Dynamical systems and Control

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Lecture 10: Incorporating State feedback and PID controller

Recap of Lec 9

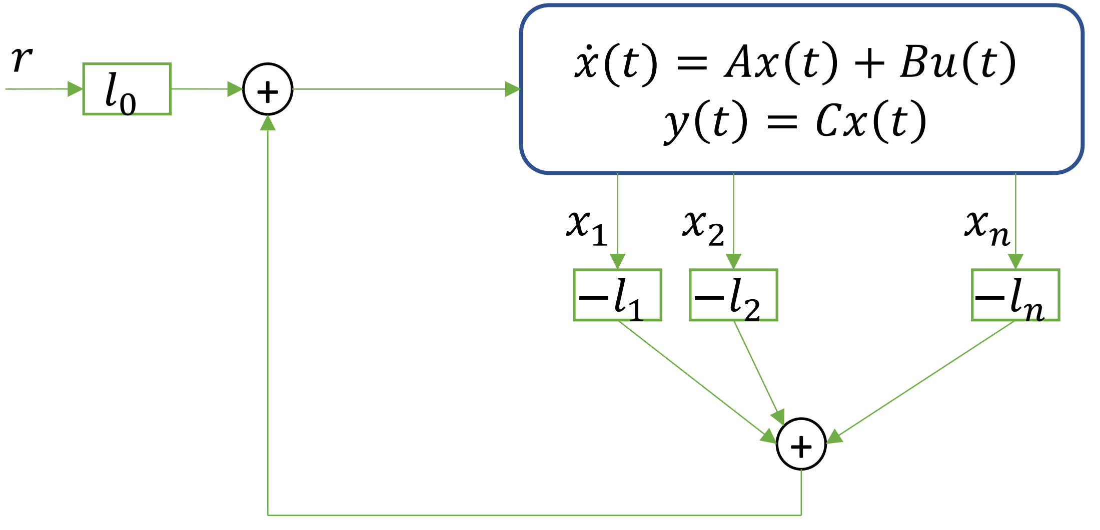
PID and state feedback

Recap of Lec 9

- State feedback

Linear state feedback

$$u(t) = -l_1 x_1(t) - l_2 x_2(t) \cdots - l_n x_n(t) + l_0 r(t)$$



$$\boxed{u(t) = -Lx(t) + l_0 r(t)}$$

Closed-loop system

$$\dot{x} = (A - BL)x + Bl_0 \mathrm{r}$$

$$y = Cx$$

Design procedure:

1. Select the desired poles
2. Design L to have the desired poles
3. Select l_0 to have a zero tracking-error for a step r

Closed-loop system

$$\dot{x} = (A - BL)x + Bl_0 \mathrm{r}$$

$$y = Cx$$

$$\longrightarrow \quad G_c(s) = C\left(sI - (A - BL)\right)^{-1} B l_0$$

Design procedure:

1. Select the desired poles
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$$y(\infty) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} sG_c(s)\frac{1}{s} = \lim_{s \to 0} G_c(s) = 1$$

$$G_c(0) = 1 \qquad \boxed{l_0 = \frac{-1}{C(A - BL)^{-1}B}}$$

Some issues

What if:

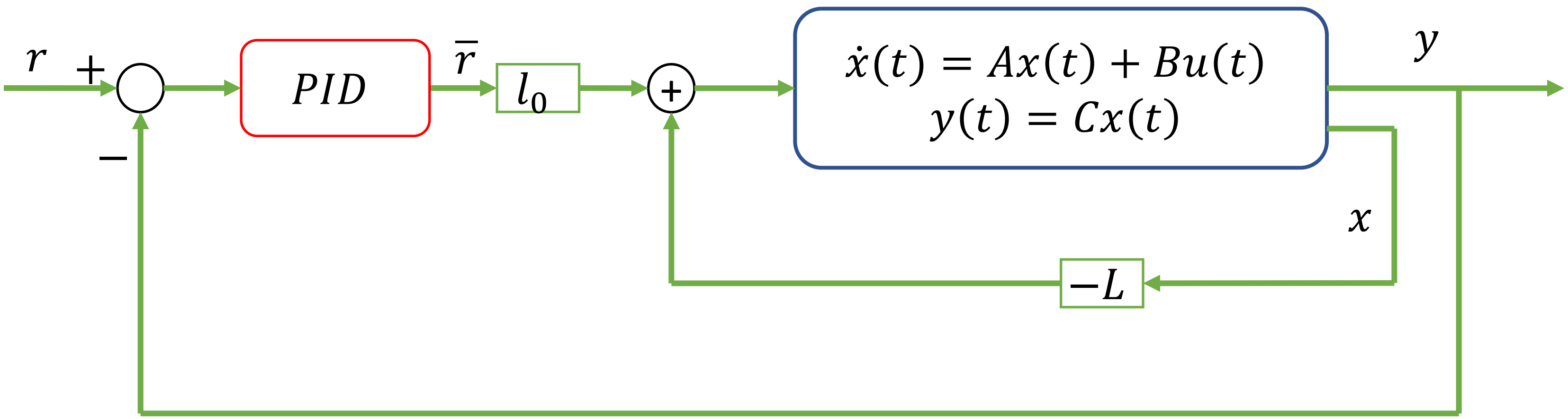
- We have error in the model
- We have disturbance

What we can do …

- Combine with PID
- Ad-hoc reasoning methods
- Extend state-feedback

PID and state-feedback

State feedback and PID

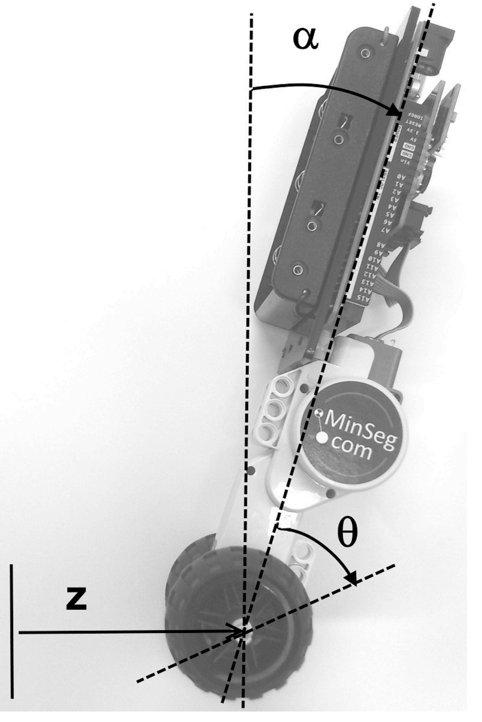


Example: Balancing MinSeg by state-feedback

Keep the robot stationary in upright position $\alpha = 0, z = 0$

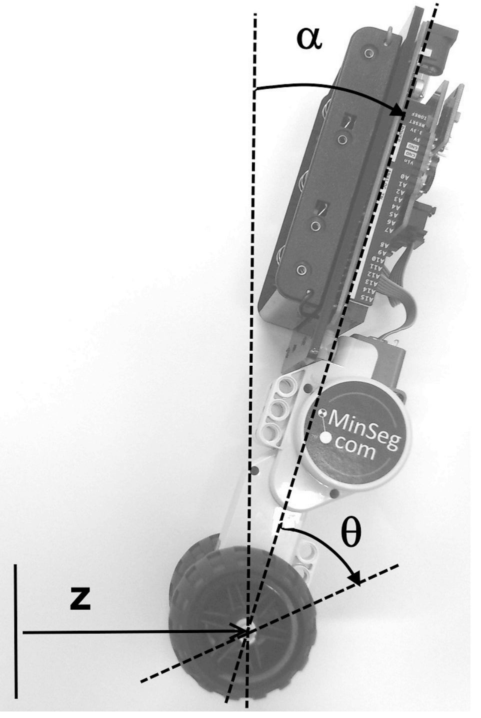
You will design a state-feedback in Lab 4 as

$$u = -l_1 z - l_2 \dot{z} - l_3 \alpha - l_4 \dot{\alpha}$$



Example: Balancing MinSeg by state-feedback

What happens in stationary point?



Example: Balancing MinSeg by state-feedback

What happens in stationary point?

We should have $\dot{z} = 0, \dot{\alpha} = 0, u = 0$

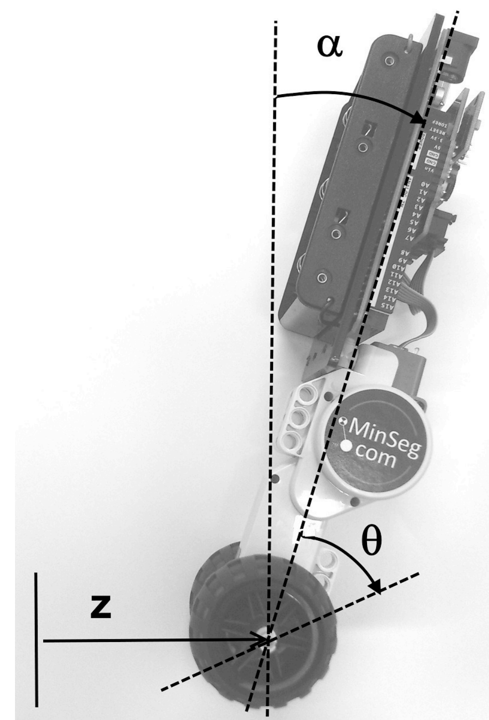
$$u = -l_1 z - l_2 \dot{z} - l_3 \alpha - l_4 \dot{\alpha} = 0 \longrightarrow \boxed{z^* = -\frac{l_3}{l_1}\alpha^*}$$

When the robot tilted in a nonzero α^*, it ends up in a nonzero position.

Change the controller to

$$u = -l_1 z - l_2 \dot{z} - l_3 (\alpha - \alpha^*) - l_4 \dot{\alpha}$$

$$\alpha^* = -K_I \int_0^t z(\tau) d\tau$$



State feedback extension

Presence of disturbance

$$\dot{x} = Ax + Bu + Fv$$

$$y = Cx$$

How to design state feedback such that $y(t) \rightarrow r(t)$ in presence of constant disturbance v?

Presence of disturbance

$\dot{x} = Ax + Bu + Fv$

$y = Cx$

Introduce a new state variable

$$x_{n+1}(t) = \int_0^t (r(\tau) - y(\tau))d\tau$$

Presence of disturbance

$\dot{x} = Ax + Bu + Fv$

$y = Cx$

Introduce a new state variable

$$x_{n+1}(t) = \int_0^t (r(\tau) - y(\tau))d\tau \longrightarrow \dot{x}_{n+1} = r(t) - y(t)$$

Extend the state space $\tilde{x} = \begin{bmatrix} x \\ x_{n+1} \end{bmatrix}$:

$$\begin{bmatrix} \dot{x} \\ \dot{x}_{n+1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_{n+1} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} F \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r = \tilde{A}\tilde{x} + \tilde{B}u + \tilde{F}v + \tilde{H}r$$

Design a state feedback

$$u = -Lx - l_{n+1}x_{n+1} = -\tilde{L}x$$

Presence of disturbance

The closed-loop system reads

Presence of disturbance

The closed-loop system reads

$$\dot{\tilde{x}} = (\tilde{A} - \tilde{B}\tilde{L})\tilde{x} + \tilde{F}v + \tilde{H}r$$

$$y = [C \quad 0]\tilde{x}$$

If $(\tilde{A} - \tilde{B}\tilde{L})$ is stable, $\dot{\tilde{x}} \to 0$ for constant external signals.

As a result, $\dot{x}_{n+1} \to 0$ and based on the definition of $\dot{x}_{n+1} = r(t) - y(t)$

$$\boxed{y \to r}$$

Example- Controlling rear steered cycle by PD

The transfer function is given by:

$$G(s) = \frac{s-2}{s^2-9}$$

Let's design a PD controller:

$$F(s) = K_P + K_D s$$

The closed-loop transfer function reads:

Photo credit: https://forum.cruzbike.com/

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$$G_c(s) = \frac{GF}{1+GF} = \frac{(K_P + K_D s)\, {}^{s-2}/_{s^2-9}}{1 + (K_P + K_D s)\, {}^{s-2}/_{s^2-9}} = \frac{(K_P + K_D s)(s-2)}{(1+K_D)s^2 + (K_P - 2K_D)s - 2K_P - 9}$$

Remember $cs^2 + as + b$ *is stable if* $a, b, c > 0$ or $a, b, c < 0$

$$\begin{cases} 1 + K_D < 0 \\ K_P - 2K_D < 0 \\ -2K_P - 9 < 0 \end{cases} \longrightarrow \begin{cases} K_D < -1 \\ K_P < 2K_D \\ K_P > -4.5 \end{cases}$$

$\boxed{\text{Select } K_P = -3, K_D = -1.25}$

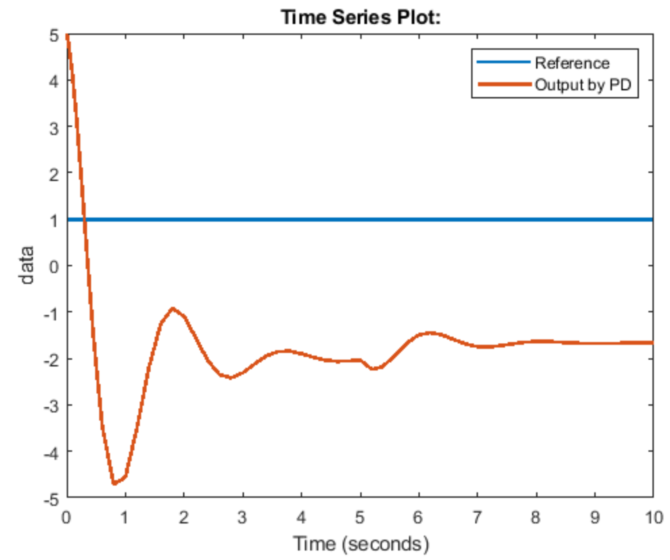
Example- Example-Controlling rear steered cycle by PD

The closed-loop system reads:

$$G_c(s) = \frac{-1.25s^2 - s + 6}{-0.25s^2 - 0.5s - 3}$$

Observe that $G_c(0) = -2$!

See the result:



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The transfer function is given by:

$$G(s) = \frac{s-2}{s^2-9}$$

The state space representation is:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 9 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v$$

$$y = \begin{bmatrix} -2 & 1 \end{bmatrix} x$$

Introduce $\dot{x}_3 = r(t) - y(t)$

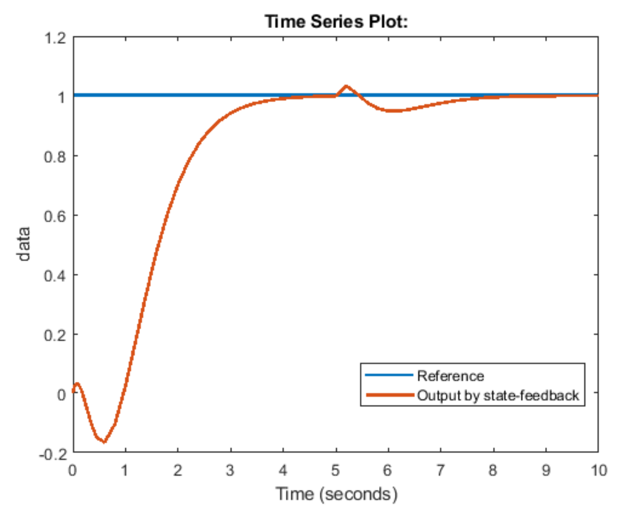
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 9 & 0 & 0 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r = \tilde{A}\tilde{x} + \tilde{B}u + \tilde{F}v + \tilde{H}r$$

Select $\tilde{L}$ to stabilize $\tilde{A} - \tilde{B}\tilde{L}$

$\boxed{place(\tilde{A}, \tilde{B}, [-2, -3, -2.5])}$

Example- Controlling rear steered cycle by state feedback

See the result



I hope you enjoyed the course

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Ask us!

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