EXAM IN MODELING AND LEARNING FOR DYNAMICAL SYSTEMS (TSRT92)

ROOM: EGYPTEN, ASGÅRD

TIME: Friday January 7th, 2022, 08.00–12.00

COURSE: TSRT92 Modeling and Learning for Dynamical Systems

CODE: DAT1

DEPARTMENT: ISY

NUMBER OF EXERCISES: 4

NUMBER OF PAGES (including cover page): 5

EXAMINER: Anders Hansson, 070-3004401

VISITS: 09.00 and 11.00

COURSE ADMINISTRATOR: Ninna Stensgård 013-282225, ninna.stensgard@liu.se

APPROVED TOOLS:

1. *L. Ljung, T. Glad & A. Hansson* ”Modeling and Identification of Dynamical Systems”
2. *T. Glad & L. Ljung*: ”Reglerteknik. Grundläggande teori”
3. *A. Hansson*: ”Optimal Experiment Design”
4. Mathematical tables and formulas (e.g.*L. Råde & B. Westergren*: ”Mathematics handbook”,*C. Nordling & J.Österman*: ”Physics handbook”, *S. Söderkvist*: ”Formler & tabeller”)
5. Calculator and Matlab

Notes in the above books are allowed.

SOLUTIONS: Linked from the course home page after the exam.

The exam can be inspected and checked out 2022-01-26, 12.30-13:00 in Ljungeln, B-building, entrance 25, A-corridore, room 2A:514.

PRELIMINARY GRADING:	grade 3	15 points
	grade 4	23 points
	grade 5	30 points

All solutions should be well motivated. Writing should be neat and clean.

Good Luck!

COMPUTER TIPS:

- To open Matlab:
  - open a terminal (right-click on the background and choose `open terminal`)
  - type

    ```
    module add prog/matlab
    matlab &
    ```

- Print out the model description and the plots requested
- Remember to write your AID number on each printed page you include
- In the identification exercise using the System Identification toolbox:
  - To print the model description: Right-click on the icon of the model you have computed and then click `Present`. The model description appears then on the Matlab main window. Copy it into a file and print it.
  - the SysId plots cannot be directly printed. You have to choose `File` → `Copy figure`, which gives an ordinary Matlab plot you can print.
- Printing in Linux:
  - A file called `file.pdf` can be printed out for instance typing in a terminal

    ```
    lp -d printername file.pdf
    ```

    (replace `printername` with the name of the printer in the room you sit in).
  - It is possible to print using `File` → `Print` in a matlab plot, but one must select the printer name writing `-Pprintername` in the `Device option` (again `printername` is the name of your printer).

1. (a) In what way is an ARX-model a special case of a Box-Jenkins-model? What pros and cons do the different models have? (3p)

   (b) Why is it usually advantageous to have a variable step length in a numerical method for solving differential equations? (3p)

   (c) For simulating models that are generated with Modelica, it can be necessary to handle DAE systems. Why is this the case? (2p)

   (d) Why is cross validation important in system identification? (2p)

2. A system has beed modeled as

$$y(t) = 0.3y(t-1) - 0.02y(t-2) + u(t) - 0.4u(t-1) \tag{1}$$

where the sampling interval is 1, but it should be approximated with a model of order one as in

$$y(t) = ay(t-1) + bu(t-1) \tag{2}$$

This can be seen as an identification experiment where (1) is the "true" stystem and where (2) is the model to be identified

   (a) It is desirabel to obtain a model that has a good fit at low frequencies. What properties should the input signal then have? Generate such an input signal and simulate (1) to obtain the corresponding output signal. (3p)

   (b) Determine the model in (2) by carrying out system identificition form the data you obtained in (a). What values do you obtain for a and b? Plot the Bode diagrams for (1) and (2). Is it true that you get a good agreement for low frequencies? (5p)

   (c) How should you proceed if you instead want to have good agreement for high frequencies? You do not have to carry out any computer calculations. It is enough to motivate your answer with a discussion. (2p)

3. We will investigate a second order linear model for identification experiments:

$$y(t) = \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + f_1 q^{-1} + f_2 q^{-2}} u(t) + e(t)$$

The following four identification experiments are carried out

(a) One performs frequency analysis at the angular frequencies ω_1 and ω_2, such that $G_0(e^{i\omega_1})$ and $G_0(e^{i\omega_2})$ are estimated, and then one fits a second order model that agrees with these points in the frequency domain.

(b) One performs an identification experiment with the input signal

$$u(t) = \cos(\omega_1 t) + \cos(\omega_2 t), \quad t = 1, 2, 3, \ldots$$

and fits a model with the structure above (i.e. an OE-model).

(c) One performs an identification experiment with the input signal

$$u(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + \cos(\omega_3 t), \quad t = 1, 2, 3, \ldots$$

and fits a model with the structure above.

(d) One finally performs an identification experiment with the input signal

$$u(t) = \cos(\omega_1 t) + \cos(\omega_2 t), \quad t = 1, 2, 3, \ldots$$

and the model

$$y(t) = \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + f_1 q^{-1} + f_2 q^{-2}} u(t) + H_*(q) e(t)$$

where H_* is a fixed noise model.

What differences are obtained between the resulting models for the four cases? Motivate your answer carefully. Assume that the measurement series are so long that you can neglect the influence of the noise. Specifically discuss how the choise of H_* in the fourth method affects the result. *Hint:* Investigate $\lim_{N\to\infty} \hat{\theta}_N$.

(10p)

4. We consider a linear regression problem where the pairs of data (x_k, y_k) with $x_k \in \mathbf{R}^n$ and $y_k \in \mathbf{R}$, $1 \leq k \leq N$, should satisfy the linear regression

$$y_k = a^T x_k$$

This means that we are able to *interpolate* the data. This is only possible if $N \leq n$. Before, when we have discussed regression, we have tacitly assumed that $N > n$. However, common practice when using artificial neural networks, is to be in this interpolation regime. We can collect all pairs of data in

$$X = \begin{bmatrix} x_1^T \\ \vdots \\ x_N^T \end{bmatrix} \in \mathbf{R}^{N \times n}; \qquad y = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} \in \mathbf{R}^N$$

Then a should satisfy

$$Xa = y$$

which has a non-unique solution since the linear system of equations is under-determined. It would then be natural to look for a solution that minimizes the norm $\|a\|_2$. We will assume that X has full row rank.

(a) Show that the minimum norm solution is given by

$$a = X^T \left(X X^T \right)^{-1} y$$

(5p)

(b) Also compute the solution of the LS problem

$$\underset{x}{\text{minimize}} \sum_{k=1}^{N} f_k(a)$$

where $f_k(x) = (y_k - a^T x_k)^2$ using the following gradient algorithm

$$a_{k+1} = a_k - \frac{\partial f_k(a_k)}{\partial a}$$

with $a_0 = 0$ for $0 \leq k \leq N-1$. You may assume that a_N satisfies $Xa_N = y$. Show that the solution a_N agrees with the solution in (a). (5p)