Dynamical systems and Control

Farnaz Adib Yaghmaie

Automatic Control (RT) Division, ISY

- Email: Farnaz.adib.Yaghmaie@liu.se
- Office: Entrance 25-27, B-huset, Campus Valla

LINKÖPING UNIVERSITY

Lecture 4: Sensor and Measurements

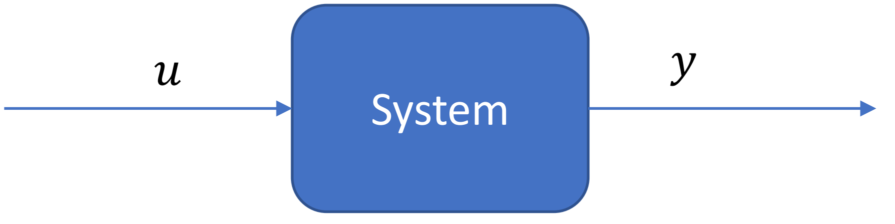
- Recap
- Sensors
- Introduction to signal processing

Dynamical systems

A quick recap of lectures 1-3

LINKÖPING UNIVERSITY

Dynamical systems



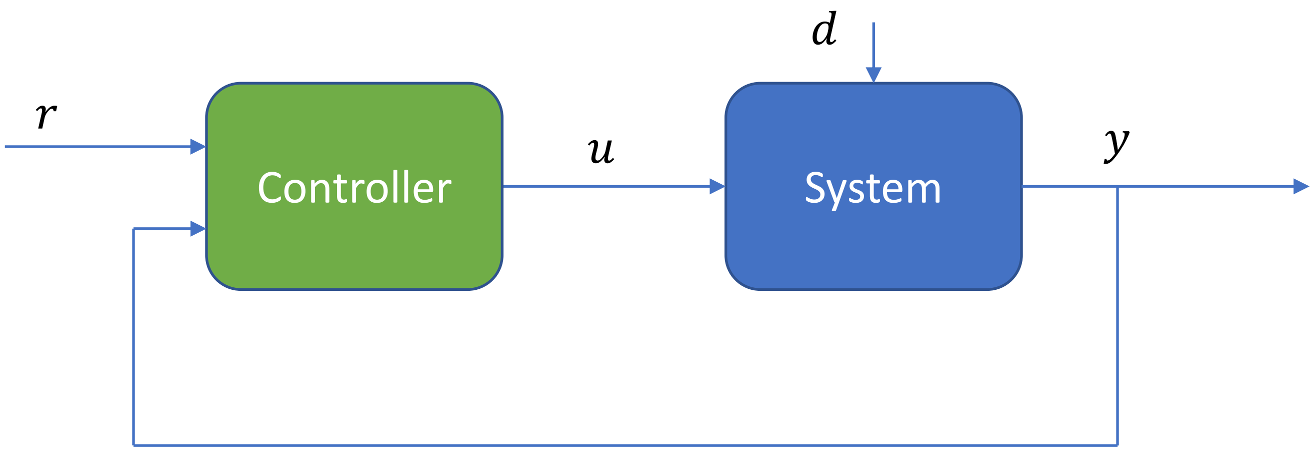
$$y^{(n)} + a_1 y^{(n-1)} + \cdots + a_n y = b_0 u^{(m)} + \cdots + b_m u$$

$$\dot{x} = Ax + Bu$$

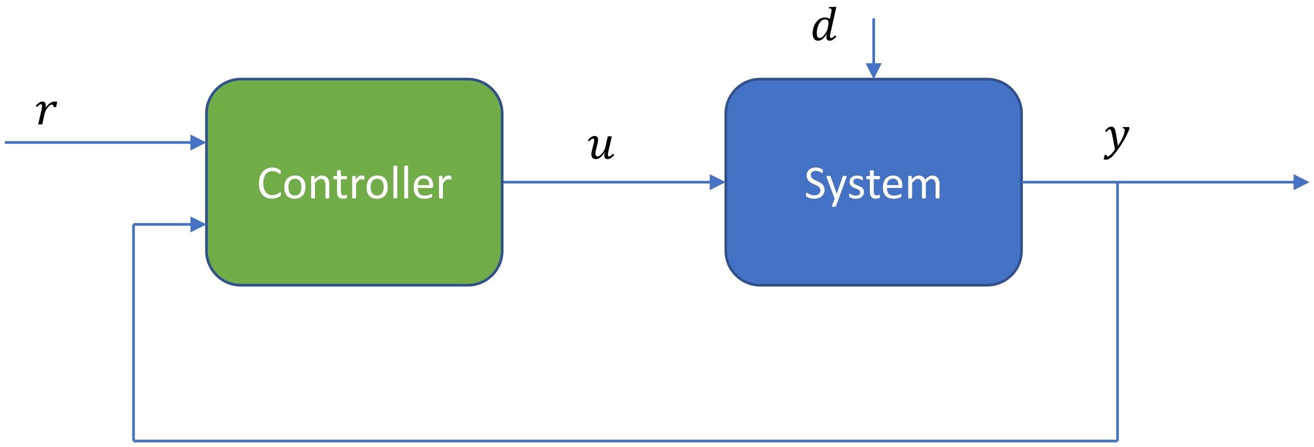
$$y = Cx + Du$$

Sensors

Introduction

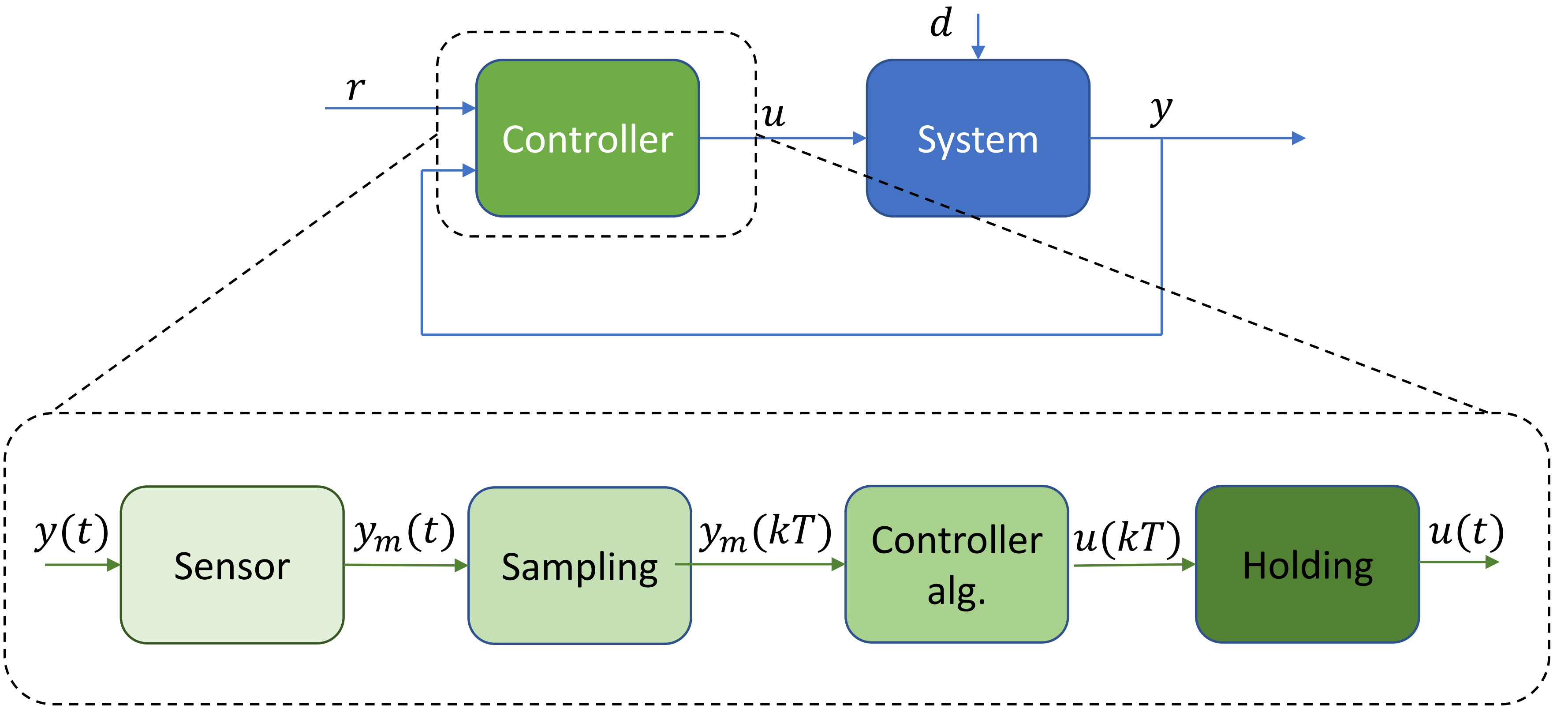


Introduction



- Generate u such that the output of the system, y, tracks a desired reference, r, in presence of disturbance d
- When we use y, we have feedback

How?



d
r
Controller
u
System
y
$y(t)$
Sensor
$y_m(t)$
Sampling
$y_m(kT)$
Controller alg.
$u(kT)$
Holding
$u(t)$

Sensors

- Accelerometers
- Gyroscopes
- GPS
- Laser, radar, sonar
- …

Photo credit: https://cdn.dxomark.com/wp-content/uploads/medias/post-90166/MicrosoftTeams-image-7.jpg

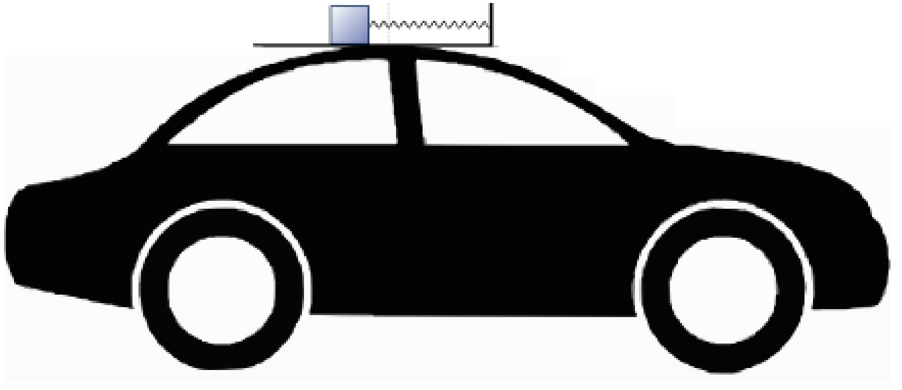
Points regarding sensors

- Sampling effect, quantization
- Calibration
- Measurement noise

Accelerometer

- Can be found on cars and modern phones.
- Used to measure distance and velocity also

$$a = \frac{kd}{M}$$

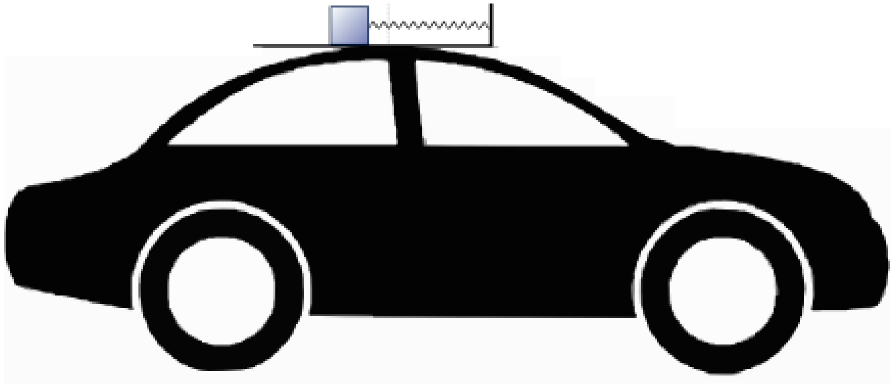


Accelerometer

- Can be found on cars and modern phones.
- Used to measure distance and velocity also

$$Ma = kd \longrightarrow a = \frac{kd}{M}$$

M: mass
a: acceleration
k: spring constant
d: extension of the spring



Position and velocity by accelerometers

$$v(t) = \int_0^t a(\tau)d\tau + v_0$$

$$x(t) = \int_0^t v(\tau)d\tau + x_0$$

- Initial conditions
- Integration
- Errors

Accelerometer-measurement error

$$a_m(t) = a(t) + b + e(t)$$

$$v_m(t) = v(t) + bt + \int_0^t e(\tau)d\tau$$

$$x_m(t) = x(t) + 0.5bt^2 + \ldots$$

Accelerometer-measurement error

$$a_m(t) = a(t) + b + e(t)$$

b: bias or calibration error
$e(t)$: rapidly varying measurement noise

$$v_m(t) = v(t) + bt + \int_0^t e(\tau)d\tau$$

$$x_m(t) = x(t) + 0.5bt^2 + \ldots$$

Measured velocity and position drift away rapidly

Gyroscopes

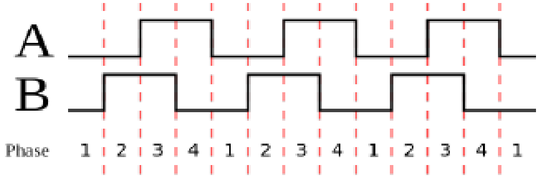
- Measure angular velocity using Coriolis effect
- Used to measure angle

$$\theta(t) = \int_0^t \omega(\tau)d\tau + \theta_0$$

- Similar principle (and issues) to accelerometers

Rotary encoder

- Measure angular velocities
- Advanced models can measure angles



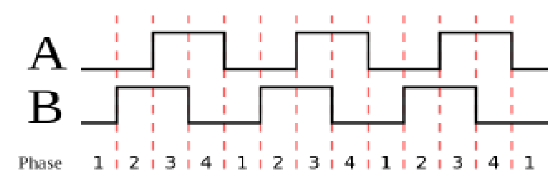
Rotary encoder

- Measure angular velocities
- Advanced models can measure angles

Let n denote the number of holes on the disc.

Resolution for 1 disk: $\frac{360}{2n}$

Resolution for 2 disks: $\frac{360}{4n}$

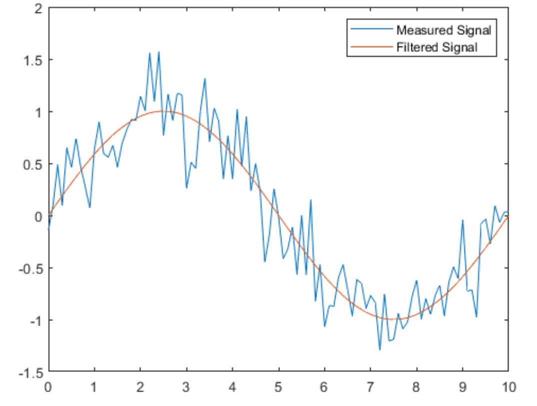


Introduction to signal processing

Concept

Given a sequence of measurements $y(kT_s)$, generate a new sequence $y_f(kT_s)$ such that

- Noise is removed
- Fast changes are removed
- Major frequency contents remain

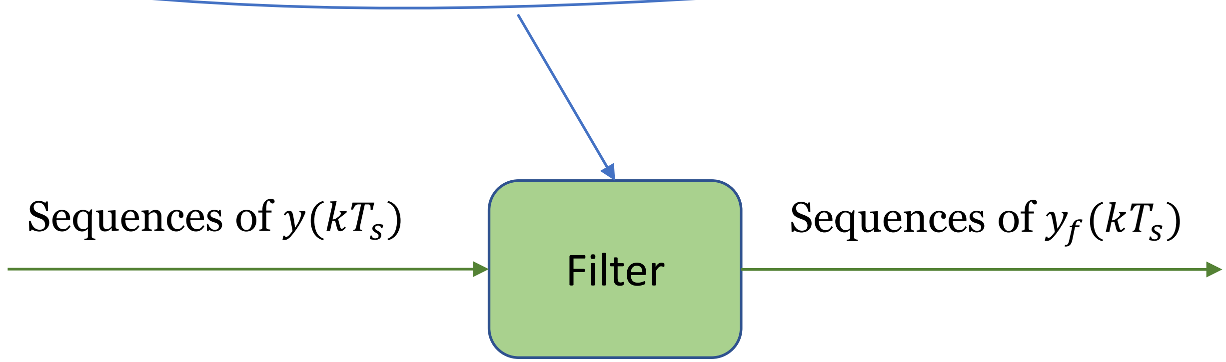


Example- Average of past 3 measurements

$$y_f(kT_s) = \frac{1}{3}[y(kT_s) + y((k-1)T_s) + y((k-2)T_s)]$$

Example- Average of past 3 measurements

$$y_f(kT_s) = \frac{1}{3}[y(kT_s) + y((k-1)T_s) + y((k-2)T_s)]$$



It filters fast changes or a wrong measurement: a low-pass filter

It is a difference equation

Example- continued

$$y_f(kT_s) = \frac{1}{3}[y(kT_s) + y((k-1)T_s) + y((k-2)T_s)]$$

Introduce a new transformation:

- z: the next
- z^{-1}: the previous

Example- continued

$$y_f(kT_s) = \frac{1}{3}[y(kT_s) + y((k-1)T_s) + y((k-2)T_s)]$$

Introduce a new transformation:

- z: the next
- z^{-1}: the previous

$$Y_f(z) = \frac{1}{3}(1 + z^{-1} + z^{-2})Y(z)$$

Transfer function in discrete-time $G(z) = \frac{1}{3}(1 + z^{-1} + z^{-2})$

Example- Another low pass filter

$$y_f(kT_s) = \alpha y_f((k-1)T_s) + (1-\alpha)y(kT_s)]$$

- $\alpha \rightarrow 1$: little effect from the measurement
- $\alpha \rightarrow 0$: much effect from the measurement

Example- Another low pass filter

$$y_f(kT_s) = \alpha y_f((k-1)T_s) + (1-\alpha)y(kT_s)]$$

- $\alpha \to 1$: little effect from the measurement $\quad y_f(kT_s) \approx y_f((k-1)T_s)$
- $\alpha \to 0$: much effect from the measurement $\quad y_f(kT_s) \approx y(kT_s)$

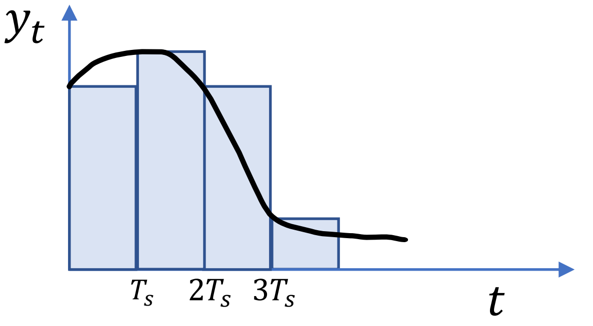
Transfer function:

$$Y_f(z) = \alpha z^{-1} Y_f(z) + (1-\alpha)Y(z) \longrightarrow \boxed{\frac{Y_f(z)}{Y(z)} = \frac{(1-\alpha)z}{z-\alpha}}$$

Example- Integrator

$$I(t) = \int_0^t y(\tau)d\tau \approx y(0)T_s + y(T_s)T_s + y(2T_s)T_s + \cdots$$

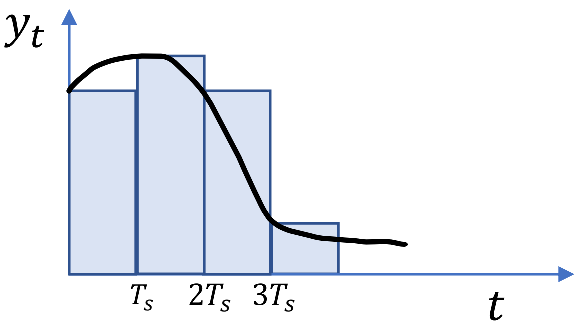
$$I(kT_s) = I\big((k-1)T_s\big) + T_s y((k-1)T_s)$$



Example- Integrator

$$I(t) = \int_0^t y(\tau)d\tau \approx y(0)T_s + y(T_s)T_s + y(2T_s)T_s + \cdots$$

$$I(kT_s) = I\big((k-1)T_s\big) + T_s y((k-1)T_s)$$



Transfer function:

$$\mathbb{I}(z) = z^{-1}\mathbb{I}(z) + T_s z^{-1} Y(z) \longrightarrow \boxed{\frac{\mathbb{I}(z)}{Y(z)} = \frac{T_s}{z-1}}$$

What do we cover next?

- Filters
- Frequency response and Bode diagram
- Alias effect

Ask us!

Farnaz Adib Yaghmaie

- Email: Farnaz.adib.Yaghmaie@liu.se

Daniel Bossér

- Email: daniel.bosser@liu.se

Filipe Marques Barbosa

- Email: filipe.barbosa@liu.se

LINKÖPING UNIVERSITY