Dynamical systems and Control

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Lecture 7: Control

- Recap
- Feedback issues
- Systems that are difficult to control
- PID tuning for simple systems

PID controller

A quick recap of lecture 6

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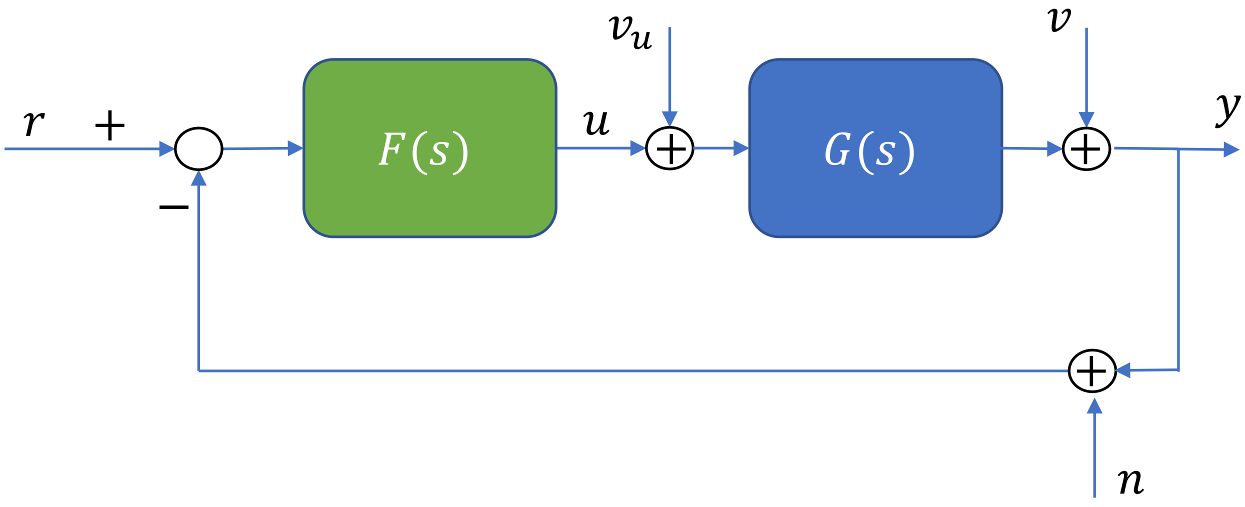
Proportional-Integrator-Derivative (PID):

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \dot{e}(t)$$

Feedback issues



$$Y(s) = \frac{F(s)G(s)}{1+F(s)G(s)}R(s) + \frac{G(s)}{1+F(s)G(s)}V_u(s) + \frac{1}{1+F(s)G(s)}V(s) - \frac{F(s)G(s)}{1+F(s)G(s)}N(s)$$

Systems that are difficult to control

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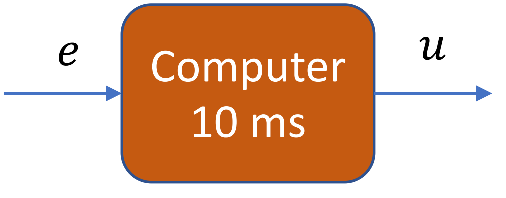
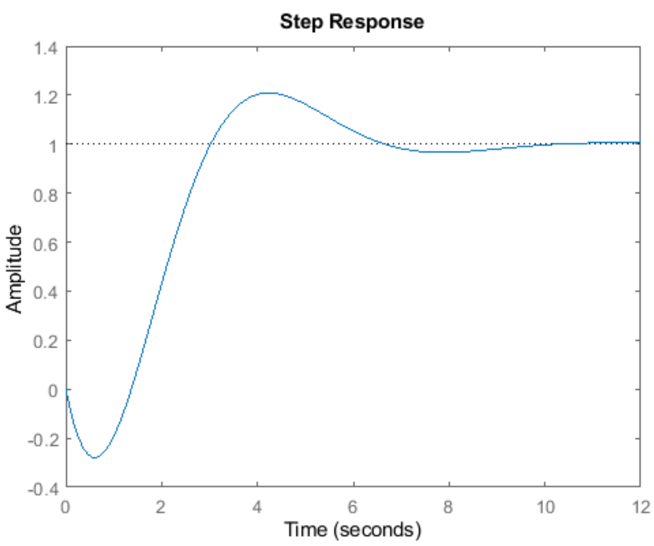
Systems with time delay



Systems with zero on RHS

The initial response can have undershoot.



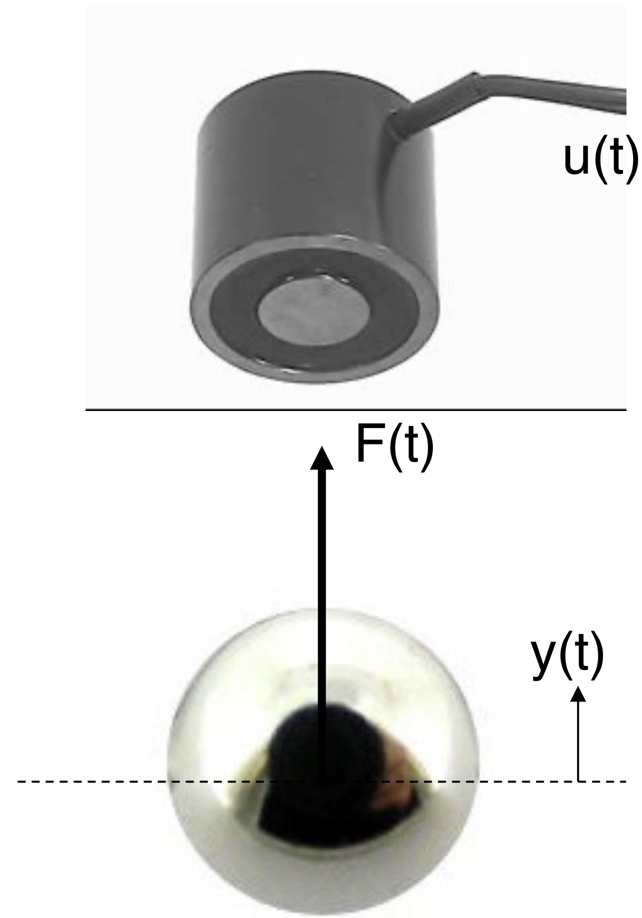
Unstable systems

We should stabilize the system.

Example-Controller design for floating ball

$y(t)$: The position of the ball
$F(t)$: Generated magnetic force
$u(t)$: Voltage to the electromagnet

$$G(s) = \frac{1}{s^2 - 1}$$



Example-Continued-P controller

P-controller: $F(s) = K_P$

The system: $G(s) = \frac{1}{s^2 - 1}$

Example-Continued-P controller

P-controller: $F(s) = K_P$

The system: $G(s) = \frac{1}{s^2 - 1}$

The transfer function of the closed-loop system: $G_c(s) = \frac{F(s)G(s)}{1 + F(s)G(s)}$

$$G_c = \frac{K_P/_{s^2-1}}{1 + K_P/_{s^2-1}} = \frac{K_P}{s^2 + K_P - 1}$$

Poles: $\lambda = \pm\sqrt{1 - K_P}$

If $1 - K_P > 0$: → Real and at least one positive

If $1 - K_P \leq 0$: → Complex with zero real port

Always unstable!

Example-Continued-PI controller

PI-controller: $F(s) = K_P + \frac{K_I}{s}$

The system: $G(s) = \frac{1}{s^2 - 1}$

Example-Continued-PI controller

PI-controller: $F(s) = K_P + \frac{K_I}{s}$

The system: $G(s) = \frac{1}{s^2 - 1}$

The transfer function of the closed-loop system: $G_c(s) = \frac{F(s)G(s)}{1 + F(s)G(s)}$

$$G_c = \frac{(K_P + \frac{K_I}{s})(\frac{1}{s^2 - 1})}{1 + (K_P + \frac{K_I}{s})(\frac{1}{s^2 - 1})} = \frac{K_P s + K_I}{s^3 + (K_P - 1)s + K_I}$$

First order: $s + c$ stable if and only if $c > 0$

Second order: $s^2 + as + b$ stable if and only if $a > 0, b > 0$

Third order: $(s + c)(s^2 + as + b)$ if stable, then all coefficients should be positive

s^2 is missing so unstable!

Example-Continued-PD controller

PD-controller: $F(s) = K_P + K_D s$

The system: $G(s) = \frac{1}{s^2 - 1}$

Example-Continued-PD controller

PD-controller: $F(s) = K_P + K_D s$

The system: $G(s) = \frac{1}{s^2 - 1}$

The transfer function of the closed-loop system: $G_c(s) = \frac{F(s)G(s)}{1 + F(s)G(s)}$

$$G_c = \frac{(K_P + K_D s)(\frac{1}{s^2 - 1})}{1 + (K_P + K_D s)(\frac{1}{s^2 - 1})} = \frac{K_P + K_D s}{s^2 + K_D s + K_P - 1}$$

Stable if $K_D > 0, K_P > 1$

PID tuning for simple systems

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How to tune PID?

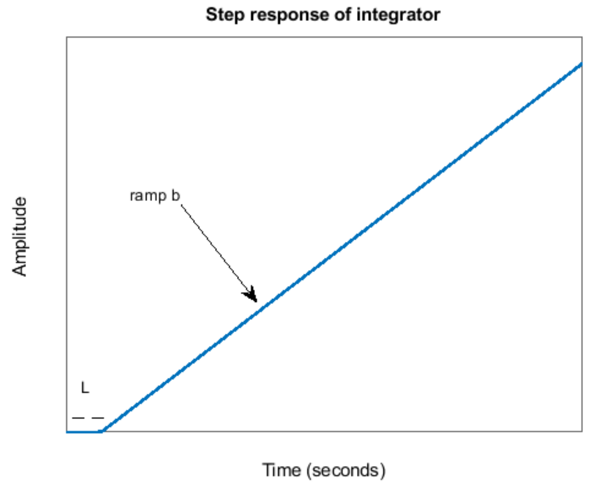
- Trial and error: Experience
- Complete model + analysis of the poles
- Simplified model + tuning rules

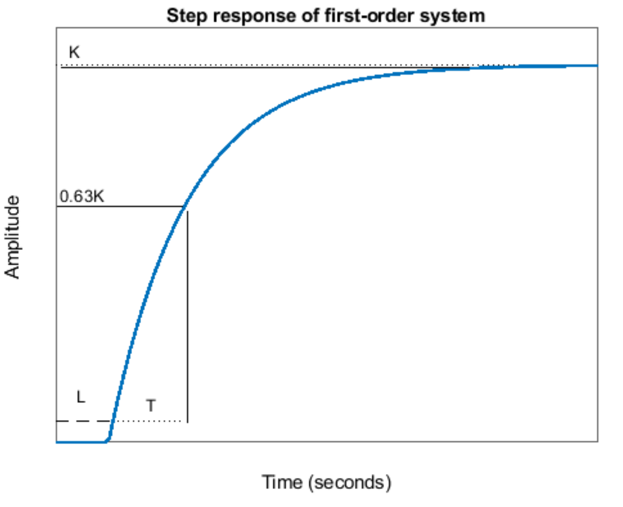
Simplified models are:

- Integrators

- First-order dynamics

Study the step response





$$G(s) = \frac{b}{s} e^{-sL}$$

$$G(s) = \frac{K_{process}}{Ts + 1} e^{-sL}$$

Ziegler-Nichols method for Integrators

- Calculate b and L
- Select the gains according to the table on p. 44 in the complementary material

For example, for a P controller: $K = \frac{1}{bL}$

Regulator	K	T_i	T_d
P	$1/(bL)$		
PI	$0.9/(bL)$	$3L$	
PID	$1.2/(bL)$	$2L$	$L/2$

λ-tunning for first-order systems

Define the desired closed loop system as

$$G_c = \frac{1}{sT_c + 1} e^{-Ls}$$

Let

$$\lambda = \frac{T_c}{T}$$

λ-tunning for first-order systems

$$G_c(s) = \frac{G(s)F(s)}{1 + G(s)F(s)} = \frac{1}{sT_c + 1} e^{-Ls}$$

λ-tunning for first-order systems

$$G_c(s) = \frac{G(s)F(s)}{1 + G(s)F(s)} = \frac{1}{sT_c + 1} e^{-Ls}$$

From the above equation, we obtain:

$$F(s) = \frac{G_c(s)}{G(s)(1 - G_c(s))}$$

Now, replace $G_c(s) = \frac{1}{sT_c+1} e^{-Ls}$ and $G(s) = \frac{K_{process}}{Ts+1} e^{-sL}$

$$F(s) = \frac{\frac{e^{-Ls}}{sT_c + 1}}{\frac{K_{process}}{Ts + 1} e^{-sL}(1 - \frac{e^{-Ls}}{sT_c + 1})} = \frac{Ts + 1}{K_{process}(sT_c + 1 - e^{-Ls})} \approx \frac{Ts + 1}{K_{process}(sT_c + 1 - 1 + Ls)}$$

$$= \frac{s + {}^{1}/_{T}}{K_{process}(s\lambda + Ls)} = \frac{T}{K_{process}(\lambda T + L)} \frac{s + {}^{1}/_{Tc}}{s} = \frac{T}{K_{process}(\lambda T + L)} (1 + \frac{1}{TS})$$

A PI controller!

$$K_P = \frac{T}{K_{process}(\lambda T + L)}$$

$$K_I = \frac{1}{K_{process}(\lambda T + L)}$$

What do we cover next?

- PID discretization
- Integral windup
- Derivative filtering

Ask us!

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