Dynamical systems and Control

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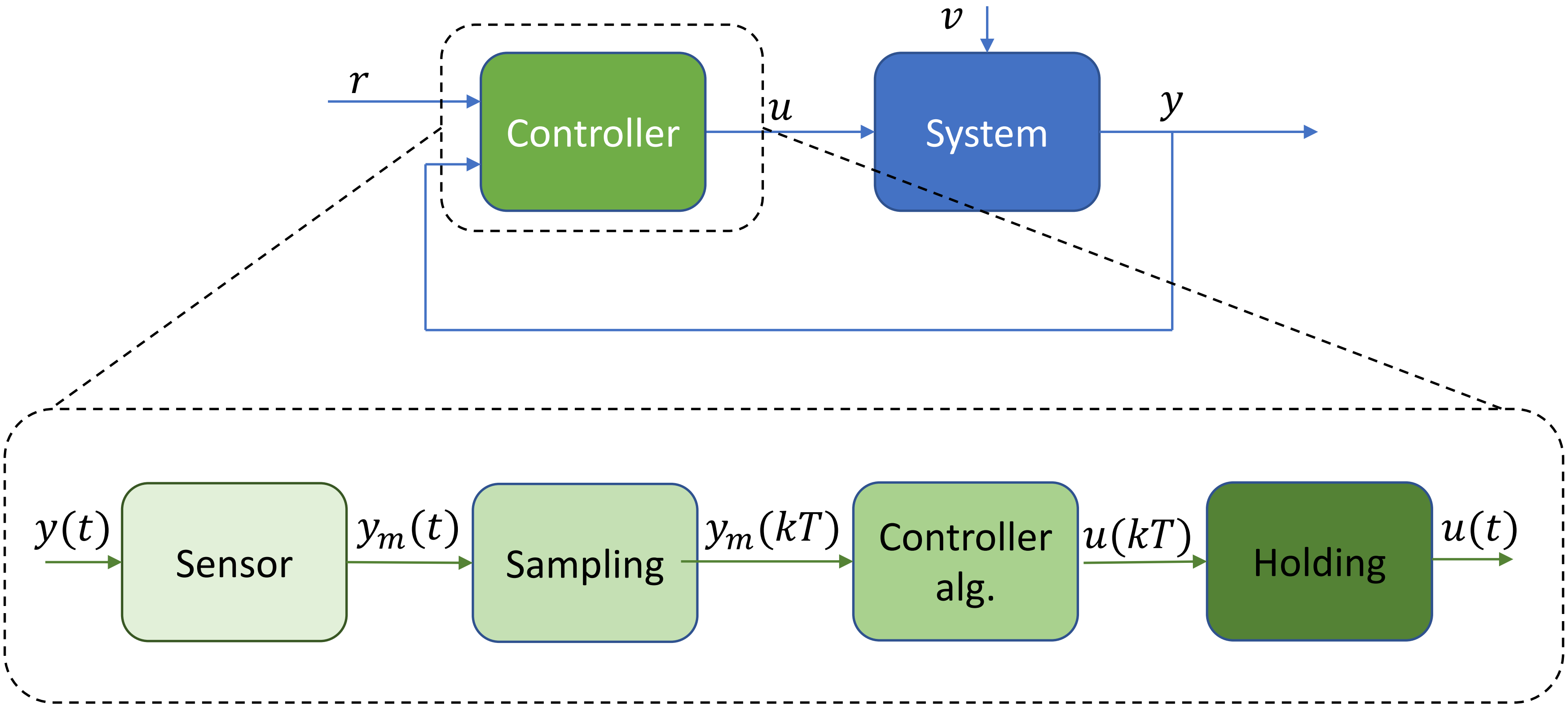
Lecture 5: Signal processing

- Recap
- Filter
- Frequency response and Bode diagram
- Alias effect

Sensors

A quick recap of lecture 4

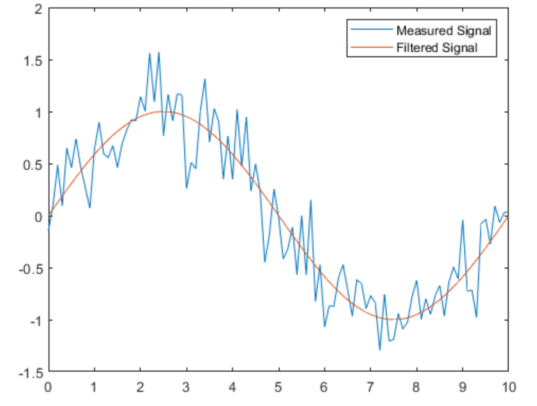
A typical control loop



Filters

Concept

Given a sequence of measurements $y(kT_s)$, generate a new sequence $y_f(kT_s)$



Needs to handle signals in discrete time!

z-transformation

Difference equations are difficult!

$$y(k) + a_1 y(k-1) + \cdots + a_n y(k-n) = b_0 u(k) \ldots + b_n u(k-n)$$

Introduce a new transformation:

$$U(z) = \mathcal{Z}\{u\{k\}\} = \sum_{k=0}^{\infty} u(k) z^{-k}$$

z-transformation

- z: the next
- z^{-1}: the previous

$$y(k) + a_1 y(k-1) + \cdots + a_n y(k-n) = b_0 u(k) \ldots + b_n u(k-n)$$

z-transformation

- z: the next
- z^{-1}: the previous

$$y(k) + a_1 y(k-1) + \cdots + a_n y(k-n) = b_0 u(k) \ldots + b_n u(k-n)$$

↓

$$Y(z) + a_1 z^{-1} Y(z) + \cdots a_n z^{-n} Y(z) = b_0 U(z) + \cdots + b_n z^{-n} U(z)$$

$$Y(z)(1 + a_1 z^{-1} + \cdots a_n z^{-n}) = U(z)(b_0 + \cdots + b_n z^{-n})$$

$$G(z) = \frac{b_0 z^n + \cdots + b_n}{z^n + a_1 z^{n-1} + \cdots + a_n}$$

Filter

$$U(z) \rightarrow \boxed{\frac{b_0 z^n + \cdots + b_n}{z^n + a_1 z^{n-1} + \cdots + a_n}} \rightarrow Y(z) = G(z)U(z)$$

is a discrete transfer function!

Use MATLAB commands butter, cheby1, … to find bs and as

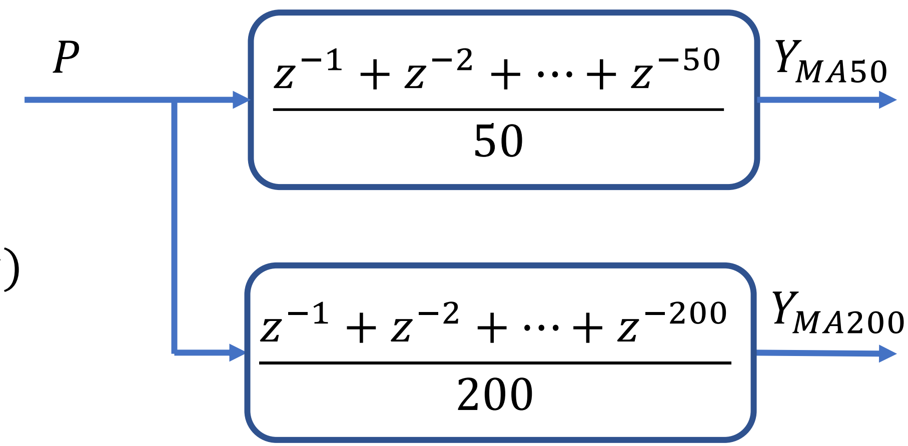
Example-How to become a billionaire?

Sell:

If $y_{MA50}(k) \le y_{MA200}(k)$

Buy:

Otherwise



Example-Continued

Filtered signals are damped

Filtered signals are after the actual signal

Frequency response and Bode diagram

Concept

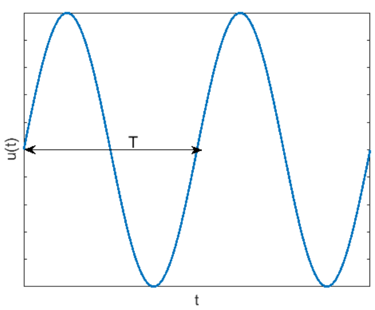
What happens, if the input is sinusoidal: $u(t) = \mathrm{A} \sin \omega t$?

Motivating example

Assume that we average over the last 3 inputs:

$$y(kT_s) = \frac{1}{3}(u(kT_s) + u\big((k-1)T_s\big) + u((k-2)T_s))]$$

Find the output if the input is $u(t) = A\sin\omega t$.



Solution:

Motivating example

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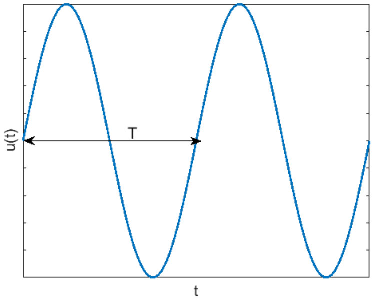


Solution: The transfer function reads

$$G(z) = \frac{1 + z^{-1} + z^{-2}}{3}$$

Motivating example-Continued

Using $u(t) = A\sin \omega t$ the output reads

(Remember $G(z) = \frac{1+z^{-1}+z^{-2}}{3}$)

Motivating example-Continued

Using $u(t) = A\sin\omega t$ the output reads

(Remember $G(z) = \frac{1+z^{-1}+z^{-2}}{3}$)

$$y(kT_s) = \frac{A}{3}(sin(\omega kT_s) + sin(\omega(k-1)T_s) + sin(\omega(k-2)T_s))]$$

Trick1 : $e^{i\varphi} = \cos\varphi + i\sin\varphi$. So, $\sin\varphi = Im\,(e^{i\varphi})$

Using trick 1, the output reads

$$y(kT_s) = \frac{A}{3} Im[e^{i\omega kT_s} + e^{i\omega(k-1)T_s} + e^{i\omega(k-2)T_s}]$$

$$= A\, Im[e^{i\omega kT_s} \frac{1 + e^{-i\omega T_s} + e^{-2i\omega T_s}}{3}]$$

$$= A\, Im[e^{i\omega kT_s} G(e^{i\omega T_s})]$$

Motivating example-Continued

Remember that $G\left(e^{i\omega T_s}\right)$ is a complex number so

Trick 2: $G\left(e^{i\omega T_s}\right) = \left|G\left(e^{i\omega T_s}\right)\right| e^{i \arg G\left(e^{i\omega T_s}\right)}$

Motivating example-Continued

Remember that $G(e^{i\omega T_s})$ is a complex number so

Trick 2: $G(e^{i\omega T_s}) = |G(e^{i\omega T_s})|e^{i \arg G(e^{i\omega T_s})}$

Using the above result, the output signal reads

$$y(kT_s) = A\ Im[e^{i\omega k T_s}|G(e^{i\omega T_s})|e^{i \arg G(e^{i\omega T_s})})]$$

$$= |G(e^{i\omega T_s})|A\ Im[e^{i(\omega T_s + \arg G(e^{i\omega T_s}))}]$$

Now, let's use Trick 1 again:

$$y(kT_s) = |G(e^{i\omega T_s})|A \sin(\omega k T_s + \arg G(e^{i\omega T_s}))$$

Bode diagram

$$y(kT_s) = \left|G\left(e^{i\omega T_s}\right)\right| A \sin\left(\omega k T_s + \arg G\left(e^{i\omega T_s}\right)\right)$$

The result is still a sin

- amplified by $\left|G\left(e^{i\omega T_s}\right)\right|$
- phase-shifted by $\arg G\left(e^{i\omega T_s}\right)$

This result holds in general!

In discrete time, we study $G\left(e^{i\omega T_s}\right)$.

In continuous time, we study $G(i\omega)$.

Bode diagram

- The bode diagram of $G(z)$:
  - Magnitude (decibel)

$$20\log\left|G\left(e^{i\omega T_s}\right)\right|$$

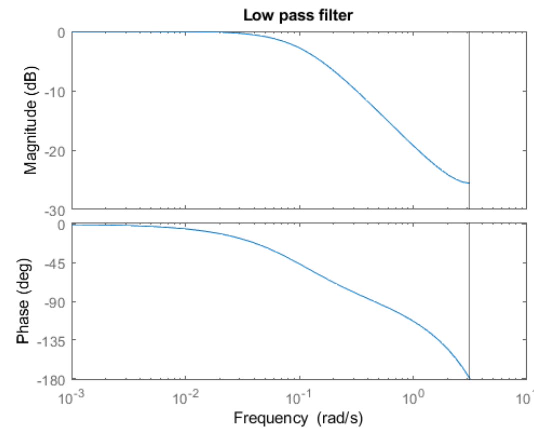
  - Phase

$$\arg(G(e^{i\omega T_s}))$$

Example

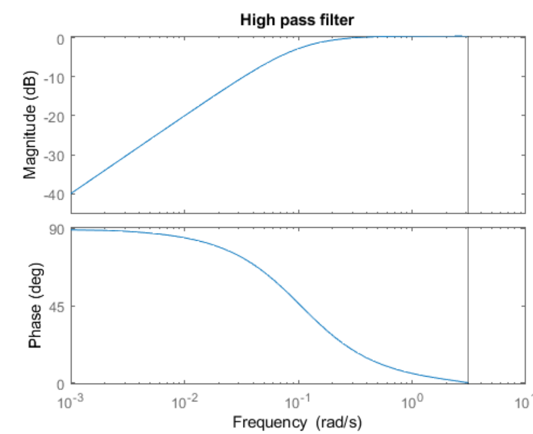
A low pass filter:

$$H_{LP}(z) = \frac{0.1}{z - 0.9}$$



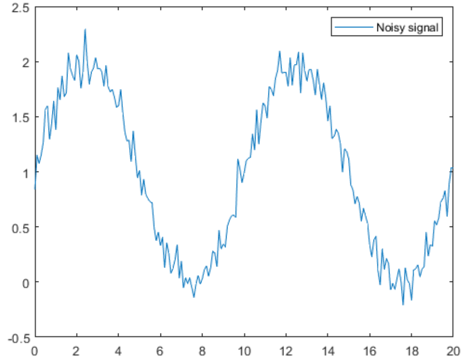
A high pass filter:

$$H_{HP}(z) = \frac{z - 1}{z - 0.9}$$



Example- remove average and high frequencies

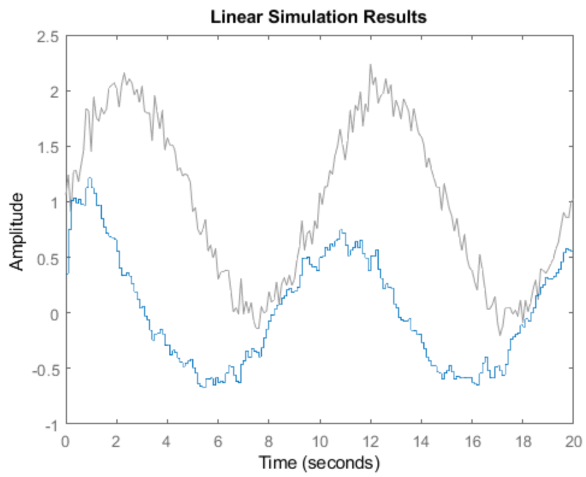
Averaging: $$L_1 = \frac{1}{20}(z^{-1} + \cdots + z^{-20})$$

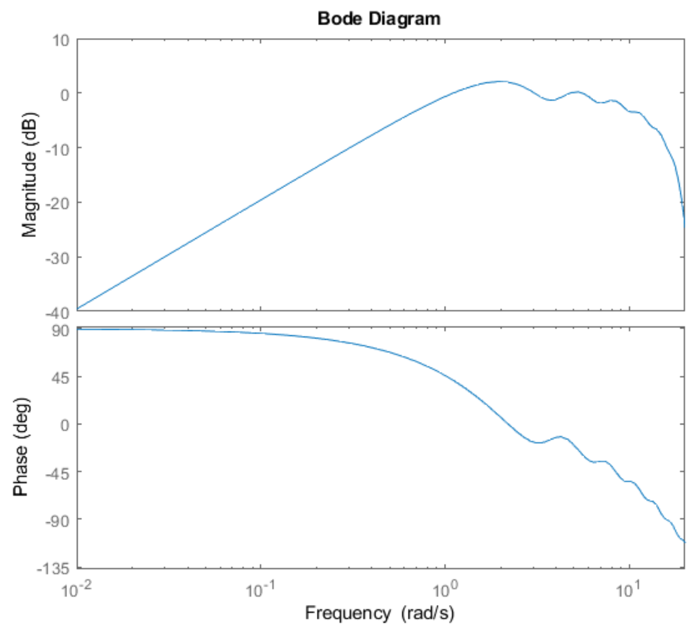


Removing average: $1 - L_1$

Low-pass filter: $$L_2 = \frac{1}{3}(z^{-1} + z^{-2} + z^{-3})$$

Example- remove average and high frequencies-Continued



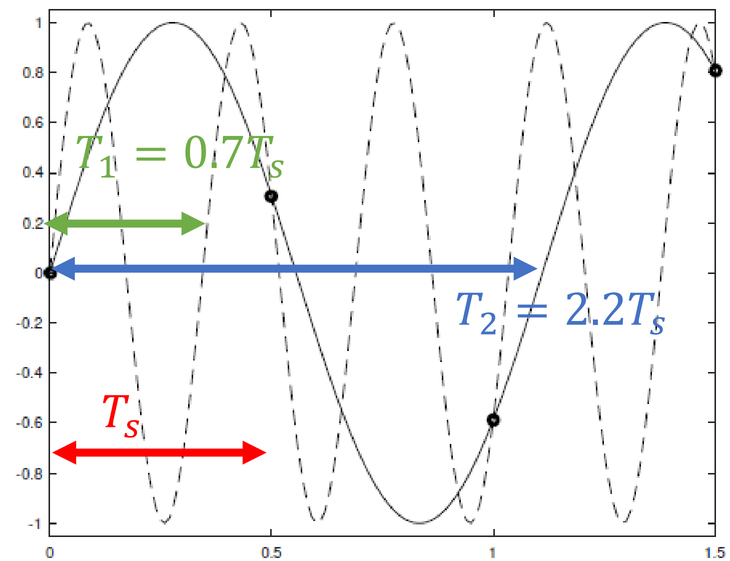


Alias effect

Alias effect

We cannot distinguish between these two sampled signals.

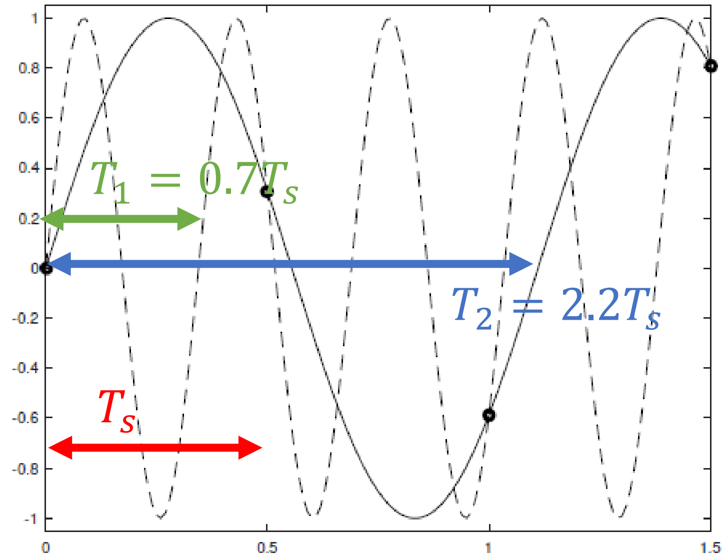
If we sample with T_s, a signal with frequency $\omega > \frac{\pi}{T_s}$ looks like a signal with $\omega < \frac{\pi}{T_s}$.



Alias effect

Sampling Theorem:

A signal with frequency up to ω_0 can be reconstructed by selecting the sampling time $T_s \leq \frac{\pi}{\omega_0}$.



What do we cover next?

- Open-loop vs. closed-loop control
- PID controller
- Analysis of the controlled system

Ask us!

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