Dynamical systems and Control

Farnaz Adib Yaghmaie

Automatic Control (RT) Division, ISY

- Email: Farnaz.adib.Yaghmaie@liu.se
- Office: Entrance 25-27, B-huset, Campus Valla

LINKÖPING UNIVERSITY

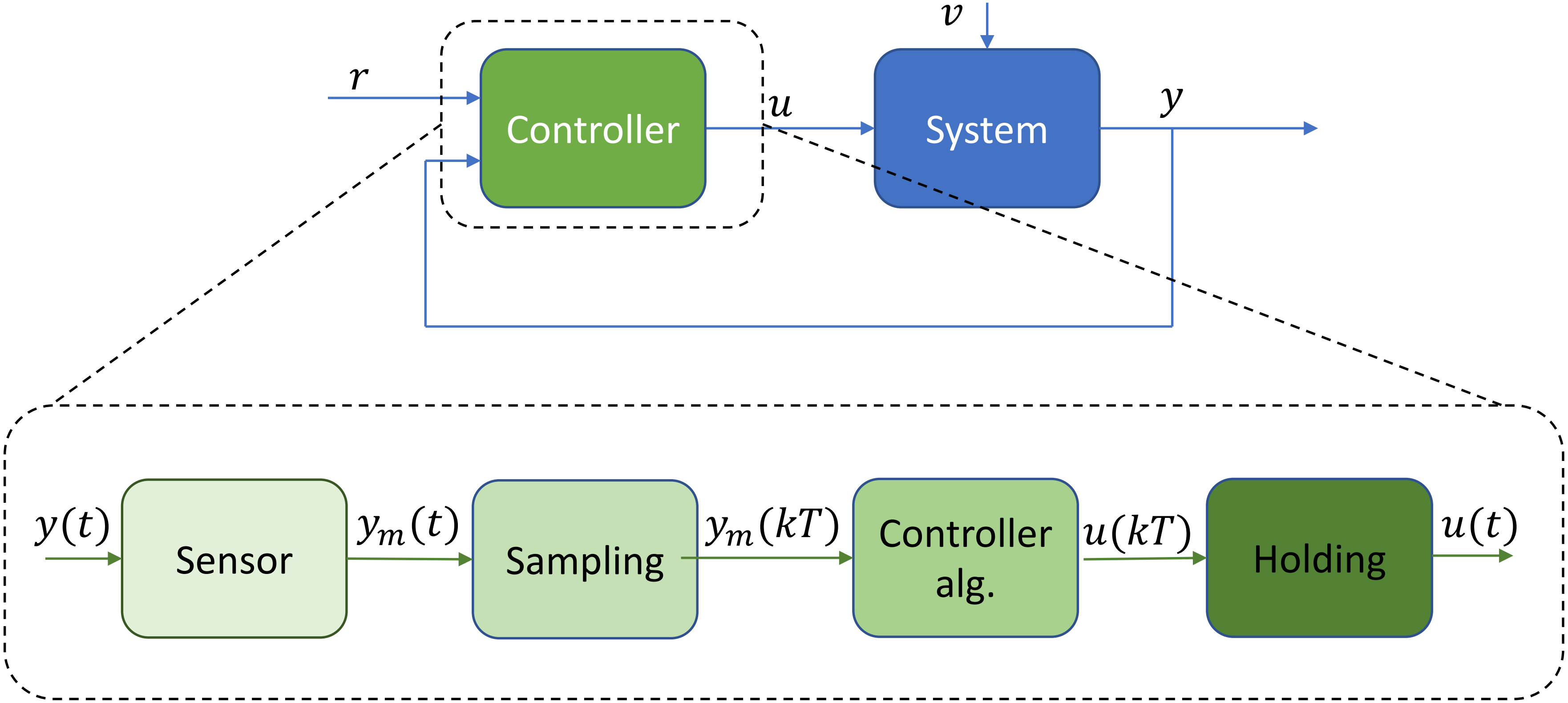
Lecture 6: Feedback

- Recap
- Open-loop vs. closed-loop control
- PID controller
- Analysis of the closed-loop system

Recap

A quick recap of lectures 1-5

A typical control loop

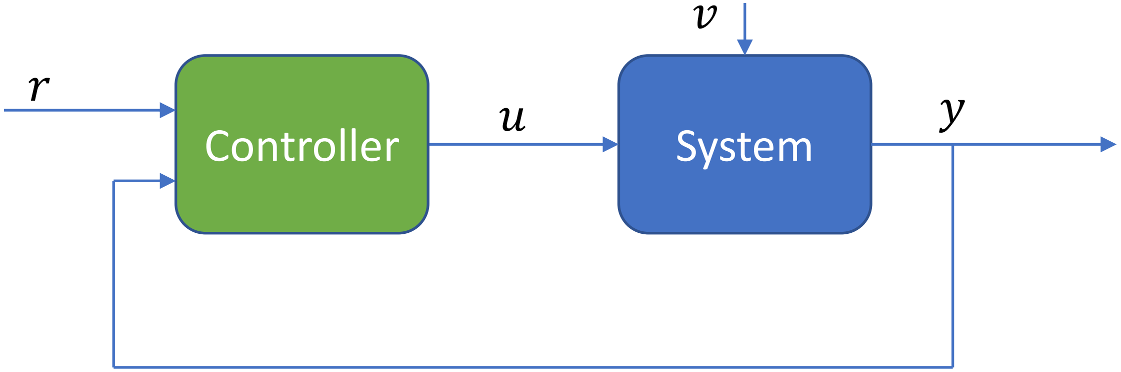


Controller

Concept

Open-loop vs. closed-loop controllers

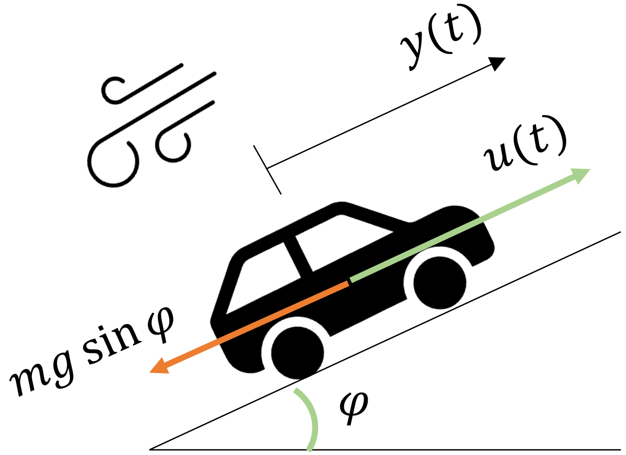
Concept



Example

The desired velocity $r(t) = 25\ {}^{m}/_{s}$. How to select $u(t)$?

Solution:



$u(t)$: Driving force

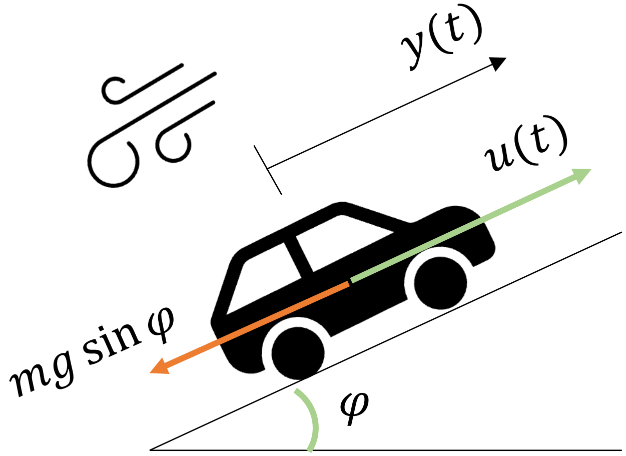
$F_a = ay(t)$: Air resistance force

$y(t)$: Velocity

$mg \sin \varphi$: Disturbance

Example

Let $m = 1000\ kg, \alpha = 200\ ^{Ns}/_{m}$. The desired velocity $r(t) = 25\ ^{m}/_{s}$. How to select $u(t)$?



Solution:

$$m\dot{y}(t) = u - mg\sin\varphi - \alpha y(t)$$

$u(t)$: Driving force

$F_a = ay(t)$: Air resistance force

$y(t)$: Velocity

$mg\sin\varphi$: Disturbance

Example-Continued

Open-loop controller:

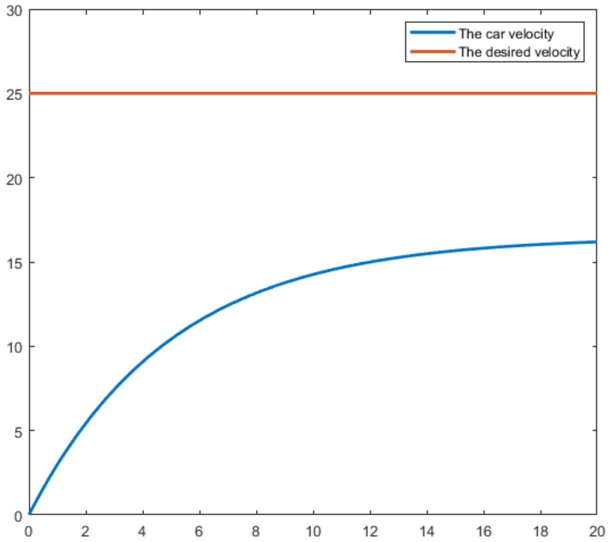
Example-Continued

Open-loop controller:

Set $y = r$ and $\dot{y} = 0$. We don't know the disturbance, so we design for $\varphi = 0$

$$u = \alpha r$$

But if $\varphi \neq 0$, we get the following plot, (here $\varphi = 10^{\circ}$)



Open-loop control

This strategy is called open-loop control:

- The performance is bad in general
- Not good if we have disturbance
- Not good if we don't know the model
- No freedom

What do we miss?

Example-Continued

Closed-loop controller:

Let’s use the velocity $y(t)$.
Define the error

$$e(t) = r(t) - y(t)$$

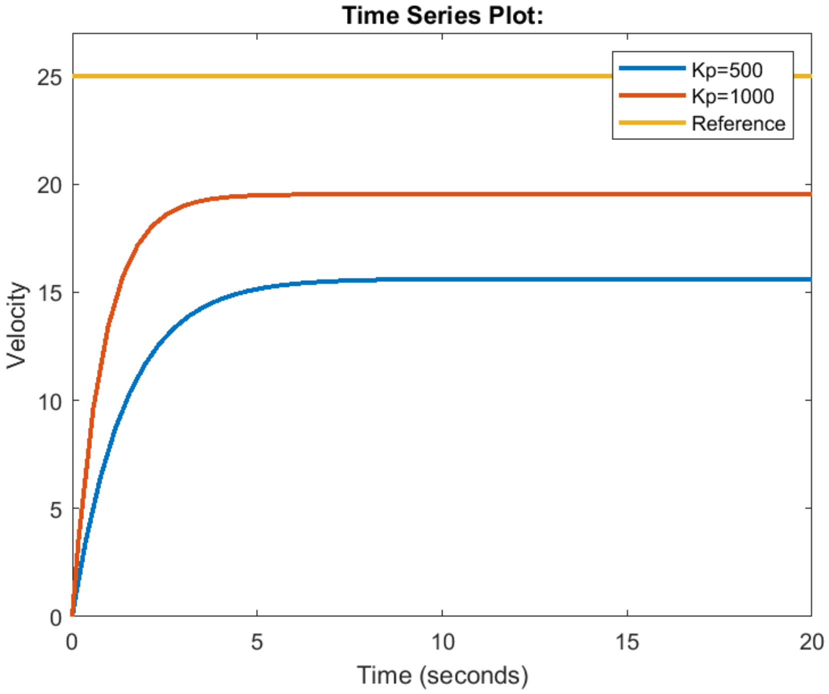
Select $u(t) = f(e(t))$

PID controller

LINKÖPING UNIVERSITY

Proportional (P):

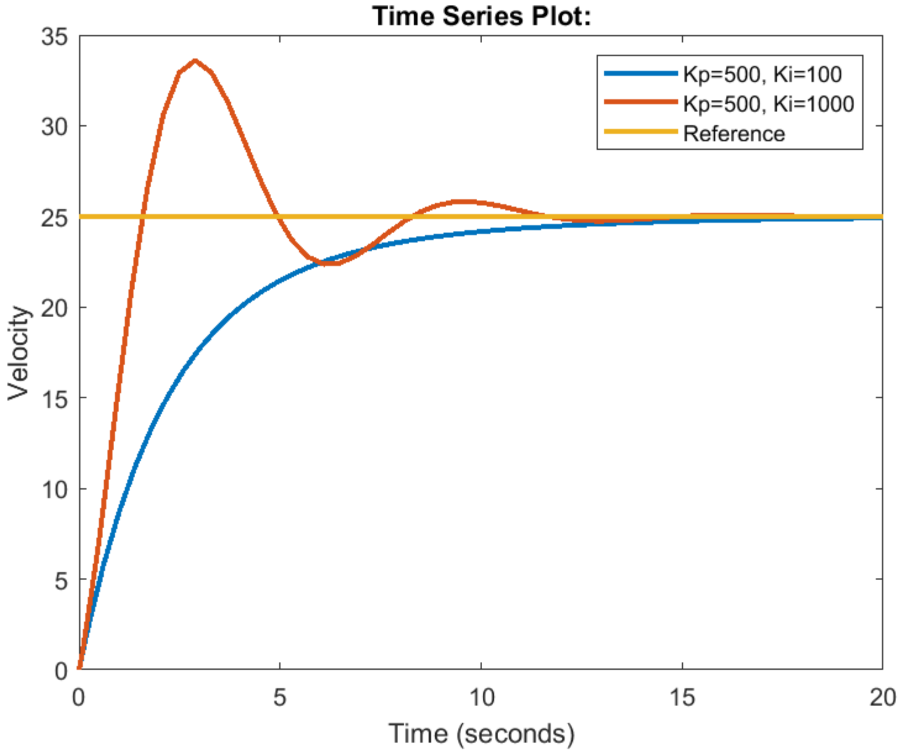
$$u(t) = K_P e(t)$$



In the example: Bigger K_P, less error. But we cannot track the reference

Proportional-Integrator (PI):

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau$$

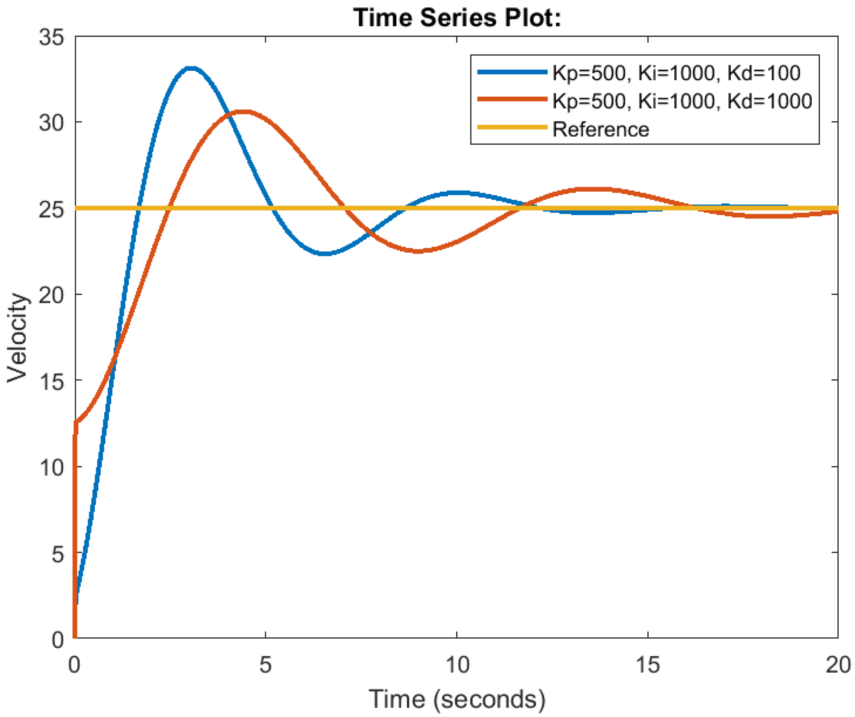


In the example: no error. But big values of K_I leads to oscillation and instability

Proportional-Integrator-Derivative (PID):

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau)d\tau + K_D \dot{e}(t)$$

Add a derivative term,
predict the future



Summary

P:

- Big K_P:
  - Less permanent error
  - Big control effort
  - Can cause oscillation

I:

- Eliminate permanent error
- Can cause oscillation, overshoot and instability

D:

- Reduce oscillation and overshoot
- Sensitive to measurement noise

Analysis of the closed-loop system

Laplace transformation of PID

P: $u(t) = K_P e(t)$

PI: $u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau$

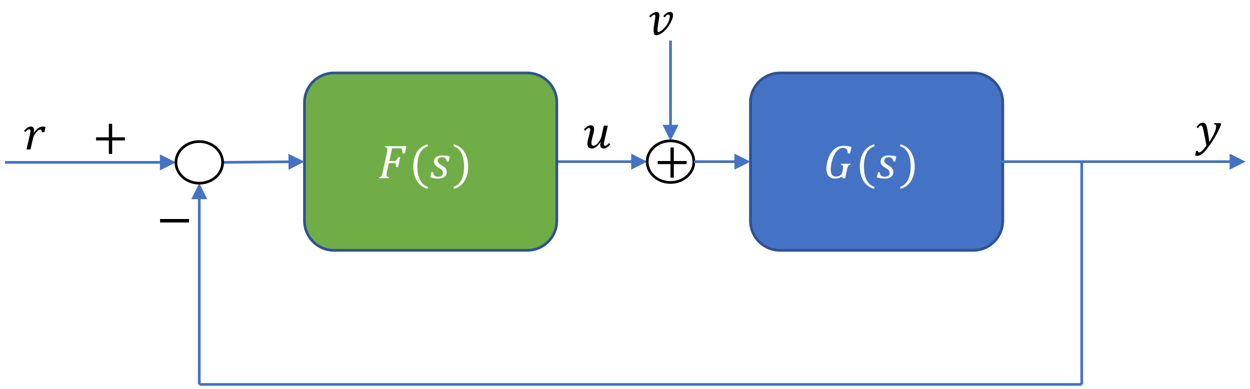
PID: $u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \dot{e}(t)$

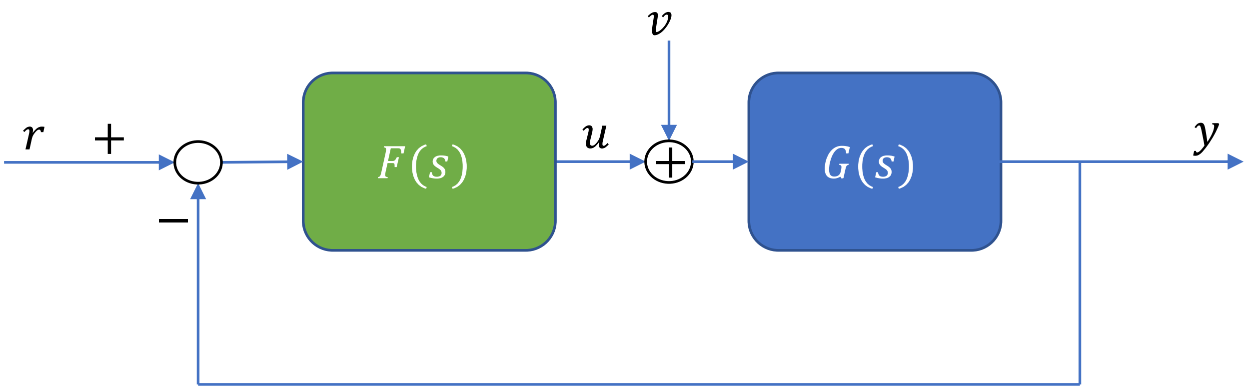
Laplace transformation of PID

$$P: \quad u(t) = K_P e(t) \longrightarrow U(s) = K_P E(s)$$

$$PI: \quad u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau \longrightarrow U(s) = (K_P + \frac{K_I}{s}) E(s)$$

$$PID: \quad u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \dot{e}(t) \longrightarrow U(s) = (K_P + \frac{K_I}{s} + K_D s) E(s)$$


r
$+$
$-$
$F(s)$
u
v
$G(s)$
y



$$Y(s) = G(s)(U(s) + V(s))$$

$$U(s) = F(s)E(s) = F(s)(R(s) - Y(s))$$

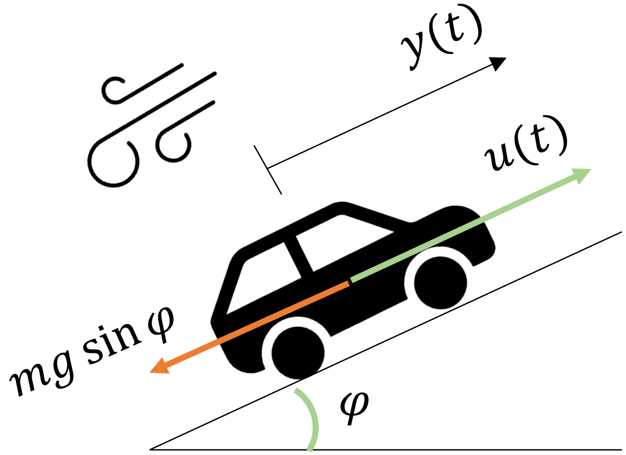
Replace the second equation in the first one

$$Y(s) = G(s)F(s)R(s) - G(s)F(s)Y(s) + G(s)V(s)$$

$$Y(s)(1 + G(s)F(s)) = G(s)F(s)R(s) + G(s)V(s)$$

$$\boxed{Y(s) = \frac{F(s)G(s)}{1 + F(s)G(s)}R(s) + \frac{G(s)}{1 + F(s)G(s)}V(s)}$$

Example

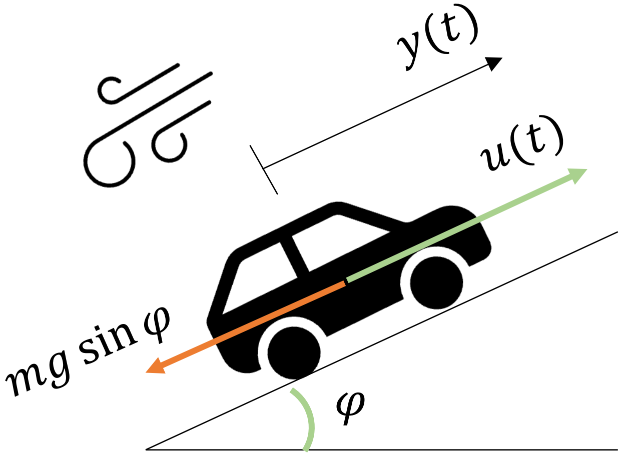


Study stability when there is no disturbance and we use a P controller.

Solution:

$$m\dot{y}(t) = u - mg\sin\varphi - \alpha y(t)$$

Example



Study stability when there is no disturbance and we use a P controller.

Solution:

$$m\dot{y}(t) = u - \alpha y(t)$$

The transfer function of the system

$$G(s) = \frac{1/m}{s + \alpha/m}$$

The transfer function of the controller

$$F(s) = K_p$$

The transfer function of the closed-loop system

$$G_c = \frac{F(s)G(s)}{1 + F(s)G(s)} = \frac{K_P \frac{1/m}{s + \alpha/m}}{1 + K_P \frac{1/m}{s + \alpha/m}} = \frac{K_P/m}{s + \alpha/m + K_P/m}$$

$\lambda = -(\frac{\alpha}{m} + \frac{K_P}{m})$:is stable for all positive K_P

What do we cover next?

- Feedback issues
- Systems that are difficult to control
- PID tuning for simple systems

Ask us!

Farnaz Adib Yaghmaie

- Email: Farnaz.adib.Yaghmaie@liu.se

Daniel Bossér

- Email: daniel.bosser@liu.se

Filipe Marques Barbosa

- Email: filipe.barbosa@liu.se

LINKÖPING
UNIVERSITY