Computer Exercises in System Identification
Closed-Loop Data

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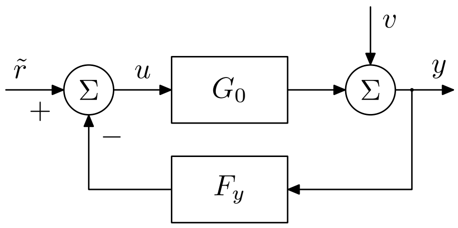
1 Closed-Loop System Identification

Experimental data can be collected in closed loop due to several reasons. For example, the open-loop system might be unstable, which essentially makes it impossible to carry out open-loop experiments. Another possible reason is that the feedback mechanism might be inherent in the system and hard or even impossible to turn off without damaging the system. However, the most common reason is probably that data have been collected during normal operation with an active feedback controller. This is the situation in many production plants, transportation systems, heating systems, water supply and treatment systems, etc., where the performance of the system depends on the use of feedback control and where the cost of turning off the controller is high. In such applications, data are usually collected 24/7 for, for example, safety and performance monitoring purposes, but not always used for system identification due to the additional challenges caused by the closed-loop setting.

In short, many of these challenges are caused by the fact that the input signal depends on the system disturbances through the feedback controller, which makes several of the methods that work well in open loop biased in closed loop. Consider the block diagram shown below.



Here, we assume that the *true system* can be written

$$y(t) = G_0(q)u(t) + \underbrace{H_0(q)e(t)}_{=v(t)},$$

where $e(t)$ is white noise with variance λ_0. The controller can be written

$$u(t) = \tilde{r}(t) - F_y(q)y(t),$$

where $\tilde{r}$ is a (filtered) reference signal,

$$\tilde{r}(t) = F_r(q)r(t).$$

The objective is to estimate $G_0(q)$ (and $H_0(q)$) despite the presence of the feedback controller $F_y(q)$. We will assume that data have been collected from the system such that the signals $u(t)$, $y(t)$ and $r(t)$ are available for $t = 1, \ldots N$.

1.1 Direct Approach

The direct approach to closed-loop system identification is to apply the same method as in open loop to the measurements of $u(t)$ and $y(t)$ and to ignore the feedback and reference signal $r(t)$. However, many common open-loop methods will result in biased estimates if used for closed-loop data. For example, this is the case for the standard subspace, spectral analysis and correlation analysis methods. An important exception is the prediction-error method, which can be applied successfully to closed-loop data provided that the dataset contains enough excitation (a varying reference signal can be used to guarantee this) and that the model structure is flexible enough such that it can describe the true system. In particular, the latter requirement means that a flexible enough noise model has to be used.

Exercises for Section 1.1

Download the dataset `closedloop.mat` and load it into Matlab with the command

```
load closedloop.mat
```

This dataset contains 10000 measurements of the input $u(t)$, output $y(t)$ and reference signal $r(t)$ from a particular closed-loop system where $G_0(q)$ and $H_0(q)$ can be described with a Box-Jenkins model with $n_b = 2$, $n_c = 1$, $n_d = 2$, $n_f = 2$ and $n_k = 1$. The sampling time is 0.1 s and the true system `truesys` is available in `closedloop.mat` for reference.

1. Import a dataset with the measured $u(t)$ and $y(t)$ into the System Identification GUI and split the data into estimation and validation data. Furthermore, import the model `truesys` into the GUI to enable convenient (but unrealistic) comparisons with the true system. Estimate

nonparametric frequency response models using spectral analysis for different frequency resolutions and evaluate the result. Remember to turn on the display of confidence intervals such that it is possible to evaluate which features of the frequency responses to pay attention to. Does the spectral analysis estimates give an accurate description of the true frequency response in this case?

2. Estimate an output-error (OE) model of the system and evaluate the result using frequency and transient responses. Note that both step and impulse responses can be viewed under transient response (and that it is often easier to interpret the impulse response). Is the model estimate accurate?

3. Estimate a Box-Jenkins (BJ) model of the system and evaluate the result using frequency and transient responses. Is the model estimate accurate?

4. Estimate a high-order ARX model of the system (with more than 50 poles and zeros). (Remember to select $n_b \gg n_a$!) Is this estimate accurate? Reduce the model order by exporting the high-order model to workspace, reducing the model order with the command

```
modelr=tf(balred(ss(model),2,...
   balredOptions('StateElimMethod','Truncate')))
```

   and then importing the new model to the System Identification GUI. Is this estimate accurate?

1.2 Two-stage Method

The two-stage method for closed-loop system identification involves the following two steps:

1. Estimate a model $\hat{G}_{ru}(q)$ of the transfer function from $r(t)$ to $u(t)$ and use it construct a new input signal

$$\hat{u}(t) = \hat{G}_{ru}(q)r(t)$$

   from $r(t)$.

2. Estimate a model of $G_0(q)$ from $\hat{u}(t)$ to $y(t)$.

Exercises for Section 1.2

1. Import a new dataset with $r(t)$ as input and $u(t)$ as output to the System Identification GUI. Estimate a model $\hat{G}_{ru}(q)$ of the transfer function from $r(t)$ to $u(t)$ and create $\hat{u}(t)$ by exporting the estimated model and using the command

   ```
   uhat=sim(Gruhat,r);
   ```

   in Matlab's workspace. Import a new dataset with $\hat{u}(t)$ as input and $y(t)$ as output and estimate a model of $G_0(q)$. Try OE and BJ model structures and see if you can find a choice that results in an accurate model of $G_0(q)$.

Solutions with Discussion

The solutions below are not necessarily the only correct ones. Depending on choices during data processing and estimation, your results may be perfectly correct even if they do not exactly agree with the discussion below.

Solutions for Section 1.1

1. More features appear when the frequency resolution is increased and the resonance peak of the true system is visible for `M=1000`. However, there is a spurious peak with low uncertainty around $\omega = 1\,\text{rad/s}$ for higher frequency resolutions (e.g. `M=100` and `M=1000`) which could have mislead us in a practical situation where the true system is unknown. This peak comes from the noise properties and is a result of the feedback in the the system.

2. The estimated OE model with correct n_b, n_f and n_k is very different from the true system, which is clear when evaluating the frequency and transient responses. The bias in the model estimate is due to the lack of a noise model in this case.

3. The estimated BJ model with correct model orders is very close to the true system. The high accuracy is obtained since the assumed noise model is able to describe the the true noise coloring.

4. The high-order ARX models with, for example, $n_a = 50$, $n_b = 100$ and $n_k = 1$ or $n_a = 100$, $n_b = 200$ and $n_k = 1$ capture some aspects of the true frequency and transient responses but are quite noisy. Reducing the model order improves the accuracy significantly.

Solutions for Section 1.2

1. A BJ model with $n_b = n_c = n_d = n_f = 5$ and $n_k = 0$ of $G_{ru}(q)$ seems to work well together with an OE model with $n_b = n_f = 2$ and $n_k = 1$ or a BJ model with $n_b = n_c = n_d = n_f = 2$ and $n_k = 1$ of $G_0(q)$. The resulting models are accurate.