EXAM IN TSRT92 MODELLING AND LEARNING FOR DYNAMICAL SYSTEMS

ROOM: Egypten, Olympen, Asgård, SU24, SU25

TIME: 2023-10-27 at 14:00–18:00

COURSE: TSRT92 Modelling and Learning for Dynamical Systems

MODULE: DAT1

DEPARTMENT: ISY

NUMBER OF EXERCISES: 5

RESPONSIBLE TEACHER: Martin Enqvist, 013-281393

VISITS: at about 15:00 och 17:00

COURSE ADMINISTRATOR: Ninna Stensgård, 013-282225, ninna.stensgard@liu.se

APPROVED TOOLS:

1. *L. Ljung, T. Glad and A. Hansson*: ”Modeling and Identification of Dynamic Systems” (Previous editions of this book, including the Swedish edition, are also allowed)
2. *T. Glad & L. Ljung*: ”Reglerteknik. Grundläggande teori”
3. Tables and formulas, for example
   L. Råde & B. Westergren: ”Mathematics handbook”
   C. Nordling & J. Österman: ”Physics handbook”
   S. Söderkvist: ”Formler & tabeller”
4. Calculator

Notes in the above books are allowed, but not complete exercise solutions.

FILES: The files needed to solve some of the exercises are available in the `exam` directory on the exam account, as well as at `/courses/tsrt92/exam2`. If you for some reason would need the original files: Open a terminal window, go to your home directory and copy the files with the command

`cp -r /courses/tsrt92/exam2 .` (N.B. The dot!)

SOLUTIONS: Will be published on the course homepage after the exam.

INSPECTION of the exam takes place on 2023-11-21 at 12.30–13.00 in Ljungeln, building B, entrance 27, corridor A to the right.

PRELIMINARY GRADE LIMITS:	grade 3	23 points
	grade 4	33 points
	grade 5	43 points

N.B. Solutions should be complete and detailed enough such that all steps (except trivial calculations) and reasoning can be followed. Insufficient motivations result in point deductions.

Good luck!

COMPUTER TIPS:

- To open Matlab: Open a terminal (right-click on the background and choose `open terminal` and type

  ```
  module add prog/matlab
  matlab &
  ```

- Print all plots, model descriptions, code, etc. that are needed to answer the questions.

- Remember to write your AID number on each page you print such that the exam guards can distribute the printout to the right person. The commands `title` and `gtext` can be useful for Matlab plots.

- In the System Identification Toolbox:

  - To print the model description: Right-click on the icon of the model you have computed and then click `Present`. The model description then appears in the Matlab command window. Copy it into a file and print it.
  - The System Identification Toolbox plots come with certain limitations but can be exported by selecting `File - Copy figure`, which gives an ordinary Matlab plot you can edit and print.

- Printing in Linux:

  - A file called `file.pdf` can, for example, be printed in a terminal by

    ```
    lp -d printername file.pdf
    ```

    (replace `printername` with the name of the printer in your room).
  - It is possible to print a Matlab plot using `File - Print`, but you must select the printer name writing `-Pprintername` in the `Device option` (again `printername` is the name of your printer).

1. (a) Consider the model

$$y(t) = \frac{q^{-3}(b_1 + b_2 q^{-1})}{1 + a_1 q^{-1} + a_2 q^{-2}} u(t) + w(t)$$

where the disturbance w is given by

$$w(t) = H(q)e(t)$$

with e a zero-mean white noise signal. For what choice of $H(q)$ do you get an ARX model? (2p)

(b) Consider the signal

$$y(t) = b_1 e(t-1) + b_2 e(t-2) + \ldots + b_N e(t-N)$$

where e is a zero-mean white noise signal, $b_k \neq 0$, $k = 1, \ldots, N$. What is the smallest value of the integer M such that, for all values of b_k, we have $R_y(\tau) = 0$ for all $|\tau| > M$? (3p)

(c) Compute the spectral density of the sampled signal $y(t)$ when it is generated as

$$y(t) = G(q)u(t) + e(t), \quad G(q) = 1 + 0.5q^{-1}$$

where $u(t)$ and $e(t)$ are uncorrelated zero-mean white noises with variances 1 and 2, respectively. The sampling time is 1 s. (3p)

(d) Consider a particular system with input $u(t)$ and output $y(t)$ and assume that data have been collected in a closed-loop experiment where a PI controller was used to calculate the input and where the output contains a significant disturbance term $w(t)$. Based on prior information, this disturbance is expected to contain one low-frequent component and one component with most of the power centered at 50 Hz. An OE model of the system from u to y is desired. Is it a good idea to estimate such a model directly from the measurements of u and y using the prediction-error method? (2p)

2. (a) The step response of the model

$$\ddot{y} + 2\dot{y} + 5y = -\dot{u} + u$$

has been simulated. Can this simulation be reused to obtain also the step response of the model

$$4\ddot{y} + 4\dot{y} + 5y = -6\dot{u} + 3u$$

without doing any further simulations? (4p)

(b) Stiff models are difficult to simulate. Why? Rank the following models according to their stiffness. (Hint: Do not forget that you can use Matlab for computing for example eigenvalues of a matrix.)

$$\text{(i)} \qquad \dot{x} = \begin{pmatrix} 1 & -6 \\ 1 & -4 \end{pmatrix} x$$

$$\text{(ii)} \qquad \dot{x} = \begin{pmatrix} 0 & 1 \\ -0.001 & -1.001 \end{pmatrix} x$$

$$\text{(iii)} \qquad \dot{x} = \begin{pmatrix} -50 & 1 \\ 2450 & -51 \end{pmatrix} x$$

(3p)

(c) Consider the DAE model

$$\begin{aligned} \dot{x}_1 &= -2x_3 \\ \dot{x}_2 &= x_1 + 3x_3 + 2u \\ 0 &= x_1 + x_2 - u \\ y &= x_2 \end{aligned}$$

where u is the input and y the output. What is the static gain of this model from u to y? (3p)

3. The data for this exercise are in a file called `sysiddata20231027.mat`, which can be loaded into Matlab with the command

```
load sysiddata20231027.mat
```

when the `exam` directory has been selected. The file contains sampled signals u and y (the sampling time is $T_s = 1\,\text{s}$).

(a) What is the best linear model you can find if you are allowed to use a maximum of 4 poles and 4 zeros overall (i.e., for instance for BJ: $n_f + n_d \leq 4$ and $n_b + n_c \leq 4$, and similarly for the other classes)? For your best model report:

- plot of the simulated model output for validation data
- parameter values and uncertainty
- residual plots
- Bode plots
- poles and zeros placement

Discuss and comment your choices and results. (5p)

(b) Show that the model fit can be improved if you estimate a nonlinear model. Explain your choices. (5p)

4. Consider the following model

$$y(t) = b_1 u(t-1) + b_2 u(t-2) + e(t)$$

where e is zero-mean white noise and b_1 and b_2 are the parameters to be identified. Assume that the true system is

$$y(t) = 0.2u(t-1) + 0.4u(t-2) + 0.5u(t-3) + w(t)$$

where w is a zero-mean white noise signal with variance λ_w.

(a) An identification experiment is performed with an input u that is a zero-mean white noise signal with variance λ_u which is uncorrelated with w. Compute the values of the parameters that minimize the prediction error $V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}(y(t) - \hat{y}(t|\theta))^2$ when $N \to \infty$. Do the parameters converge to the true values? Is the model unbiased? (5p)

(b) How is the parameter convergence affected if w is not white? (1p)

(c) How is the parameter convergence affected if u is not white? (2p)

(d) How is the parameter convergence affected if w is not white and data have been collected in closed loop? (2p)

5. Consider the following DAE model:

$$\dot{x}_1 = 1 + x_2 \tag{1}$$

$$\dot{x}_3 = 0 \tag{2}$$

$$x_1 x_3 - x_2(2 + x_3) + t = 0 \tag{3}$$

(a) What is the differentiability index for this model? (Hint 1: What is the implication of (2)? Hint 2: There are two cases to consider.) (7p)

(b) Is there any difference in the index if we replace (3) with the following equation:

$$x_1 x_3 - 2x_2 + t = 0 \tag{4}$$

Explain your answer. (3p)