Simultaneous Localization and Mapping (SLAM): EKF SLAM

Sensor Fusion

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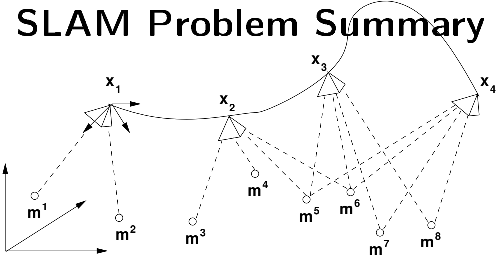
SLAM Problem Summary



- *Simultaneous localization and mapping* (SLAM) is the problem of finding ones position, x_k, in a map, $\mathbf{m}$, while the map is built. Both parts must be considered simultaneous.
- Model:

$$z_k = \begin{pmatrix} x_{k+1} \\ \mathbf{m}_{k+1} \end{pmatrix} = \begin{pmatrix} f(x_k, v_k) \\ \mathbf{m}_k \end{pmatrix}, \qquad \mathrm{Cov}(v_k) = Q$$

$$y_k^i = h(x_k, \mathbf{m}_k^{c_k^i}) + e_k^i, \qquad \mathrm{Cov}(e_k^i) = R, \quad i = 1, \dots, I_k.$$

- Solve using the extended Kalman filter yields the EKF SLAM.

EKF SLAM Model

- Assume a linear(-ized) model

$$
\begin{aligned}
x_{k+1} &= F x_k + G v_k & \mathrm{Cov}(v_k) &= Q \\
\mathbf{m}_{k+1} &= \mathbf{m}_k \\
y_k &= H_k^x x_k + H_k^{\mathbf{m}}(c_k^{1:I_k})\mathbf{m}_k + e_k, & \mathrm{Cov}(e_k) &= R.
\end{aligned}
$$

- The map is represented by $\mathbf{m}_k$.
- The index $c_k^{1:I_k}$ relate the observed landmark i to a map landmark j_i, which affects the measurement model.
- We assume the association to be solved.
- The state and its covariance matrix

$$
\hat{z} = \begin{pmatrix} \hat{x} \\ \hat{\mathbf{m}} \end{pmatrix}, \qquad P = \begin{pmatrix} P^{xx} & P^{x\mathbf{m}} \\ P^{\mathbf{m}x} & P^{\mathbf{mm}} \end{pmatrix}.
$$

Basic Kalman Filter Steps

Applying the Kalman filter and utilizing the structure yields.

Time update:

$$\hat{z}_{k|k-1} = \begin{pmatrix} F & 0 \\ 0 & I \end{pmatrix} \hat{z}_{k-1|k-1},$$

$$P_{k|k-1} = \begin{pmatrix} F_k P^{xx}_{k-1|k-1} F_k^T + G_k Q_k G_k^T & F_k P^{x\mathbf{m}}_{k-1|k-1} \\ P^{\mathbf{m}x}_{k-1|k-1} F_k^T & P^{\mathbf{mm}}_{k-1|k-1} \end{pmatrix}$$

Measurement update:

$$S_k = H_k^x P^{xx}_{k|k-1} H_k^{xT} + H_k^{\mathbf{m}} P^{\mathbf{mm}}_{k|k-1} H_k^{\mathbf{m}T} + H_k^{\mathbf{m}} P^{\mathbf{m}x}_{k|k-1} H_k^{xT} + H_k^x P^{x\mathbf{m}}_{k|k-1} H_k^{\mathbf{m}T} + R_k$$

$$K_k^x = (H_k^x P^{xx}_{k|k-1} + H_k^{\mathbf{m}} P^{\mathbf{m}x}_{k|k-1}) S_k^{-1}$$

$$K_k^{\mathbf{m}} = (H_k^x P^{x\mathbf{m}}_{k|k-1} + H_k^{\mathbf{m}} P^{\mathbf{mm}}_{k|k-1}) S_k^{-1}$$

$$\varepsilon_k = y_k - H_k^x \hat{x}_{k|k-1} - H_k^{\mathbf{m}} \hat{\mathbf{m}}_{k|k-1}$$

$$\hat{z}_{k|k} = \hat{z}_{k|k-1} + \begin{pmatrix} K_k^x \\ K_k^{\mathbf{m}} \end{pmatrix} \varepsilon_k$$

$$P_{k|k} = P_{k|k-1} - \begin{pmatrix} K_k^x \\ K_k^{\mathbf{m}} \end{pmatrix} S_k^{-1} \begin{pmatrix} K_k^x \\ K_k^{\mathbf{m}} \end{pmatrix}^T$$

Kalman Filter Problems

- All elements in $P_{k|k}^{\mathbf{mm}}$ are affected by the measurement update.
- It turns out that the cross-correlations are essential for performance.
- No simple turn-around.

Information Filter Reformulation

- Focus on sufficient statistics and information matrix

$$\imath_{k|l} = \mathcal{I}_{k|l}\,\hat{z}_{k|l}$$

$$\mathcal{I}_{k|l} = P_{k|l}^{-1} = \begin{pmatrix} P_{k|l}^{xx} & P_{k|l}^{x\mathbf{m}} \\ P_{k|l}^{\mathbf{m}x} & P_{k|l}^{\mathbf{mm}} \end{pmatrix}^{-1} = \begin{pmatrix} \mathcal{I}_{k|l}^{xx} & \mathcal{I}_{k|l}^{x\mathbf{m}} \\ \mathcal{I}_{k|l}^{\mathbf{m}x} & \mathcal{I}_{k|l}^{\mathbf{mm}} \end{pmatrix}.$$

- Measurement update trivial

$$\imath_{k|k} = \imath_{k|k-1} + H_k^T R_k^{-1} y_k$$

$$\mathcal{I}_{k|k} = \mathcal{I}_{k|k-1} + H_k^T R_k^{-1} H_k.$$

Note:

The measurement update is sparse!!!

Information Filter Algorithm (1/4)

Initialization:

$$
\begin{aligned}
\imath^{x}_{1|0} &= 0_{3\times 1}\\
\imath^{\mathbf{m}}_{1|0} &= 0_{0\times 0}\\
\mathcal{I}^{xx}_{1|0} &= 0_{3\times 3}\\
\mathcal{I}^{\mathbf{m}x}_{1|0} &= 0_{0\times 3}\\
\mathcal{I}^{\mathbf{mm}}_{1|0} &= 0_{0\times 0}
\end{aligned}
$$

Note:

The information form allows for representing no prior knowledge with zero information (infinite covariance).

Information Filter Algorithm (2/4)

1. **Associate** a map landmark $j = c_k^i$ to each observed landmark j, and construct the matrix $H_k^{\mathbf{m}}$. This step includes data gating for outlier rejection and track handling to start and end landmark tracks.
2. **Measurement update:**

$$\imath_{k|k}^{x} = \imath_{k|k-1}^{x} + {H_k^{x}}^T R_k^{-1} y_k$$

$$\imath_{k|k}^{\mathbf{m}} = \imath_{k|k-1}^{\mathbf{m}} + {H_k^{\mathbf{m}}}^T R_k^{-1} y_k$$

$$\mathcal{I}_{k|k}^{xx} = \mathcal{I}_{k|k-1}^{xx} + {H_k^{x}}^T R_k^{-1} H_k^{x}$$

$$\mathcal{I}_{k|k}^{x\mathbf{m}} = \mathcal{I}_{k|k-1}^{x\mathbf{m}} + {H_k^{x}}^T R_k^{-1} H_k^{\mathbf{m}}$$

$$\mathcal{I}_{k|k}^{\mathbf{mm}} = \mathcal{I}_{k|k-1}^{\mathbf{mm}} + {H_k^{\mathbf{m}}}^T R_k^{-1} H_k^{\mathbf{m}}$$

Note:

$H_k^{\mathbf{m}}$ is very thick, but contains mostly zeros.
The low-rank sparse corrections influencing only a fraction of the matrix elements.

Information Filter Algorithm (3/4)

3. **Time update:**

$$
\begin{aligned}
\bar{\mathcal{I}}^{xx}_{k|k-1} &= F_k^{-1}\,\mathcal{I}^{xx}_{k-1|k-1}\,F_k^{-T}\\
\bar{\mathcal{I}}^{x\mathbf{m}}_{k|k-1} &= F_k^{-1}\,\mathcal{I}^{x\mathbf{m}}_{k-1|k-1}\\
M_k &= G_k\left(G_k^T F_k^{-1}\,\mathcal{I}^{xx}_{k-1|k-1}\,F_k^{-T} + Q_k^{-1}\right)^{-1} G_k^T,\\
\mathcal{I}^{xx}_{k|k-1} &= \bar{\mathcal{I}}^{xx}_{k|k-1} - \bar{\mathcal{I}}^{xx}_{k|k-1} M_k \bar{\mathcal{I}}^{xx}_{k|k-1}\\
\mathcal{I}^{x\mathbf{m}}_{k|k-1} &= \bar{\mathcal{I}}^{x\mathbf{m}}_{k|k-1} - \bar{\mathcal{I}}^{xx}_{k|k-1} M_k \bar{\mathcal{I}}^{x\mathbf{m}}_{k|k-1},\\
\mathcal{I}^{\mathbf{mm}}_{k|k-1} &= \bar{\mathcal{I}}^{\mathbf{mm}}_{k|k-1} - \bar{\mathcal{I}}^{\mathbf{m}x}_{k|k-1} M_k\, G_k^T \bar{\mathcal{I}}^{x\mathbf{m}}_{k|k-1}\\
\imath^{x}_{k|k-1} &= \left(I - \mathcal{I}^{xx}_{k|k-1}\,G_k Q_k G_k^T F_k^T\right)\imath^{x}_{k-1|k-1}\\
\imath^{\mathbf{m}}_{k|k-1} &= \imath^{\mathbf{m}}_{k-1|k-1} - \mathcal{I}^{\mathbf{m}x}_{k|k-1}\,G_k Q_k G_k^T F_k^T \imath^{x}_{k|k-1}
\end{aligned}
$$

Note:

Now, $\mathcal{I}^{\mathbf{mm}}_{k|k-1}$ is corrected with the outer product of $\bar{\mathcal{I}}^{\mathbf{m}x}_{k|k-1}$ which gives a full matrix. Many of the elements in $\mathcal{I}^{\mathbf{m}x}_{k|k-1}$ are close to zero and may be truncated!

Information Filter Algorithm (4/4)

4. **Estimate extraction:**

$$\begin{aligned} P_{k|k} &= \mathcal{I}_{k|k}^{-1}, \\ \hat{x}_{k|k} &= P_{k|k}^{xx}\imath_{k|k}^{x} + P_{k|k}^{x\mathbf{m}}\imath_{k|k}^{\mathbf{m}}, \\ \hat{\mathbf{m}}_{k|k} &= P_{k|k}^{\mathbf{m}x}\imath_{k|k}^{x} + P_{k|k}^{\mathbf{mm}}\imath_{k|k}^{\mathbf{m}}. \end{aligned}$$

Here is another catch, the information matrix needs to be inverted! The pose is needed at all times for linearization and data gating. How to proceed?

Idea:

Solve

$$\imath = \mathcal{I}\,\hat{z},$$

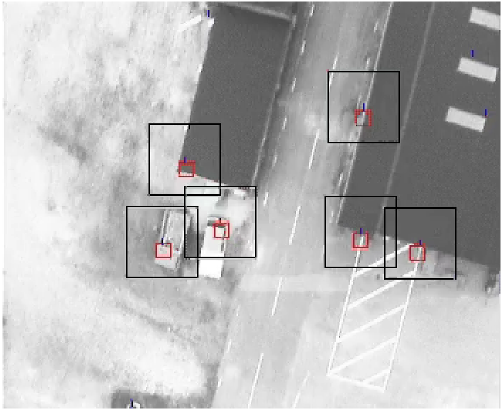
directly using a gradient search algorithm initialized at previous estimate.

Summary of Properties

- EKF SLAM scales well in state dimension.
- EKF SLAM scales badly in landmark dimension, though natural approximations exist for the information form.
- EKF SLAM is not robust to incorrect associations.

EKF SLAM Illustration

- Airborne simultanous localization and mapping (SLAM) using a UAV with camera producing image features.
- Research collaboration with IDA.



http://youtu.be/VQlaCHl3Yc4

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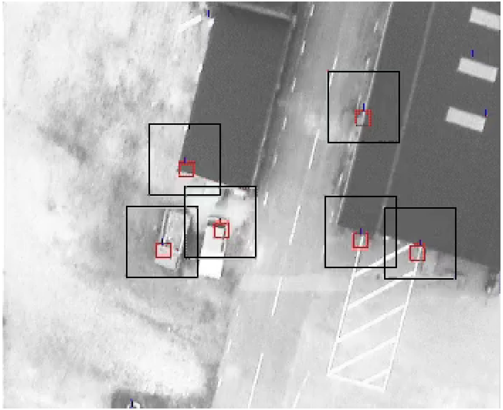

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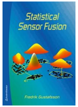
Summary

The simultaneous localization and mapping (SLAM) problem has been solved using an extended Kalman filter in two different ways:

- EKF filter form.
- Information filter form.

Properties:

- Scales well with state dimension, but poorly with number of landmarks.
- Using information for has some benefits.
- Proper landmark associations are essential!



Section 11.2