be made to allow for this simplification? (2p)

   (e) What is a stiff differential equation? Why is such a differential equation difficult to simulate? (2p)

2. (a) Consider the system below.

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -15 & 0 \\ 1 & 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 10 \\ 2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 3 & 0.5 & 1 \end{bmatrix} x(t)$$

Determine an approximative second order system that neglects fast dynamics. (3p)

(b) Using an electromagnet you can make a metal ball hover in the air. Suppose the electromagnet is positioned such that the magnetic force is directed upwards. If the metal ball is placed under the magnet, the ball is hovering, if the magnetic force exactly balances the gravitational force acting on the ball. The force from the magnet is given by

$$F_m(t) = kI(t)r(t)^{-2}$$

where $I(t)$ is the current through the coil of the electromagnet, $r(t) \geq 0$ is the distance to from the magnet to the ball, and where k is a constant.

  i. Determine a state-space model that describes the system with the current $I(t)$ as input signal and the distance $r(t)$ as output signal. (3p)

  ii. Suppose $I(t) \equiv I_0 > 0$. What is the stationary point of the system? (1p)

  iii. Linearize the system around the stationary point. What are the poles of the transfer function of the linearized system? (3p)

3. In the file **ex100111_3.mat** there are 10000 data samples in the vectors **y** och **u**, collected with the sampling time $T = 0.2$ s. In this exercise you should fit linear black-box models and then validate and compare the models. It is known that the system is resonant and that the disturbances are dominated by high frequencies.

  (a) In the figure below the first 50 s of the signals are plotted. Suppose you want to determine a linear model. What is your first action and why? (2p)

4
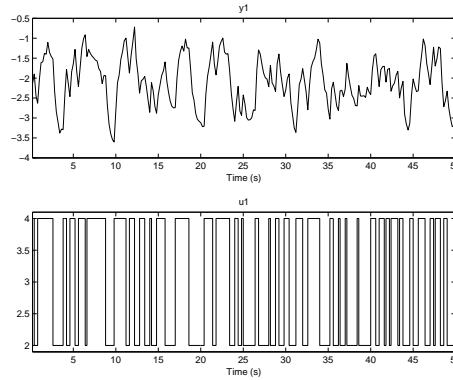
When considering the following sub-exercises you should divide the data set in estimation data and validation data, and you should also do any pre-processing as suggested in Exercise 3a. Do not forget to **attach plots and matlab code** to support your reasoning.

(b) Perform spectral analysis (preferably using `SPAFDR`) that shows the resonant behavior of the system and the properties of the disturbances. (2p)

(c) Estimate black-box models of the types ARX, ARMAX and OE that fits the collected data well without being of unnecessary high order.

For simplicity you can put $n_k = 1$ and use the same degree for nominator and denominator polynomials, i.e., $n_a = n_b$ (ARX), $n_a = n_b = n_c$ (ARMAX) and $n_b = n_f$ (OE).

Discuss the degrees of the polynomials for the different models and how they affect the fit on validation data (`Model output`). (3p)

(d) Choose a few of the models from Exercise 3c (the best for each model type and possibly one more in order to be able to compare models with the same number of poles).

Try to reason about how the model types differ with respect to poles and zeros, the frequency responses, and the noise spectrum. Also compare with the spectral analysis in Exercise 3b. (3p)

4. You are going to create an expression that describes the pressure p in the shock wave formed when an explosive material detonates under water. The pressure may depend on a number of factors:

- The initial pressure p_0 generated by the explosion itself. This pressure depends on the chemical properties of the explosive.

- The radius of the shock wave r.

- The mass of the explosive m.

- The density of water ρ.

- The compression module of water E. The compression module for a substance describes how much the surrounding pressure has to be increased around the volume of the substance to achieve a given relative volume decrease. The compression module is given by

$$E = -V \frac{\partial p_{\mathrm{sur}}}{\partial V}$$

where p_{sur} is the surrounding pressure and V is the volume.

(a) How many independent dimensionless variables can be formed from the listed variables, including p? (7p)

(b) What structure will an expression for p have? (3p)