EXAM IN MODELING AND LEARNING FOR DYNAMICAL SYSTEMS (TSRT92)

ROOM: ??????????

TIME: Friday October 29th, 2021, 14.00–18.00

COURSE: TSRT92 Modeling and Learning for Dynamical Systems

CODE: DAT1

DEPARTMENT: ISY

NUMBER OF EXERCISES: 4

NUMBER OF PAGES (including cover page): 6

EXAMINER: Anders Hansson, 070-3004401

VISITS: 15.00 and 17.00

COURSE ADMINISTRATOR: Ninna Stensgård 013-282225, ninna.stensgard@liu.se

APPROVED TOOLS:

1. *L. Ljung, T. Glad & A. Hansson* ”Modeling and Identification of Dynamical Systems”
2. *T. Glad & L. Ljung*: ”Reglerteknik. Grundläggande teori”
3. *A. Hansson*: ”Optimal Experiment Design”
4. Mathematical tables and formulas (e.g.*L. Råde & B. Westergren*: ”Mathematics handbook”,*C. Nordling & J.Österman*: ”Physics handbook”, *S. Söderkvist*: ”Formler & tabeller”)
5. Calculator and Matlab

Notes in the above books are allowed.

SOLUTIONS: Linked from the course home page after the exam.

The exam can be inspected and checked out 2021-11-18, 12.30-13:00 in Ljungeln, B-building, entrance 25, A-corridore, room 2A:514.

PRELIMINARY GRADING:	grade 3	15 points
	grade 4	23 points
	grade 5	30 points

All solutions should be well motivated. Writing should be neat and clean.

Good Luck!

COMPUTER TIPS:

- To open Matlab:
  - open a terminal (right-click on the background and choose `open terminal`)
  - type

    ```
    module add prog/matlab
    matlab &
    ```

- Print out the model description and the plots requested
- Remember to write your AID number on each printed page you include
- In the identification exercise using the System Identification toolbox:
  - To print the model description: Right-click on the icon of the model you have computed and then click `Present`. The model description appears then on the Matlab main window. Copy it into a file and print it.
  - the SysId plots cannot be directly printed. You have to choose `File` → `Copy figure`, which gives an ordinary Matlab plot you can print.
- Printing in Linux:
  - A file called `file.pdf` can be printed out for instance typing in a terminal

    ```
    lp -d printername file.pdf
    ```

    (replace `printername` with the name of the printer in the room you sit in).
  - It is possible to print using `File` → `Print` in a matlab plot, but one must select the printer name writing `-Pprintername` in the `Device option` (again `printername` is the name of your printer).

1. (a) Mention 3 possible reasons for non-identifiability (i.e. failure in identifying the parameters) of a model. [2p]

(b) Alice and Bob must solve a black-box system identification problem. Alice uses a program that can solve only systems of linear equations, while Bob has a program based on Gauss-Newton method. Which classes of models among ARMAX, ARX, OE, and BJ, can Alice, respectively, Bob use with their software program? [2p]

(c) Compute the spectrum of the system

$$y(t) = G(p)u(t) + e(t), \qquad G(p) = \frac{p+\alpha}{p+\beta}$$

where α, $\beta > 0$, $u(t)$ and $e(t)$ are uncorrelated white noises of variances, resp. 1 and 2. [2p]

(d) For the system

$$\begin{aligned} \dot{x}(t) &= -3x(t) - u^2(t) \\ y(t) &= x^2 \end{aligned}$$

compute the static relationship between u and y. [2p]

(e) The system

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix} x$$

must be simulated with the Euler method $x_{n+1} = x_n + hf(x_n)$. For what values of h is the method stable? [2p]

2. Consider the following "true" system

$$y(t) + 0.4y(t-1) = 0.2u(t) + v(t)$$

where $v(t)$ is a zero-mean white noise of variance 2. We aim at fitting ARX models

$$y(t)+a_1y(t-1)+\ldots+a_{n_a}y(t-n_a) = b_1u(t)+\ldots+b_{n_b}u(t-n_b+1)+e(t)$$

of different order n_a and n_b using the prediction error minimization method on the function

$$V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}(y(t) - \hat{y}(t|\theta))^2$$

The input u is a white noise of zero mean and variance 1 and uncorrelated with v.

(a) Assume that $n_a = n_b = 1$. To what values are the estimates of a_1 and b_1 converging when $N \to \infty$? What is the variance of these estimates as a function of N? [4p]

(b) Assume instead that $n_a = 2$ and $n_b = 1$. To what values are the estimates of a_1, a_2 and b_1 converging when $N \to \infty$? What is the variance of these estimates as a function of N? [4p]

(c) What can you say in general for the estimated values of the parameters and for the variance of these estimates when we vary $n_a \geq 1$, $n_b \geq 1$ for this system? [2p]

3. The data for this exercise are in a file called `sysid_data_20211029.mat`. To load it into your Matlab workspace type `load sysid_data_20211029.mat` at the Matlab prompt.

   Inside `sysid_data_20211029.mat` you will find the sampled signals u and y (the sample time is $T_s = 0.1$).

   (a) Do the data show any sign of resonances? [2p]

   (b) Construct one or more appropriate black-box models fitting the data, with the constraint that the total number of poles in the input-output transfer function is less or equal to 3 (i.e., $n_a \leq 3$ for ARX and ARMAX, $n_f \leq 3$ for OE and BJ). Report:

   - plot of the fitted model vs. validation data
   - parameter values and uncertainty
   - residual plot
   - Bode plots
   - poles and zeros placement

   Discuss and comment your choices and results. [8p]

4. Consider the following ARX model

$$y(t) + ay(t-1) = bu(t-1) + e(t)$$

We asume that the LS method is used to estimate the parameters $\theta = \begin{bmatrix} a & b \end{bmatrix}^T$ based on data $(y(t), u(t))$ collected for $t = 0, \dots, N$. The LS criterion is

$$\frac{1}{2} \sum_{t=1}^{N} (\hat{y}(t) - y(t))^2$$

where $\hat{y}(t)$ is the predictor. The LS estimate $\hat{\theta}$ satisfies the normal equations

$$X^T X \hat{\theta} = X^T Y$$

for some matrix X and some vector Y.

(a) Show that

$$X = \begin{bmatrix} -y(0) & u(0) \\ -y(1) & u(1) \\ \vdots & \vdots \\ -y(N-1) & u(N-1) \end{bmatrix}; \qquad Y = \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(N) \end{bmatrix}$$

[2p]

(b) Show that

$$X^T X = \begin{bmatrix} \alpha & -\beta \\ -\beta & \gamma \end{bmatrix}; \qquad X^T Y = \begin{bmatrix} -\delta \\ \eta \end{bmatrix}$$

where

$$\alpha = \sum_{t=0}^{N-1} y^2(t); \quad \beta = \sum_{t=0}^{N-1} u(t)y(t); \quad \gamma = \sum_{t=0}^{N-1} u^2(t)$$

and

$$\delta = \sum_{t=1}^{N} y(t-1)y(t); \quad \eta = \sum_{t=1}^{N} u(t-1)y(t)$$

[3p]

(c) Now assume that data has been collected when feedback is present so that $u(t) = -ky(t)$ for some constant $k \neq 0$. Show that

$$X^T X = \alpha \begin{bmatrix} 1 & k \\ k & k^2 \end{bmatrix}; \qquad X^T Y = -\delta \begin{bmatrix} 1 \\ k \end{bmatrix}$$

where α and δ are defined as before. Compute all solutions to the normal equations for this case. [5p]