SENSOR FUSION

Filter Banks

Sensor Fusion

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Kalman Filter

The Kalman filter is the exact solution to the Bayesian filtering recursion for linear Gaussian model

$$x_{k+1} = F_k x_k + G_k v_k, \qquad \mathrm{Cov}(v_k) = Q_k$$
$$y_k = H_k x_k + e_k, \qquad \mathrm{Cov}(e_k) = R_k,$$

assuming $\mathrm{E}(v_k) = 0$, $\mathrm{E}(e_k) = 0$, and mutual independence.

Kalman Filter (KF) Algorithm

Time update:

$$\hat{x}_{k+1|k} = F_k \hat{x}_{k|k}$$
$$P_{k+1|k} = F_k P_{k|k} F_k^T + G_k Q_k G_k^T$$

Meas. update:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - \hat{y}_k)$$
$$P_{k|k} = P_{k|k-1} - K_k P_{k|k-1}$$
$$\hat{y}_k = H_k \hat{x}_{k|k-1} \qquad \varepsilon_k = y_k - \hat{y}_k$$
$$K_k = P_{k|k-1} H_k^T S^{-1} \qquad S_k = H_k P_{k|k-1} H_k^T + R_k$$

Kalman Filter: limitations

- **Only linear models**
  Addressed by *extended Kalman filter* (EKF), *unscented Kalman filter* (UKF), etc.
- **Only uni-modal posterior:** the estimate is only a mean and covariance
  Solved using *point-mass filter* (PMF), *particle filter* (PF), or filter banks.

Systems with distinct modes

E.g., a commercial aircraft that either flies in a straight line or makes coordinated turns; or inlier and outlier measurements.
Different approaches:

- Use high process and/or measurement noise to "hide" the different system behaviors, at the cost of loss of filter performances.
- Use bank of filters considering different mode possibilities, estimating both mode and state. With many enough components, this can be an arbitrarily good approximation.

Models Combining Several Behaviors

Jump Markov Linear (JML) Model

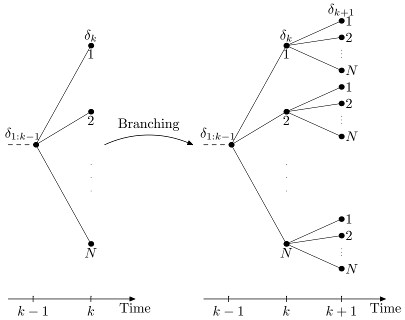
$$
\begin{aligned}
x_{k+1} &= F(\delta_k)x_k + w_k(\delta_k) & w_k(\delta_k) &\sim \mathcal{N}\big((0, Q(\delta_k)\big) \\
y_k &= H(\delta_k)x_k + e_k(\delta_k) & e_k(\delta_k) &\sim \mathcal{N}\big((0, R(\delta_k)\big) \\
(\delta_k|\delta_{k-1}) &\sim p(\delta_k|\delta_{k-1})
\end{aligned}
$$

where δ_k is a discrete valued Markov process, typically given by the transition matrix Π ($\Pi^{\delta_{k-1}\delta_k} = \Pr(\delta_k|\delta_{k-1})$), to indicate the current mode of the model.

- Well-defined modes.
- Given the mode sequence, the system is linear Gaussian.

Filter Bank

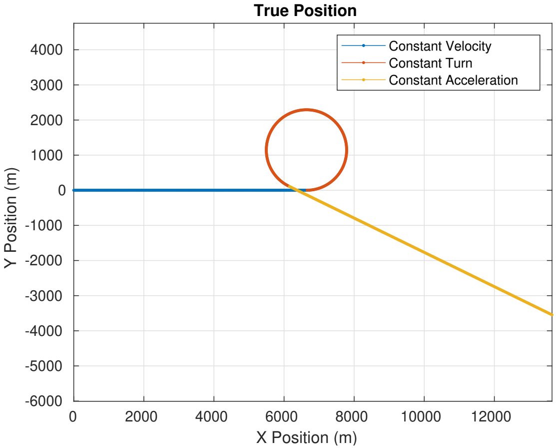
- Both the state x_k and the mode δ_k must be estimated.
- Conditioned on the mode sequence $\delta_{1:k}$ the estimate of x_k is given by the Kalman filter.
- The process of enumerating all possible mode sequences in the next step is called branching.
- A filter bank is an estimator that maintains a KF for each "interesting" mode sequence, with matching probability, $\omega_{k|k}^{(\delta_{1:k})}$.
- The resulting posterior estimate is a weighted sum of all filters in the filter bank.



Branching between time k and $k+1$. Each mode sequence at time k gives rise to N new sequences at time $k+1$.

Illustration (1/3)

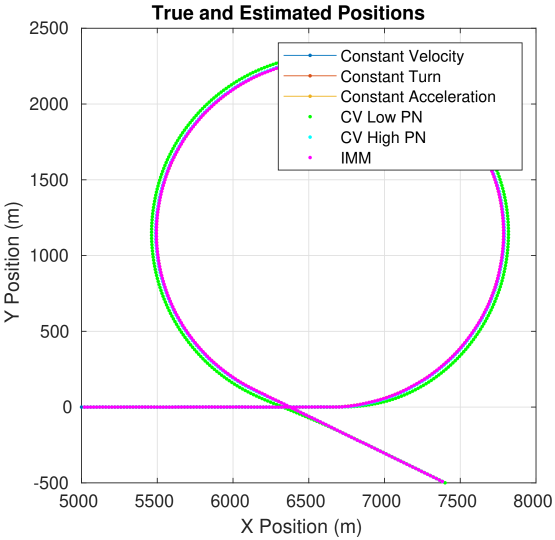
- Simulated trajectory with CV, CT, and CA segments.
- Position measurements.
- Compared filters:
  - KF with CV low process noise.
  - KF with CV high process noise.
  - Filter bank (interacting multiple model, IMM, filter) with CV, CT, and CA models.



Simulated trajectory

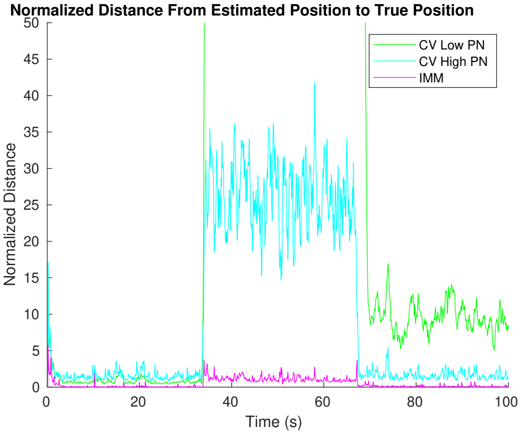
Example taken from MATLAB Sensor Fusion and Tracking toolbox.

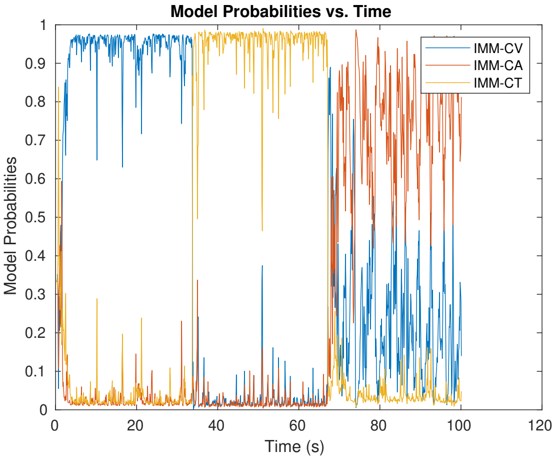
Illustration (2/3)



- The low process noise KF clearly cannot keep up.
- The high process noise KF, keeps up better but is slightly noisier than the IMM filter.
- Differences not very visible in this plot.
- The predominant models in the IMM matches the simulated trajectory well.

Illustration (3/3)





Algorithm Details

- Each KF is maintained independently in the filter bank, assuming the specific mode sequence.
- Equations to update the filter probabilities/weights

$$\omega_{k+1|k}^{(\delta_{1:k+1})} = p(\delta_{k+1}|\delta_k)\omega_{k|k}^{(\delta_{1:k})}$$

$$\omega_{k|k}^{(\delta_{1:k})} = \frac{p(y_k|\delta_{1:k}, y_{1:k-1})\omega_{k|k-1}^{(\delta_{1:k})}}{\sum_{\delta_{1:k}} p(y_k|\delta_{1:k}, y_{1:k-1})\omega_{k|k-1}^{(\delta_{1:k})}} = \frac{\mathcal{N}(y_k|\hat{y}_k^{(\delta_{1:k})}, S_k^{(\delta_{1:k})})\omega_{k|k-1}^{(\delta_{1:k})}}{\sum_{\delta_{1:k}} \mathcal{N}(y_k|\hat{y}_k^{(\delta_{1:k})}, S_k^{(\delta_{1:k})})\omega_{k|k-1}^{(\delta_{1:k})}}$$

- Resulting Gaussian mixture posterior distribution

$$p(x_k|y_{1:k}) = \sum_{\delta_{1:k}} \omega_{k|k}^{(\delta_{1:k})} p(x_k|y_{1:k}, \delta_{1:k}) = \sum_{\delta_{1:k}} \omega_{k|k}^{(\delta_{1:k})} \mathcal{N}(x_k|\hat{x}_{k|k}^{(\delta_{1:k})}, P_{k|k}^{(\delta_{1:k})})$$

- The MMSE given the individual KF estimates with mean and covariance $(\hat{x}^{(\delta)},\ P^{(\delta)})$ becomes:

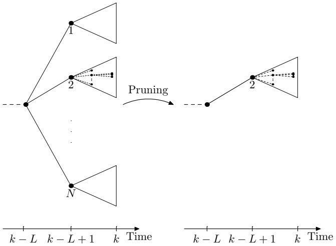
$$\hat{x} = \sum_{\delta} \omega^{(\delta)} \hat{x}^{(\delta)}$$

$$P = \sum_{\delta} \omega^{(\delta)} \big(P^{(\delta)} + \underbrace{(\hat{x}^{(\delta)} - \hat{x})(\hat{x}^{(\delta)} - \hat{x})^T}_{\text{Spread of the mean}}\big).$$

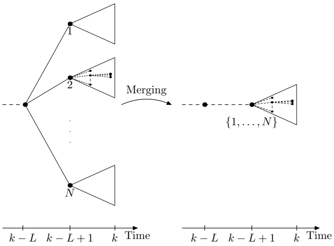
Filter Bank: problem

- Filter banks grows with combinatorial complexity, hence it quickly becomes unmanageable.
- Common approximations:

  Pruning: Drop unlikely branches,
  Merging: Combine branches with recent common heritage.

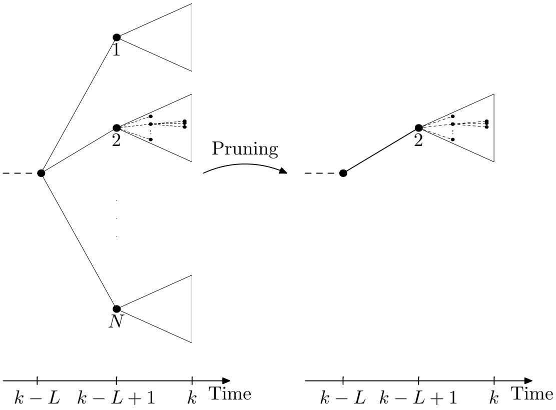


Pruning



Merging

Filter Bank Approximation: pruning



- Prune branches with low probability:
  - Mode sequences with too low probability.
  - "Trees" with too low accumulated probability since L steps back.
- After reducing the filter bank to suitable size, re-normalize the remaining weights, $\delta \in \Delta$, such that

$$\sum_{\delta \in \Delta} \omega^{(\delta)} = 1.$$

Filter Bank Approximation: merging

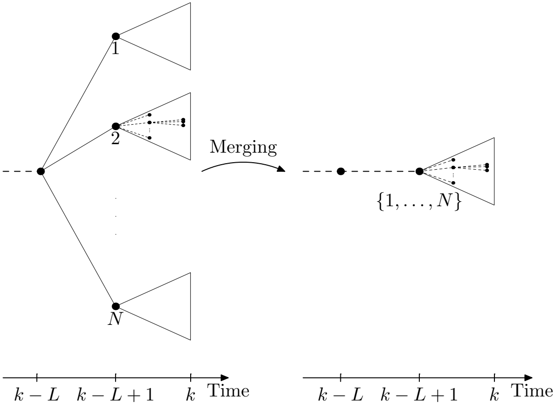
- Reduce the filter bank by combining mode sequences that have recently been similar.
- The weight of the merged mode sequences, $\delta \in \Delta$, are add up to the weight of the merged branch, δ',

$$\omega^{(\delta')} = \sum_{\delta \in \Delta} \omega^{(\delta)}.$$

- The mean and covariance becomes

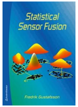
$$\hat{x}^{(\delta')} = \frac{1}{\omega^{(\delta')}} \sum_{\delta \in \Delta} \omega^{(\delta)} \hat{x}^{(\delta)}$$

$$P^{(\delta')} = \frac{1}{\omega^{(\delta')}} \sum_{\delta \in \Delta} \omega^{(\delta)} \Big(P^{(\delta)} + (\hat{x}^{(\delta)} - \hat{x})(\hat{x}^{(\delta)} - \hat{x})^T \Big).$$



Summary

- **Jump Markov linear (JML) models**; models which behave differently based on a discrete mode, which evolves according to a Markov process.
- **Jointly estimate state and mode** using a bank of filters:
  - **Enumerate all possible mode sequences**, and run a regular filter for each in parallel.
  - **Compute the probability** for each mode sequence.
  - The posterior is a **weighted sum of the solutions** for each mode sequence.
- **Reduce the computational complexity:** pruning and merging.
- Kalman filter banks can, contrary to the Kalman filter, handle multi-modal posterior distributions.
- Famous algorithms: **Generalized pseudo-Bayesian** (GPB) and **interacting multiple models** (IMM) filters.



Chapter 10