Dynamical systems and Control

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Lecture 8: PID implementation

- Recap
- PID discretization
- Integral windup
- Derivative filtering

Recap of Lec 6-7

PID controller

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \dot{e}(t)$$

or

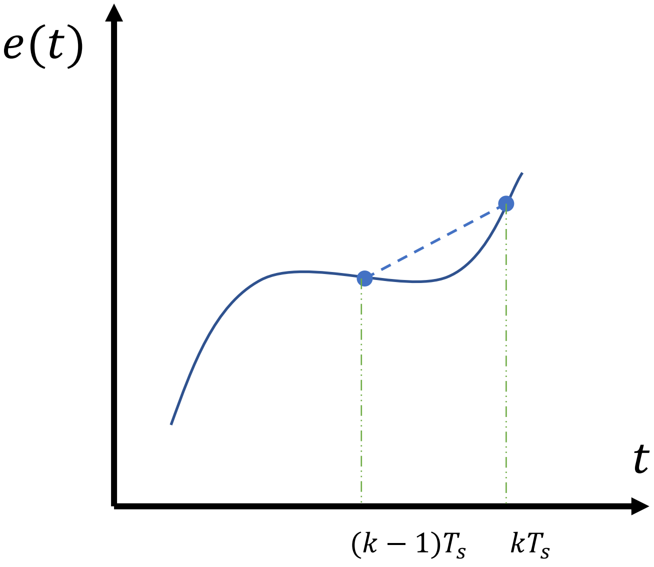
$$u(t) = K\left(e(t) + \frac{1}{T_I}\int_0^t e(\tau) d\tau + T_D \dot{e}(t)\right)$$

PID discretization

Approximation of derivative

Euler method: $\dot{e} \approx \frac{e_k - e_{k-1}}{T_s}$

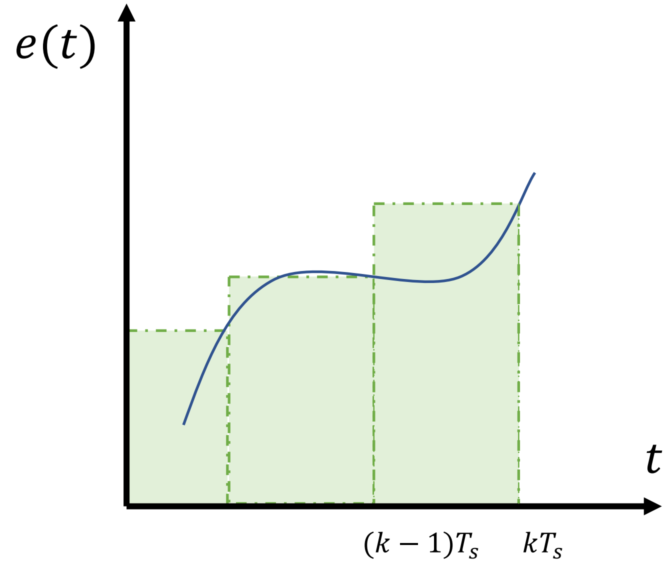
(Note $e_k = e(kT_s)$)



Approximation of integral

$$s(t) = \int_0^t e(\tau)d\tau$$

$$s_k = s_{k-1} + T_s e_k$$



Discretized PID controller

Define the error: $e_k = r_k - y_k$

Build the integral: $s_k = s_{k-1} + T_s e_k$

Build the derivative: $\dot{e}_k \approx \dfrac{e_k - e_{k-1}}{T_s}$

$$u_k = K\left(e_k + \frac{1}{T_I} s_k + T_D \frac{e_k - e_{k-1}}{T_s}\right)$$

What if the control input is constrained?

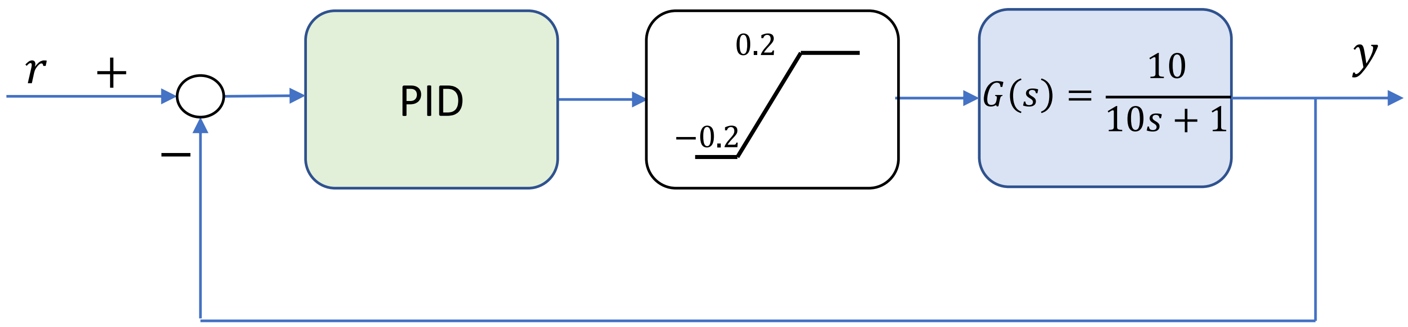
The input to a system is not unlimited!

$$v_k = K(e_k + \frac{1}{T_I} s_k + T_D \frac{e_k - e_{k-1}}{T_s})$$

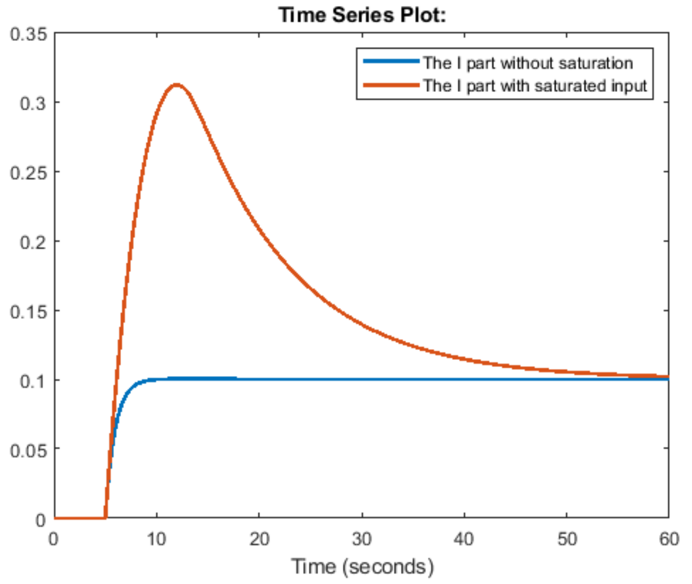
$$u_k = \begin{cases} u_{max}, & v_k > u_{max} \\ v_k, & u_{min} < v_k < u_{max} \\ u_{min}, & v_k \leq u_{min} \end{cases}$$

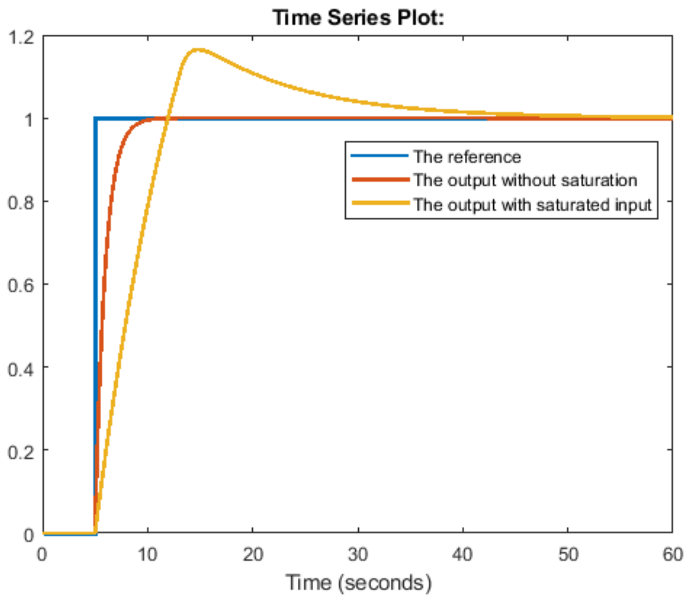
Integral windup

What is integral windup?



Example


Time Series Plot:
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
10
20
30
40
50
60
Time (seconds)
The I part without saturation
The I part with saturated input


Time Series Plot:
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
Time (seconds)
The reference
The output without saturation
The output with saturated input

Strategies 1- Conditional update

Turn off integral update when the control is saturated

If $u_{min} < v_k < u_{max}$

$$I_k = I_{k-1} + K \frac{T_s}{T_I} e_k$$

Else

$$I_k = I_{k-1}$$

End

Strategies 2- Modification of integral

Modify the integral term when control is near to saturation

$$I_k = I_{k-1} + K\frac{T_s}{T_I}e_k$$

$$v_k = Ke_k + I_k + K\frac{T_d}{T_s}(e_k - e_{k-1})$$

$$u_k = \begin{cases} u_{max}, & v_k > u_{max} \\ v_k, & u_{min} < v_k < u_{max} \\ u_{min}, & v_k \leq u_{min} \end{cases}$$

$$\boxed{I_k = I_k + \frac{T_s}{T_t}(u_k - v_k)}$$

The new part!

T_t is called tracking constant

Derivative filtering

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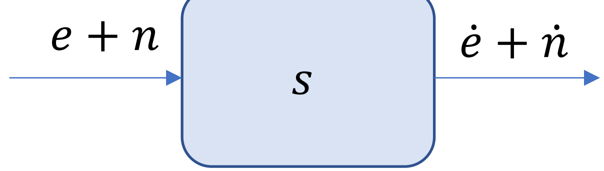
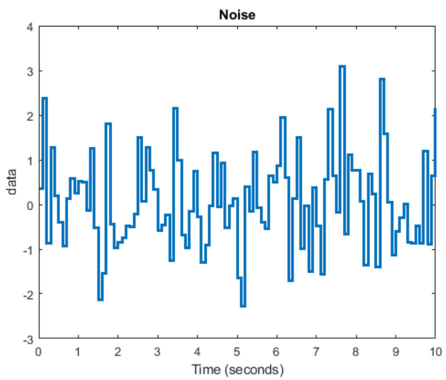
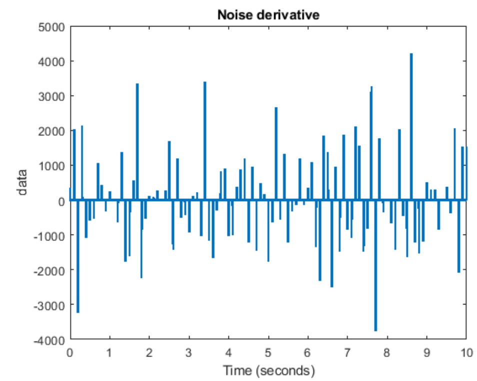
What is the issue with derivative?







Remedy

Replace s with $\frac{s}{\alpha s+1}$

The derivative becomes

$$D(s) = \frac{s}{\alpha s + 1} E(s)$$

Quiz: Can you write the transfer function in z domain?

Remedy

Replace s with $\frac{s}{\alpha s+1}$

The derivative becomes

$$D(s) = \frac{s}{\alpha s + 1} E(s)$$

Derive the differential equation

$$\alpha s D(s) + D(s) = sE(s) \xrightarrow{\text{Inverse Laplace}} \alpha \dot{d}(t) + d(t) = \dot{e}(t)$$

Approximate the derivative by Euler

$$\alpha \left(\frac{d_k - d_{k-1}}{T_s} \right) + d_k = \frac{e_k - e_{k-1}}{T_s}$$

$$\boxed{d_k = \frac{\alpha}{\alpha + T_s} d_{k-1} + \frac{1}{\alpha + T_s} (e_k - e_{k-1})}$$

Quiz: Can you write the transfer function in z domain?

What do we cover next?

- Issues with PID
- State feedback

Ask us!

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