Dynamical systems and Control

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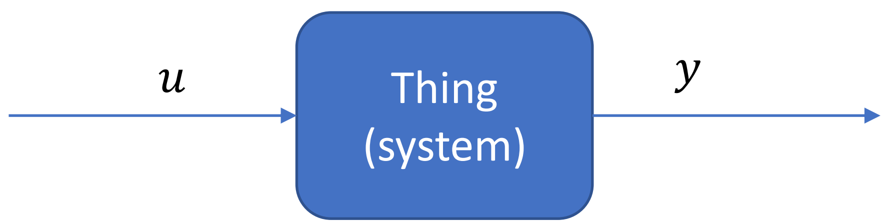
Lecture 2: Differential Equations

- Recap
- Laplace transformation
- Stability
- Characteristics of first and second-order systems

Model of systems

A quick recap of lecture 1

Recap



$$y^{(n)} + a_1 y^{(n-1)} + \cdots + a_n y = b_0 u^{(m)} + \cdots + b_m u$$

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

Characteristic equation

$$y^{(n)} + a_1 y^{(n-1)} + \cdots + a_n y = b_0 u^{(m)} + \cdots + b_m u$$

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

Laplace transformation

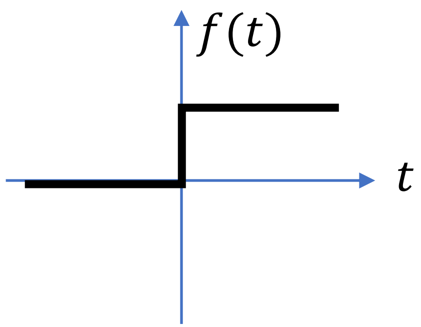
Why Laplace?

- Differential equations are difficult!

$$F(s) = \mathcal{L}\big(f(t)\big) = \int_0^{+\infty} e^{-st} f(t)dt$$

Example-Step function

$$f(t) = \begin{cases} 1, & t \geq 1 \\ 0, & otherwise \end{cases}$$



Example-Derivative

$$\mathcal{L}(\frac{d}{dt}f(t)) \ =$$

Example-Integral

$$\mathcal{L}(\int_0^t f(\tau)d\tau) =$$

Hint: use the Laplace of derivative!

Useful identities

$$\mathcal{L}\left(\frac{d^n}{dt^n}f(t)\right) =$$

$$\mathcal{L}\big(af(t) + bh(t)\big) =$$

$$\lim_{t\to+\infty} f(t) = \lim_{s\to 0} sF(s)$$

See p. 232-233 in the book.

Transfer function

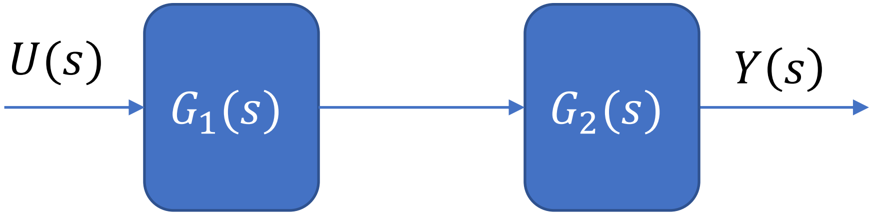
$$y^{(n)} + a_1 y^{(n-1)} + \cdots + a_n y = b_0 u^{(m)} + \cdots + b_m u$$

$$s^n Y + a_1 s^{n-1} Y + \cdots + a_n Y = b_0 s^m U + \cdots + b_m U$$

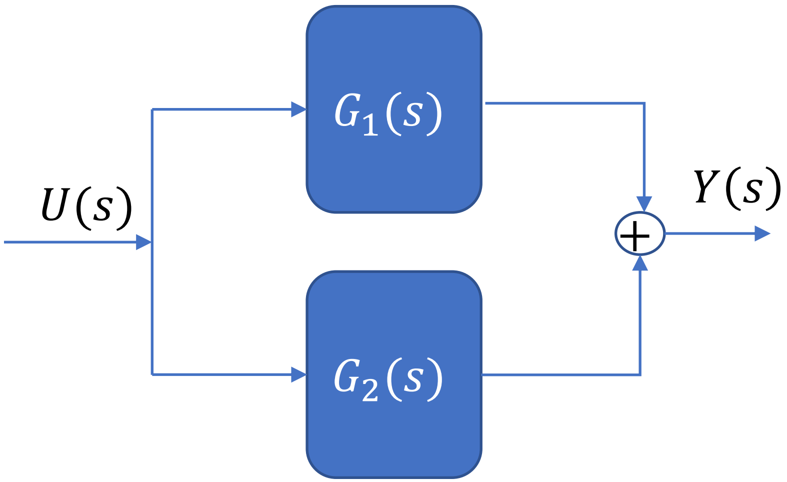
Example- Boeing 747

$$\ddot{y} + 0.93\dot{y} + 1.76y = -1.2\dot{u} + 28.31u$$

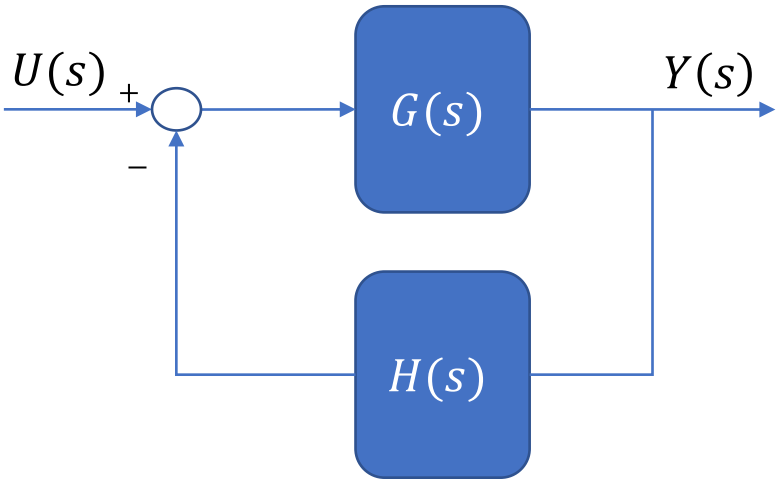
Series config.



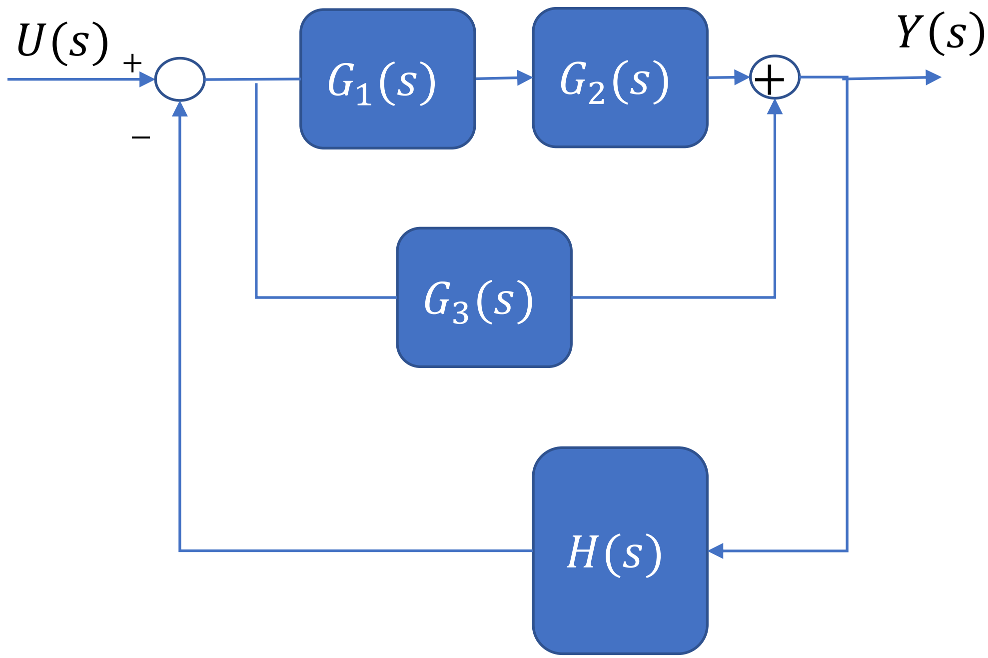
Parallel config.



Feedback config.



Example



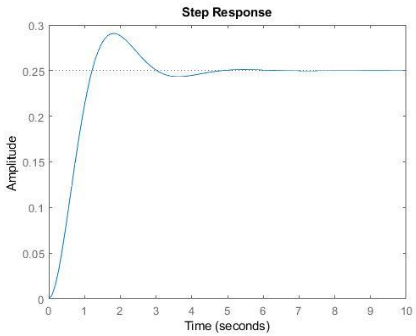
Characteristic equation

$$G(s) = \frac{N(s)}{D(s)}$$

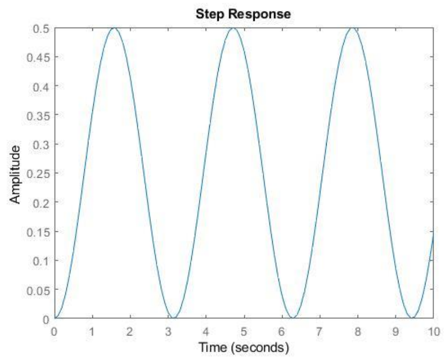
- Characteristic equation: $D(s) = 0$
- Poles: Roots of $D(s) = 0$
- Zeros: Roots of $N(s) = 0$

Stability

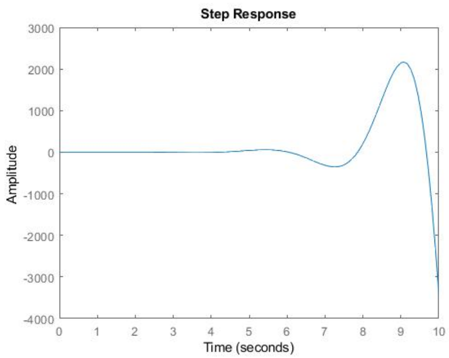
Stability concept



stable



unstable
(marginally stable)



unstable

Stable:

If all poles have negative real parts

Unstable:

otherwise

Characteristics of first and second-order systems

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First-order system

$$T\dot{y}(t) + y(t) = Ku(t) \quad \longleftrightarrow \quad G(s) = \frac{K}{Ts + 1}$$

- T: Time constant
- K: Static gain

First-order system- Solution

$$y(t) = KC(1 - e^{-\frac{1}{T}t})$$

Second-order system

$$G(s) = \frac{a_2}{s^2 + a_1 s + a_2}$$

Second-order system- Solution

$$y(t) = \frac{1}{a_2} + c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

Second-order system- Solution

Real and negative poles:

Second-order system- Solution

Real and at least one positive:

Second-order system- Solution

Complex poles:

Second-order system- Solution

Complex with positive real part:

Complex with negative real part:

Complex with zero real part:

What do we cover next?

- Transfer function of state-space model
- Controllability and observability
- Linearization

Ask us!

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