Simultaneous Localization and Mapping (SLAM): problem formulation

Sensor Fusion

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Simultaneous Localization and Mapping

Localization concerns the estimation of pose from known landmarks.

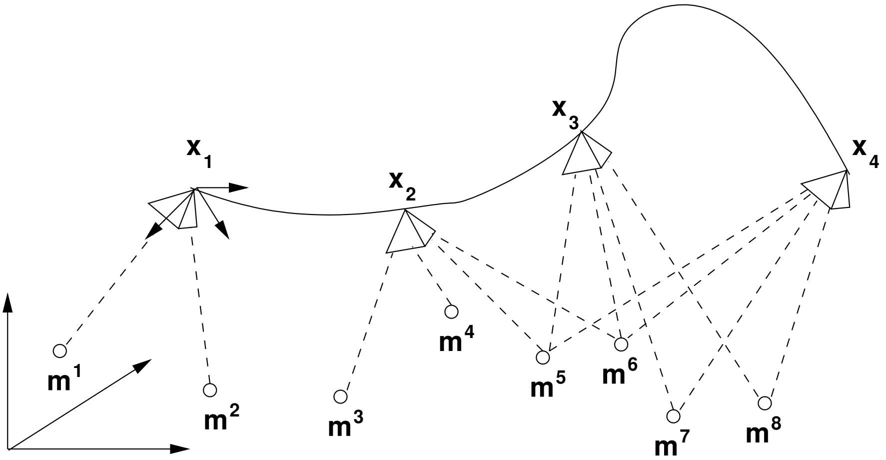
Navigation concerns estimation of pose, velocities and other states from known landmarks.

Mapping concerns the estimation of landmark positions from known values of pose.

SLAM concerns the joint estimation of pose and landmark positions.

- The map is considered static.
- The position of the sensor is dynamic.
- Separately localization and mapping are straightforward, combining the two is more involved.

Problem Illustration



Original SLAM Application

- A ground robot with three state components: $x = (X, Y, \psi)^T$.
- Robot measures speed and turn rate: $u = (v, \dot{\psi})^T$.
- Simple dynamics.
- A map comprising stacked positions of landmarks, $\mathbf{m} = (m_X^1, m_Y^1, \dots, m_X^M, m_Y^M)^T$.
- Sensors:
  1. Ranging sensor (sonar, laser scanner, radar) measures distance to obstacles (walls, furniture); tens to hundreds of landmarks.
  2. Vision (camera, Kinect, stereo camera) provides detections (corners, markers, patterns) as potential landmarks; thousands or tens of thousands of landmarks.

SLAM Model

General SLAM model

$$z_k = \begin{pmatrix} x_{k+1} \\ \mathbf{m}_{k+1} \end{pmatrix} = \begin{pmatrix} f(x_k, v_k) \\ \mathbf{m}_k \end{pmatrix}, \qquad \mathrm{Cov}(v_k) = Q$$

$$y_k^i = h(x_k, \mathbf{m}_k^{c_k^i}) + e_k, \quad \mathrm{Cov}(e_k) = R, \quad i = 1, \dots, I_k.$$

- The map is represented by $\mathbf{m}_k$, and assumed constant (*i.e.*, no process noise).
- I_k landmarks are assumed observed at time k.
- The $j = c_k^i$ associates the observed landmark i to the map landmark j, as needed in the measurement model.
- Association is crucial for some sensors (laser, radar, etc.), but less of a problem some applications (camera using image features, microphones using designed pings).

Typical Measurement Model

Typically the measurement is a function of the relative position of a landmark, $x_k - \mathbf{m}^{c_k^i}$, *i.e.*,

$$y_k^i = h(x_k - \mathbf{m}^{c_k^i}) + e_k^i.$$

Example

Measurement of relative position of the landmarks (known heading):

$$y_k^i = x_k - \mathbf{m}^{c_k^i} + e_k^i, \quad i = 1, 2$$

$$y_k = \begin{pmatrix} I & 0 & & & \cdots & 0 & \overbrace{-I}^{\text{map pos. } c_k^1} & 0 & \cdots & 0 \\ I & 0 & \cdots & 0 & \underbrace{-I}_{\text{map pos. } c_k^2} & 0 & \cdots & & & 0 \end{pmatrix} \begin{pmatrix} x_k \\ \mathbf{m} \end{pmatrix} + \begin{pmatrix} e_k^1 \\ e_k^2 \end{pmatrix}$$

Solving the SLAM Problem

Common algorithms

- Batch methods
  - Bundle adjustment
  - Structure from motion
  - GraphSLAM
- Filtering solutions
  - **EKF SLAM:** EKF (information form) on augmented state vector z_k.
  - **FastSLAM:** MPF on augmented state vector z_k.

Challenges

- Limited view; all landmarks cannot be seen at the same time.
- False detections (outliers); y_k^i does not match any landmark m^j.
- False associations; landmarks get mixed up.
- Complexity.

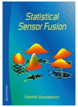
Summary

- *Simultaneous localization and mapping* (SLAM) is the problem of finding ones position, x_k, in a map, $\mathbf{m}$, while the map is built. Both parts must be considered simultaneous.
- Model:

$$z_k = \begin{pmatrix} x_{k+1} \\ \mathbf{m}_{k+1} \end{pmatrix} = \begin{pmatrix} f(x_k, v_k) \\ \mathbf{m}_k \end{pmatrix}, \qquad \mathrm{Cov}(v_k) = Q$$

$$y_k^i = h(x_k, \mathbf{m}_k^{c_k^i}) + e_k, \qquad \mathrm{Cov}(e_k) = R, \qquad i = 1, \ldots, I_k.$$

- Common solutions:
  - EKF SLAM: EKF (information form) on augmented state vector z_k.
  - FastSLAM: MPF on augmented state vector z_k.



Section 11–11.1