

# EXAM IN MODELING AND LEARNING FOR DYNAMICAL SYSTEMS (TSRT92)

ROOM: EGYPTEN

TIME: Monday, August 22, 08.00–12.00

COURSE: TSRT92 Modeling and Learning for Dynamical Systems

CODE: DAT1

DEPARTMENT: ISY

NUMBER OF EXERCISES: 4

NUMBER OF PAGES (including cover page): 5

EXAMINER: Anders Hansson, 070-3004401

VISITS: 09.00 and 11.00

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APPROVED TOOLS:

1. *L. Ljung, T. Glad & A. Hansson* "Modeling and Identification of Dynamical Systems"
2. *T. Glad & L. Ljung*: "Reglerteknik. Grundläggande teori"
3. *A. Hansson*: "Optimal Experiment Design"
4. Mathematical tables and formulas (e.g. *L. Råde & B. Westergren*: "Mathematics handbook", *C. Nordling & J. Österman*: "Physics handbook", *S. Söderkvist*: "Formler & tabeller")
5. Calculator and Matlab

Notes in the above books are allowed.

SOLUTIONS: Linked from the course home page after the exam.

The exam can be inspected and checked out 2022-09-15, 12.30-13:00 in Ljungeln, B-building, entrance 25, A-corridore, room 2A:514.

PRELIMINARY GRADING: grade 3 15 points  
grade 4 23 points  
grade 5 30 points

All solutions should be well motivated. Writing should be neat and clean.

*Good Luck!*

## COMPUTER TIPS:

- To open Matlab:
  - open a terminal (right-click on the background and choose **open terminal**)
  - type

```
module add prog/matlab
matlab &
```
- Print out the model description and the plots requested
- Remember to write your AID number on each printed page you include
- In the identification exercise using the System Identification toolbox:
  - To print the model description: Right-click on the icon of the model you have computed and then click **Present**. The model description appears then on the Matlab main window. Copy it into a file and print it.
  - the SysId plots cannot be directly printed. You have to choose **File** → **Copy figure**, which gives an ordinary Matlab plot you can print.
- Printing in Linux:
  - A file called `file.pdf` can be printed out for instance typing in a terminal

```
lp -d printername file.pdf
```

(replace `printername` with the name of the printer in the room you sit in).
  - It is possible to print using **File** → **Print** in a matlab plot, but one must select the printer name writing `-Pprintername` in the **Device** option (again `printername` is the name of your printer).

1. (a) Describe shortly how a NARX model that is based on a neural network looks like. In what way is it different from an ARX model? (2p)
- (b) How can you see from a step response experiment if a system is linear or nonlinear? (2p)
- (c) The system

$$\begin{aligned}\dot{x}_1(t) &= -350x_1(t) + 70u(t) \\ \dot{x}_2(t) &= 10x_1(t) - 3x_2(t) + u(t) \\ y(t) &= 5x_1(t) + 4x_2(t)\end{aligned}$$

has a slowly varying input signal  $u(t)$ . Suggest a simpler model which has approximately the same output signal. (2p)

- (d) The system

$$\ddot{y}(t) + 4\dot{y}(t) + 3y(t) = u(t)$$

has been simulated with a step as input signal, and now we would like to study the step response of

$$\ddot{y}(t) + 8\dot{y}(t) + 12y(t) = u(t)$$

Can we use the results from the first simulation if we apply scaling for this purpose? (2p)

- (e) Suppose that

$$y(t) = G(p)u(t) + e(t), \quad G(p) = \frac{1}{p+1}$$

where both  $u(t)$  and  $e(t)$  are white noise with variance 1, and also independent of one another. Compute the spectrum for  $y$ . (2p)

2. A physical system is described by

$$y(t) = G_0(q)u(t) + w(t)$$

where  $w$  is white noise independent of  $u$  and

$$G_0(q) = \frac{q^{-1} + 0.92q^{-2}}{1 - 1.71q^{-1} + 0.79q^{-2}}$$

The system is identified by minimizing the criterion

$$V_N(\theta) = \frac{1}{N} \sum_{t=1}^N (y(t) - \hat{y}(t|\theta))^2$$

(a) Suppose that we fit an OE-model

$$y(t) = \frac{b_1q^{-1} + b_2q^{-2}}{1 + a_1q^{-1} + a_2q^{-2}}u(t) + e(t) \quad (1)$$

where the input signal  $u(t)$  is white noise. What values do you obtain for the estimated parameter  $b_1$ ,  $b_2$ ,  $a_1$  och  $a_2$  as  $N \rightarrow \infty$ ? Motivate your answer! (2p)

(b) If instead a static model is fitted

$$y(t) = bu(t) + e(t)$$

using a constant input signal, what value of the estimated parameter  $b$  do you obtain as  $N \rightarrow \infty$ ? Motivate your answer! (2p)

(c) Suppose that you should identify an OE-model as in (1), but that you are unsure about how to carry out the identification experiment so that you obtain as good model as possible using finite data. You have the following three data sets to choose from, **z1**, **z2**, **z3** (load the file `ex091012_4c.mat`) which all have the same maximum value of the input signal, but different frequency content.

Estimate an OE-model as in (1) for each of the data sets and motivate clearly which data set is most suitable to use for identification. The resulting model should be used for controller design where the closed loop bandwidth should be 3 rad/s. (6p)

3. (a) A DAE  $F(\dot{x}, x, t) = 0$  can be solved using backward differences

$$F((x_n - x_{n-1})/h, x_n, t_n) = 0$$

Suppose this is done for the system

$$\begin{aligned}\dot{x}_1(t) &= x_2(t) \\ x_1(t) &= t^2\end{aligned}$$

What will the error be? (3p)

- (b) A company is holding a stock of preserve tins in a warehouse. Every day tins are delivered to and collected from the warehouse. On day  $k$   $u_{in}(k)$  tins are delivered and  $u_{out}(k)$  tins are collected. Unfortunately, the warehouse is not well maintained, so the tins are collected at random.

- Determine a state space model for the warehouse, where  $n(k)$  is the number of tins at day  $k$  (before delivery and collection) and  $x(k)$  is the mean value of the age of a tin in days since production (i.e. determine mathematical expressions for  $n(k+1)$  and  $x(k+1)$  as functions of  $n(k)$ ,  $x(k)$ ,  $u_{in}(k)$  and  $u_{out}(k)$ ). You may assume that the tins that are delivered have age zero. You may also assume that the tins that are collected have age equal to the mean value of their ages.
- What are the stationary points of the dynamical system?
- What mean values of the age do we obtain for the stationary points if we instead assume that only the oldest tins are collected from the warehouse?

(7p)

4. Consider the DAE model

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_3 + u_1 \\ \dot{x}_2 &= -x_2 - x_3 \\ 0 &= x_1 - x_2 + u_2\end{aligned}$$

where  $u_1$  and  $u_2$  are input signals.

- (a) What index does the system have? Motivate! (5p)  
 (b) We introduce a state feedback

$$u_2 = \ell_1 x_1 + \ell_2 x_2 + \ell_3 x_3$$

Give values for  $\ell_1$ ,  $\ell_2$  and  $\ell_3$  such that the resulting closed loop system has index=1. (5p)