Safe Fusion

Sensor Fusion

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Repetition: the Fusion Formula

The fusion formula for two independent estimates is

$$
\begin{aligned}
\mathrm{E}(\hat{x}_1) = \mathrm{E}(\hat{x}_2) &= x, \\
\mathrm{Cov}(\hat{x}_1) = P_1, \quad \mathrm{Cov}(\hat{x}_2) &= P_2 \Rightarrow \\
\hat{x} &= P\left(P_1^{-1}\hat{x}_1 + P_2^{-1}\hat{x}_2\right), \\
P &= \left(P_1^{-1} + P_2^{-1}\right)^{-1}.
\end{aligned}
$$

Safe fusion applies when the estimates are not independent, and the dependence is unknown.

Fusion with Known Dependence

With known cross-correlation P_{12}, the known facts can be summarized as

$$\mathrm{E}\begin{pmatrix}\hat{x}_1\\ \hat{x}_2\end{pmatrix} = \begin{pmatrix}x\\ x\end{pmatrix},$$

$$\operatorname{Cov}\begin{pmatrix}\hat{x}_1\\ \hat{x}_2\end{pmatrix} = \begin{pmatrix}P_1 & P_{12}\\ P_{21} & P_2\end{pmatrix}.$$

Note: correlation is the same as second order properties of a stochastic variable. For Gaussian variables, uncorrelated ($P_{12} = 0$) is the same as independence.
If we want to find a *linear* combination that is *unbiased* then the final estimate has the form

$$\hat{x} = L\hat{x}_1 + (I - L)\hat{x}_2$$

Of all linear combinations L, the most natural is the one that minimizes the covariance

$$L^{\mathrm{opt}} = \arg\min_L \operatorname{Cov}\big(L\hat{x}_1 + (I - L)\hat{x}_2\big)$$

This is a well-defined mathematical problem for the BLUE (Best Linear Unbiased Estimator)

Safe Fusion (Unknown Dependence)

- The fusion formula hinges on that the estimates are independent. If they are dependent, e.g. based on overlapping information, then the result will be too optimistic. That is, the covariance and confidence intervals will be too small.
- Better to have conservative covariance and thus confidence intervals.
- A very conservative approach is to take either $\hat{x}_1$ or $\hat{x}_2$ as the final estimate. That is, ignore one of them (the worst one).
- Safe fusion (or covariance intersection algorithm) provides a less conservative method.

Safe fusion

Given two unbiased estimates $\hat{x}_1, \hat{x}_2$ with information $\mathcal{I}_1 = P_1^{-1}$ and $\mathcal{I}_2 = P_2^{-1}$ (pseudo-inverses if singular covariances), respectively. Compute the following:

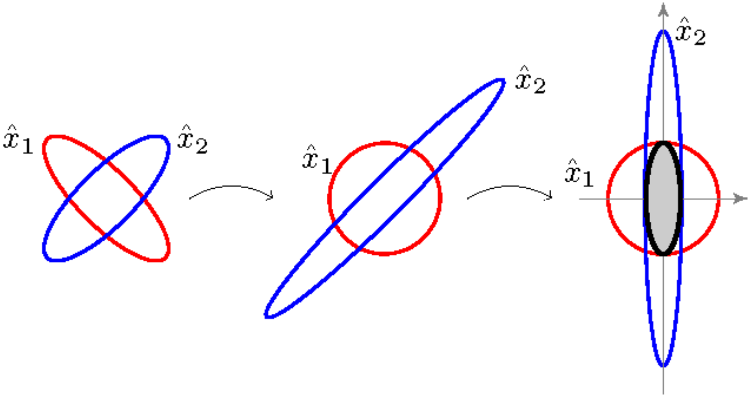
1. SVD: $\mathcal{I}_1 = U_1 D_1 U_1^T$.
2. SVD: $D_1^{-1/2} U_1^T \mathcal{I}_2 U_1 D_1^{-1/2} = U_2 D_2 U_2^T$.
3. Transformation matrix: $T = U_2^T D_1^{1/2} U_1$.
4. State transformation: $\hat{\hat{x}}_1 = T\hat{x}_1$ and $\hat{\hat{x}}_2 = T\hat{x}_2$. The covariances of these are $\mathrm{Cov}(\hat{\hat{x}}_1) = I$ and $\mathrm{Cov}(\hat{\hat{x}}_2) = D_2^{-1}$, respectively.
5. For each component $i = 1, 2, \ldots, n_x$, let

$$\hat{\hat{x}}^i = \hat{\hat{x}}_1^i, \quad D^{ii} = 1 \quad \text{if} \quad D_2^{ii} < 1,$$
$$\hat{\hat{x}}^i = \hat{\hat{x}}_2^i, \quad D^{ii} = D_2^{ii} \quad \text{if} \quad D_2^{ii} > 1.$$

6. Inverse state transformation:

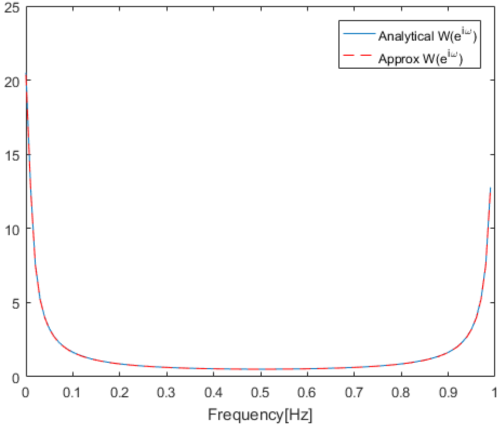
$$\hat{x} = T^{-1}\hat{\hat{x}}, \quad P = T^{-1} D^{-1} T^{-T}$$

Transformation steps



Sensor network example, cont'd

```
X3=fusion(X1,X2);      % WLS
X4=fusion(X1,X3);      % X1 used twice
X5=safefusion(X1,X3);
plot2(X3,X4,X5)
```



Summary Safe Fusion

We have two dependent estimates

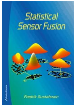
$$\mathrm{E}\begin{pmatrix}\hat{x}_1\\ \hat{x}_2\end{pmatrix} = \begin{pmatrix}x\\ x\end{pmatrix},$$

$$\operatorname{Cov}\begin{pmatrix}\hat{x}_1\\ \hat{x}_2\end{pmatrix} = \begin{pmatrix}P_1 & P_{12}\\ P_{21} & P_2\end{pmatrix}.$$

The BLUE solution for the case of known correlation $P_{12} = P_{21}^T$ has the form

$$\hat{x} = L^{\mathrm{opt}}\hat{x}_1 + (I - L^{\mathrm{opt}})\hat{x}_2$$
$$L^{\mathrm{opt}} = \arg\min_L \operatorname{Cov}\big(L\hat{x}_1 + (I - L)\hat{x}_2\big)$$

Safe fusion gives a conservative P that tries to avoid double counting information without knowing the correlation.



Section 2.3.5