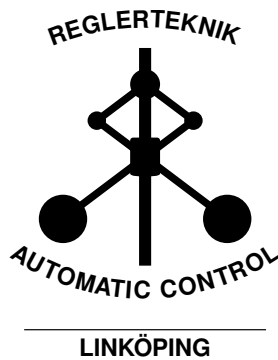


# Statistical Sensor Fusion — Lab 2

## Orientation Estimation using Smartphone Sensors

This version: 2023-06-02



Name: \_\_\_\_\_

P-number: \_\_\_\_\_

Date: \_\_\_\_\_

Passed: \_\_\_\_\_

# 1 Introduction

Navigation is one of the first and one of the most important applications of sensor fusion. At the same time, it is quite challenging both from a complexity and a numerical point of view. The core in any navigation system is the orientation filter that integrates inertial information from gyroscopes and accelerometers with magnetometer measurements and other supporting sensors that relate to the orientation of the platform with respect to the world.

This laboration aims at providing theoretical insights into the design and implementation of such filters, and also to give practical experience in issues such as tuning, disturbance rejection and sensitivity.

The laboration is split into two parts, an introductory session and an examination session, separated by approximately one week of independent work. The introductory session is intended to help you get started with the laboration tasks, sort out any practical issues with data collection and to provide you an opportunity to ask questions about the lab. During the examination session, you present your findings to a lab assistant and are expected to be able to answer questions about the lab in general.

**Note:** Section 4 contains preparatory exercises that should be completed before turning up at the **introductory** laboratory session. You do need the results of these exercises to complete the lab.

## 2 The Sensor Fusion Android app

During the laboration you will work with data that you collect online with an Android smartphone (technically any Android device with suitable sensors that can be connected to Internet) using an app that streams the measurements from the phone to a PC, where it can be accessed for in example Matlab. The application is available for free from Google Play (<https://goo.gl/0qNyU>) under the name *Sensor Fusion*.



First time launching the Sensor Fusion app you end up in its home screen (Figure 1(a)), from which all the app's basic functionality can be reached.

**Select Sensor** Clicking “Select Sensor” yields a new view similar to the one in Figure 1(b). Exactly what exact sensors show up depends on what sensors are available in your phone, this varies between different brands and models. When selecting a sensor, sensor readings from it are visualized. Figure 1(c) shows the result from clicking the “Gyroscope” button. The gyroscope measurements are visualized as three curves in a graph, together with the norm of the measurements.

**Log Data** Clicking “Log Data” brings up a view (Figure 2(a)) from which it is possible to stream data over internet or save it to a file on the device to be read out and processed at a later time. What sensor data is collected is determined by checking the respective check box in the sensor list. The check boxes “Stream” and “Log” determine if the collected measurements

should be streamed over internet and/or saved to a log file, respectively. The details about where to stream the data and the log file are displayed in the field below and can be changed by clicking the red gear at the bottom right of the screen. Data collection is started and stopped using the button at the bottom of the page. (Note that before starting to stream data a receiver must be available, or the connection will be automatically shut down.)



**Sensor list** Clicking “Sensor list” brings up a dialog (Figure 2(b)) enumerating all sensory measurements that are available on request by an Android app. Note that some of the listed sensors are physical sensors, *e.g.*, accelerometers and magnetometers; whereas others are virtual sensors (the measurements are obtained from fusing other sensors), *e.g.*, orientation and gravity. (The orientation sensor will be used to benchmark the orientation filter you construct in this lab.)

**About/Help** This displays some information about the app.



Clicking the red gear in any view where it is present brings up the preferences view (Figure 2(c)). Changes made in this view are stored in the device. The most important fields are the Measurement>Frequency which determine the measurement rate (default: 100 Hz); the Network settings which are needed to determine where data is streamed; and the Log settings which decide where the log file is created.

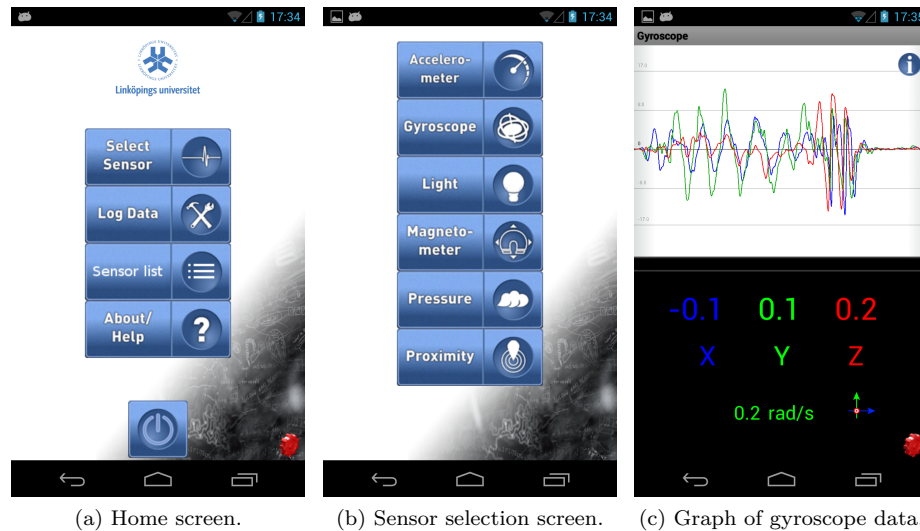


Figure 1: Screen shots from the Sensor Fusion app.

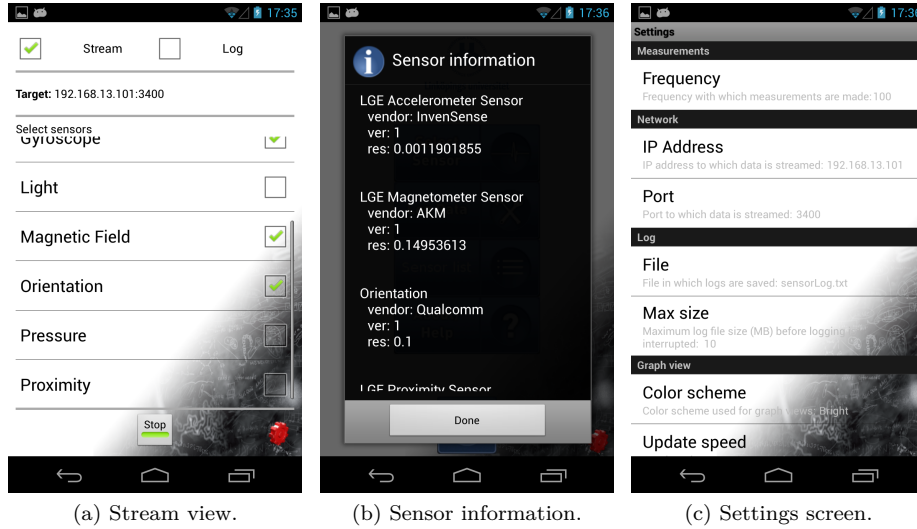


Figure 2: Screen shots from the Sensor Fusion app.

### 3 Summary of Relevant Theory

This section provides relevant background theory needed to perform this lab. The presentation here could be considered complimentary to the description in Chapter 13 of the textbook, and can in many cases replace it for the purposes of this lab.

#### 3.1 Representing Rotations using Quaternions

In this exercise two inertial frames are considered:

- The *world frame*,  $\mathcal{W}$ , which is fixed to the earth, is aligned with its  $x$ -axis pointing east, its  $y$ -axis pointing north, and its  $z$ -axis pointing up. This way an orthogonal right-hand system is formed, sometimes denoted an ENU-system (east-north-up-system).
- The *sensor frame*,  $\mathcal{S}$ , is centered in the smartphone and moves with it. Ideally it would be centered in the accelerometer to avoid measuring accelerations due to pure rotations, but as the exact location of it is unknown, the frame is considered positioned in the center of the phone. Laying with the screen face up the  $x$ -axis points right, the  $y$ -axis points forward, and the  $z$ -axis points up. This is the same coordinate system used in both the Android and the iOS APIs for writing apps.

See Figure 3 for an illustration of how the world frame,  $\mathcal{W}$ , and the sensor frame,  $\mathcal{S}$ , relate to each other.

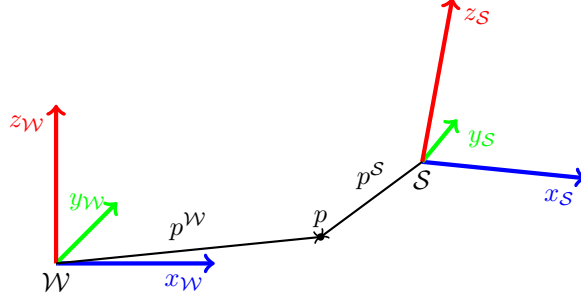


Figure 3: Illustration of the two inertial frames used here; the world fixed frame,  $\mathcal{W}$ , and the sensor (phone) fixed frame,  $\mathcal{S}$ .

The world and sensor systems are related via a linear transformation, which can be described mathematically as

$$p^{\mathcal{W}} = R^{\mathcal{W}/\mathcal{S}} p^{\mathcal{S}} + t^{\mathcal{W}/\mathcal{S}}, \quad (1)$$

where  $p^{\mathcal{S}}$  is a point expressed in the sensor frame and  $p^{\mathcal{W}}$  is the same point expressed in the world frame. The relative rotation between the two frames is given by  $R^{\mathcal{W}/\mathcal{S}}$ , the rotation aligns the world frame with the sensor frame. The translation between centers of the coordinate frames is given by  $t^{\mathcal{W}/\mathcal{S}}$ ; however, as only the rotation is of interest in this lab,  $t^{\mathcal{W}/\mathcal{S}}$  will not be considered further.

The focus of this lab is to estimate the rotation  $R^{\mathcal{W}/\mathcal{S}}$  based on the sensor measurements available from a smartphone. The rotation describes how to rotate points in the sensor frame,  $\mathcal{S}$ , to describe them in the world frame,  $\mathcal{W}$ . It can also be interpreted as the rotation that is applied to the world frame to align it with the sensor system.

A rotation can be described in many ways. One way is to use a rotation matrix. This is a description most of us are familiar with and know well how to manipulate. However, it uses 9 values to represent 3 degrees of freedom. Hence, it is not a minimal representation, which makes estimating the rotation matrix difficult. Another alternative is to use Euler angles, *i.e.*, a representation in terms of three consecutive rotations around predefined axes. Even though represented by three values, Euler angles are not unique — several combinations of rotations result in the same final rotation — the representation has discontinuities in the parameters, and suffers from the gimbal lock effect. These are all factors that make the Euler angles unsuitable for estimation purposes. A third alternative is to use a unit length quaternion representation. This representation suffers less from the problems of the two other representations and is therefore a popular choice for orientation estimation.

The quaternion representation of a rotation can be interpreted as an axis

angle representation of the rotation according to

$$q = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \cos(\frac{1}{2}\alpha) \\ \sin(\frac{1}{2}\alpha) \begin{pmatrix} \hat{v}_x \\ \hat{v}_y \\ \hat{v}_z \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \cos(\frac{1}{2}\alpha) \\ \sin(\frac{1}{2}\alpha)\hat{v} \end{pmatrix},$$

where  $\alpha$  is a positive rotation around the axis of rotation  $\hat{v}$ , assumed to be of unit length, *i.e.*,  $\|\hat{v}\| = 1$ . Note, this interpretation clearly shows that the quaternion representation is in fact not unique,  $q$  and  $-q$  represent the same rotation. (To show this is a straightforward application of trigonometric identities on the axis angle representation.) However, this is much less of a problem than in the case with Euler angles. In most cases,  $q_0 \geq 0$  is assumed and enforced to get an unambiguous representation.

The following mathematical relations are useful when working with orientation estimation using the quaternion representation:

- The formula to convert from a unit length quaternion to a rotation matrix,

$$R = Q(q) = \begin{pmatrix} 2q_0^2 - 1 + 2q_1^2 & 2q_1q_2 - 2q_0q_3 & 2q_1q_3 + 2q_0q_2 \\ 2q_1q_2 + 2q_0q_3 & 2q_0^2 - 1 + 2q_2^2 & 2q_2q_3 - 2q_0q_1 \\ 2q_1q_3 - 2q_0q_2 & 2q_2q_3 + 2q_0q_1 & 2q_0^2 - 1 + 2q_3^2 \end{pmatrix}. \quad (2a)$$

This is implemented in  $\mathbf{Qq}(q)$  in Appendix A.2.3. Note how all occurrences of  $q_i$  are in pairs, hence, changing the sign on all quaternion components does not change the resulting rotation matrix  $Q(q) = Q(-q)$ , as noted previously.

The reverse operation is obtained as

$$q = Q^{-1}(R) = \frac{1}{2} \begin{pmatrix} \sqrt{1 + R_{11} + R_{22} + R_{33}} \\ \frac{\text{sgn}(R_{32} - R_{23})\sqrt{1 + R_{11} - R_{22} - R_{22}}}{2} \\ \frac{\text{sgn}(R_{13} - R_{31})\sqrt{1 - R_{11} + R_{22} - R_{22}}}{2} \\ \frac{\text{sgn}(R_{21} - R_{12})\sqrt{1 - R_{11} - R_{22} + R_{22}}}{2} \end{pmatrix}, \quad (2b)$$

which is a relatively numerically stable expression compared to the alternatives at the cost of slightly more costly operations.

- The angle,  $\Delta$ , between two unit length quaternions  $q^0$  and  $\hat{q}$  can be calculated as

$$\Delta = 2 \arccos \left( \left| \sum_{i=0}^3 q_i^0 \hat{q}_i \right| \right). \quad (3)$$

This way  $\Delta$  describes the difference between  $\hat{q}$  and  $q^0$  as the angle  $\hat{q}$  must be turned to obtain  $q^0$ .

- The time derivative of a quaternion, in terms of angular velocities,  $\omega$ , (where the quaternion represents  $R^{\mathcal{W}/\mathcal{S}}$  and  $\omega$  is given in the sensor frame)

can be shown to be

$$\dot{q} = \frac{1}{2}S(\omega)q = \frac{1}{2} \begin{pmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{pmatrix} \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} \quad (4a)$$

$$= \frac{1}{2}\bar{S}(q)\omega = \frac{1}{2} \begin{pmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}. \quad (4b)$$

The functions  $S(\omega)$  and  $\bar{S}(q)$  are implemented in `Somega(omega)` and `Sq(q)`, in Appendices A.2.5 and A.2.4, respectively.

- Differentiating  $Q(q)$  with respect to  $q$  is a bit more tricky as the result is a three-dimensional tensor. Differentiating with respect to one component at the time yields:

$$\frac{dQ(q)}{dq_0} = 2 \begin{pmatrix} 2q_0 & -q_3 & q_2 \\ q_3 & 2q_0 & -q_1 \\ -q_2 & q_1 & 2q_0 \end{pmatrix}, \quad (5a)$$

$$\frac{dQ(q)}{dq_1} = 2 \begin{pmatrix} 2q_1 & q_2 & q_3 \\ q_2 & 0 & -q_0 \\ q_3 & q_0 & 0 \end{pmatrix}, \quad (5b)$$

$$\frac{dQ(q)}{dq_2} = 2 \begin{pmatrix} 0 & q_1 & q_0 \\ q_1 & 2q_2 & q_3 \\ -q_0 & q_3 & 0 \end{pmatrix}, \quad (5c)$$

$$\frac{dQ(q)}{dq_3} = 2 \begin{pmatrix} 0 & -q_0 & q_1 \\ q_0 & 0 & q_2 \\ q_1 & q_2 & 2q_3 \end{pmatrix}. \quad (5d)$$

The quaternion derivative is implemented in `dQdq(q)`, in Appendix A.2.6.

- The discrete time update of a quaternion based on given angular velocities,  $\omega_k$ , and process noise,  $w_k$  in (7), can be approximated with

$$\begin{aligned} q_{k+1} &= e^{\frac{1}{2}S(\omega_k+w_k)T} q_k \\ &= \left( \cos\left(\frac{\|\omega_k+w_k\|T}{2}\right) I + \frac{T}{2} \cdot \frac{\sin\left(\frac{\|\omega_k+w_k\|T}{2}\right)}{\frac{\|\omega_k+w_k\|T}{2}} S(\omega_k+w_k) \right) q_k \end{aligned} \quad (6a)$$

$$\approx \left( I + \frac{1}{2}S(\omega_k)T \right) q_k + \frac{T}{2}\bar{S}(q_k)w_k. \quad (6b)$$

### 3.2 Sensor Fusion

The basic sensor fusion model is

$$x_{k+1} = f(x_k, u_k, w_k), \quad (7a)$$

$$y_k = h(x_k, u_k, e_k), \quad (7b)$$

where  $x_k$  is the state at time  $k$ ,  $u_k$  known (or measured) input to the system,  $w_k$  is process noise,  $y_k$  is the measurements (or system output), and  $e_k$  is the measurement noise. Here, the goal is to estimate the orientation, represented by the unit quaternion  $q_k$ . The sensors provide 3D measurements of:

- Angular rates  $\omega_k$ , denoted  $y_k^\omega$  when used as measurements, at sampling rate  $f_s^\omega$ .
- Accelerations  $a_k$ , denoted  $y_k^a$  when used as measurements, at sampling rate  $f_s^a$ .
- Magnetic field  $m_k$ , denoted  $y_k^m$  when used as measurement, at sampling rate  $f_s^m$ .

When designing a filter to estimate the phone's orientation there are several things to take into consideration: how to divide the measurements in input and output, which sensor biases, if any, to estimate, and how to handle asynchronous measurements.

There are many ways to divide the measured quantities in inputs  $u_k$  and outputs  $y_k$  in (7). For clarity, everything that is measured is referred to as measurements, and can be used as inputs or outputs depending on the chosen filter structure. The main structures are:

1. The simplest model is to let

$$x_k = q_k, \quad u_k = \omega_k, \quad y_k = (y_k^{a,T} \quad y_k^{m,T})^T, \quad (8a)$$

and not to attempt any bias estimation.

2. The most complex model is to let

$$x_k = (q_k^T \quad \omega_k^T \quad b_k^{\omega,T} \quad b_k^{a,T})^T, \quad y_k = (y_k^{\omega,T} \quad y_k^{a,T} \quad y_k^{m,T})^T, \quad (8b)$$

and estimate both the accelerometer bias,  $b^a$ , and the gyroscope bias,  $b^\omega$ .

3. An intermediate model is

$$x_k = (q_k^T \quad b_k^{\omega,T} \quad b_k^{a,T})^T, \quad u_k = \omega_k, \quad y_k = (y_k^{a,T} \quad y_k^{m,T})^T, \quad (8c)$$

where the gyroscope measurement is considered as input, but the biases are estimated.

4. Since the accelerometer measures both the specific force  $F_k$  (that accelerates the platform) and the gravity  $g^0$ , another alternative is to include position and velocity in the state, and to let acceleration either be a state or an output. That is, to also estimate  $t^{\mathcal{W}/\mathcal{S}}$ . This is a much harder problem which needs some sort of absolute measurement to be observable, or the estimate will quickly start to drift.



A more philosophical question is what the difference of an input and output is from a filtering perspective. There are many explanations to this degree of freedom. One specific aspect here is the following:

- If the angular rates are integrated as inputs in the time update, the motion affects the mean of the state estimate directly.
- If the angular rates are used in the measurement update, this new information is weighted together with the prior in a Bayesian fashion. The mean of the orientation state does not take the full step according to the measurement. There will also be a delay in information, because it takes one time update until the derivative affects the angular state.

The conclusion is that we prefer to consider  $u_k = \omega_k$  as an input. Both  $a_k$  and  $m_k$  are subject to disturbances, and outlier rejection is most naturally implemented in the measurement update. This is another reason why we advocate the first approach above.

A remaining question is whether an attempt should be made to estimate the biases in the sensors,  $b^\omega$  and  $b^a$  above.

The multi-rate, or asynchronous, measurement aspect is an issue in practice if all sensors have different rates. Consider case 1 above. If the sample rate  $f_s^\omega$  is an integer multiple of  $f_s^a$  and  $f_s^m$ , then the measurement update can be performed at regular cycles after the time update. In the general case, the piece-wise constant assumption has to be used, and the time update is matched to the next output observation.

## 4 Preparations

The following exercises should be completed **before the introductory laboratory session**. The results are needed before you can get started with the lab. Hence, it is important that you come well prepared to the introductory laboratory session in order to be able to get as useful help as possible.

Given the discussion above, use a state-space model with the rotation  $R^{W/S}$  represented as a quaternion as state vector.

### 4.1 Dynamic Equation

The dynamic model is then given by (6a). Before the lab session, write a Matlab function `[x, P] = tu_qw(x, P, omega, T, Rw)` that implements the time update function, where `omega` is the measured angular rate, `T` the time since the last measurement, and `Rw` the process noise covariance matrix. For your convenience, the functions  $\bar{S}(q)$  and  $S(\omega)$  are available in App. A.2.4 and A.2.5, respectively.

Furthermore, write a similar function to handle the time update in the case that there is no angular rate measurement available.

## 4.2 Acceleration Measurement Equation

The measurements from the accelerometer can be modeled using the relation

$$y_k^a = Q^T(q_k)(g^0 + F_k) + e_k^a, \quad (9)$$

where no specific force is assumed present (*i.e.*,  $F_k = 0$ ),  $g^0$  is the nominal gravity vector, and  $e_k^a$  is the measurement noise.

Before the lab session, prepare a Matlab function `[x, P] = mu_g(x, P, yacc, Ra, g0)` that implements the accelerometer measurement update, where `yacc` is a shorthand for  $y_k^a$  and `Ra` is the measurement noise covariance matrix. You can let  $g^0$  be a constant in your implementation. The functions to compute  $Q(q)$  and  $dQ(q)/dq$  are available in App. A.2.3 and A.2.6, respectively.

## 4.3 Magnetic Field Measurement Equation

Before the lab session prepare a Matlab function `[x, P] = mu_m(x, P, ymag, Rm, m0)` that implements a magnetometer measurement update, where `ymag` is a shorthand for  $y_k^m$ .

**Hint:** The magnetometer measures the earth magnetic field (possibly including disturbances) in sensor coordinates,

$$y_k^m = Q^T(q_k)m^0 + e_k^m, \quad (10)$$

where  $m^0$  is the earth magnetic field in world coordinates. By definition there should be no magnetic field component in the west-east direction. Hence, the nominal magnetic field can be derived from magnetic field measurements as the phone lies horizontally without knowledge of the direction of north using the relation

$$m^0 = \begin{pmatrix} 0 & \sqrt{m_x^2 + m_y^2} & m_z \end{pmatrix}^T, \quad (11)$$

where  $m$  is the measured magnetic field.

## 4.4 Quaternion Normalization

In order for the quaternion representation of the orientation to work, it is essential that the quaternion is of unit length, *i.e.*,  $\|q_k\| = 1$ . The extended Kalman filter does not automatically maintain this quaternion property. Therefore, before the lab session, figure out a way to ensure that the quaternion keeps unit length.

## 5 Exercises

The following exercises should be completed before the examination session. The passages prefixed by “**Check-point:**” should be presented to a lab assistant to

pass the lab. To make reporting as efficient as possible, `labreport.m` is provided. By using it you make sure that you have the documentation the lab assistant will ask for, and that you do not miss any important aspects of the lab. Hence, fill out the report template as you work with the lab, and have it available at the examination session. For all of the checkpoints, be prepared to explain what you've done, observed and the assumptions that you've made.

Consider the structure (8a) and make use of the code skeleton in Appendix A.1.



1. **Connect the phone with your lab computer:**

- Download all the supporting Matlab files and the `sensorfusion.jar` file from: <https://bit.ly/3qqdTKT>. Extract the files in the zip-archive to your lab folder.
- Run the `startup` function to initialize the environment correctly. Note, this must be done each time Matlab is restarted.
- Connect the smartphone to eduroam.
- Determine the IP of your lab computer. One way to do this is to run `showIP` in Matlab.
- Enter the lab computer's IP into the app's configuration view (see Figure 2(c).) At the same time, make sure the port is set to 3400 and that the measurement frequency is 100 Hz.
- Always start your Matlab program before initiating the streaming from the smartphone, it acts as a server to which the client on the phone tries to connect.

2. **Get to know your data:** Spend a few minutes to play around with the app and the sensors in the "Select Sensor" view to get an initial feeling for the data you are going to work with.

Use `filterTemplate` to collect a few seconds of data and compute mean and variance for the accelerometer, gyroscope, and magnetometer. Also, analyze the measurements to see if the noise is Gaussian and if it has any trends. What do the results tell you? Your results should be used to tune the filter.

*Hint:* Unavailable measurements are marked with a NaN (not a number) value. Make sure to take this into consideration when computing means etc. The function `isnan(x)` can be used to find NaN values; the average acceleration can be computed as:

```
mean(meas.acc(:, ~any(isnan(meas.acc), 1)), 2)
```

**Check-point:** Be prepared to show the lab assistant:

- A histogram of the measurements for each sensor and axis.
- A plot of the signals over time. If there are trends figure out a way to deal with these.
- The determined average acceleration vector, angular velocity vector, and magnetic field, and their respective covariance matrices.

3. **Add the EKF time update step:** Start to implement an EKF to estimate the orientation by adding the *time update*. (Make a copy of `filterTemplate.m` and add the time update step you wrote in Section 4.1 as part of your lab preparations.) Use the previously estimated mean and variance as input to the filter and use it to compensate for bias and noise levels.

You should now have a very reactive estimate with no absolute orientation. What happens if you start the filter with the phone on the side instead of face up on the desk? Why? Shake the phone and explain what happens.

**Check-point:** Be prepared to show the lab assistant a plot of the resulting orientation as well as orientation error (as compared to the orientation measurement). Be ready to report your findings guided by the questions above.

4. **Add the EKF accelerometer measurement update step:** Add the *accelerometer update* to the EKF (use `mu_g(x, P, yacc, Ra)` you wrote in Section 4.2 preparing for the lab). Now, the orientation should be correct up to a rotation in the horizontal plane. Test the sensitivity to specific forces (body accelerations) with different experiments. As an example, slide the device quickly back and forth on the horizontal surface of a table.
5. **Add accelerometer outlier rejection:** Add a simple *outlier rejection* algorithm for  $a_k$ , based on the assumption that  $\|a_k\| \approx g = 9.81$  without specific force. Use the orientation view's `setAccDist` function to indicate when the accelerometer measurements are considered disturbed/outliers.

Repeat the same experiments as in the previous two steps.

**Check-point:** Be prepared to show plots of the resulting orientation as well as orientation error (as compared to the orientation measurement) with and without outlier rejection. Be ready to explain what you observed when shaking and sliding your phone on the desk with and without outlier rejection.

6. **Add the EKF magnetometer measurement update step:** Implement a *magnetometer update*, based on your preparations in Section 4.3. You should now have a orientation filter that behaves much like the one implemented in the phone. What happens if you introduce a magnetic disturbance? (The power supply in the monitor can provide a suitable disturbance.)
7. **Add magnetometer outlier rejection:** Try to come up with a way to perform *outlier rejection* for magnetic disturbances, *e.g.*, by estimating the strength of the magnetic field and reject measurements when the field strength differs too much from what you expect. Implement the outlier rejection, and evaluate if it helps. Use the orientation view's `setMagDist` function to indicate when the magnetic field measurements are considered disturbed/outliers. What assumptions do you rely on? When are they

reasonable? What happens now when you introduce a magnetic disturbance?

Use the `q2euler` function (Appendix A.2.2) and plot the Euler angles of both your orientation filter and the built in filter in the phone.

**Check-point:** Be prepared to show plots of the resulting orientation as well as orientation error (as compared to the orientation measurement) with and without outlier rejection. Be ready to explain what you observed when using the filter with and without a magnetic disturbance. Report your findings guided by the questions above.

8. **Test your filter without gyroscope measurements:** Turn off the gyroscope measurements and evaluate the filter; slide the phone on the desk, shake it, and introduce magnetic disturbances.

**Check-point:** Show plots of the resulting orientation as well as orientation error (as compared to the orientation measurement) without gyroscope measurements. Show the filter running without gyroscope measurements to the assistant.

9. **If you are interested and have time:**

- Compare the difference between using (6b) and the exact form (6a).
- Evaluate the remaining combinations of measurements in the filter: only accelerometer, only magnetometer, gyroscope and magnetometer.

10. **Wipe your account details from the phone:** Once you have passed the lab, wipe your login details for eduroam from the phone. Got to the network settings screen again, press the eduroam entry until you get the option to “Forget network” and do so.

## A Provided Code

The following functionality is provided in the zip-archive <https://bit.ly/3qqdTKT>. Make sure to run the included `startup` function to initialize the package before doing anything else. Without this step it is not possible to establish a connection with the phone. When using `filterTemplate` always start the Matlab program before initializing the streaming from the phone as the Matlab program acts as a server for the data from the phone



### A.1 Code Skeleton

```
1 function [xhat, meas] = filterTemplate(fname, calAcc, calGyr, calMag)
   % FILTERTEMPLATE Filter template
   %
   % This is a template function for how to collect and filter data
5  % sent from a smartphone live. As you implement your own orientation
   % estimate, it will be visualized in a simple illustration. If the
```

```

% orientation estimate is checked in the Sensor Fusion app, it will be
% displayed in a separate view.
%
10 % [xhat, meas] = FILTERTEMPLATE() starts a server that the Sensor Fusion
% app can connect to and runs the implemented filter. Interrupting the code
% yields the result in xhat and meas.
%
% [xhat, meas] = FILTERTEMPLATE(fname) uses the data in fname as the
15 % measurement source instead of a stream. The function returns when the
% data file has been completely parsed.
%
% Input
% -----
20 % fname: string
%         Path to data file
%
% calAcc/calGyr/calMag: struct with fields:
%         m: bias mean
25 %         R: bias covariance
%
% Output
% -----
% xhat: struct with fields:
30 %         t: timestamp (1 x T)
%         x: state estimate for corresponding timestamp (nx x T)
%         P: state covariance for corresponding timestamp (nx x nx x T)
%
% meas: struct with fields:
35 %         t: timestamp (1 x T)
%         acc: accelerometer measurement for corresponding timestamp (3 x T)
%         gyr: gyroscope measurement for corresponding timestamp (3 x T)
%         mag: magnetometer measurement for corresponding timestamp (3 x T)
%         orient: orientation quaternion from the phone for corresponding
40 %         timestamp (4 x T)
%
% Measurements not available are marked with NaNs.

%% Setup necessary infrastructure
45 import('se.hendeby.sensordata.*'); % Used to receive data.

DISPLAY_FREQ = 10; % [Hz] Frequency of update of the visualization

%% Filter settings
50 t0 = []; % Initial time (initialize on first data received)
nx = 4;
% Add your filter settings here.

% Current filter state.
55 x = [1; 0; 0; 0];
P = eye(nx, nx);

% Saved filter states.
xhat = struct('t', zeros(1, 0),...
60         'x', zeros(nx, 0),...
         'P', zeros(nx, nx, 0));

meas = struct('t', zeros(1, 0),...

```

```

65         'acc', zeros(3, 0),...
        'gyr', zeros(3, 0),...
        'mag', zeros(3, 0),...
        'orient', zeros(4, 0));
try
    %% Create data link
70    err_hint = ['Unsuccessful loading of sensordata.jar!\n',...
        'Make sure to run startup.m before attempting to run this function.'];
    if nargin == 0 || isempty(fname)
        server = StreamSensorDataReader(3400);
        err_hint = ['Unsuccessful connecting to client!\n',...
75        'Make sure to start streaming from the phone *after* starting this function.'];
    else
        server = FileSensorDataReader(fname);
        err_hint = sprintf(['Unsuccessful reading data from file!\n',...
            'Make sure ''%s'' is the correct path to the log file.'], fname);
80    end
    % Makes sure to resources are returned.
    sentinel = onCleanup(@() server.stop());

    server.start(); % Start data reception.
85    clear err_hint
catch e
    e = e.addCause(MException(sprintf('%s: UnableToConnect', mfilename), err_hint));
    rethrow(e);
end
90
% Used for visualization.
figure(1);
subplot(1, 2, 1);
ownView = OrientationView('Own filter', gca); % Used for visualization.
95 ownView.activateKeyboardCallback;
googleView = [];
tnextdisp = 0; % Next time to update the visualization

%% Filter loop
100 % Repeat while data is available and q hasn't been pressed
while server.status() && ~ownView.quit
    % Get the next measurement set, assume all measurements
    % within the next 5 ms are concurrent (suitable for sampling
    % in 100Hz).
105    data = server.getNext(5);

    if isnan(data(1)) % No new data received
        continue;
    end
110    t = data(1)/1000; % Extract current time

    if isempty(t0) % Initialize t0
        t0 = t;
    end
115
    gyr = data(1, 5:7)';
    if ~any(isnan(gyr)) % Gyro measurements are available.
        % Do something
    end
120

```

```

acc = data(1, 2:4)';
if ~any(isnan(acc)) % Acc measurements are available.
    % Do something
end
125
mag = data(1, 8:10)';
if ~any(isnan(mag)) % Mag measurements are available.
    % Do something
end
130
orientation = data(1, 18:21)'; % Google's orientation estimate.

% Visualize result
if t >= tnextdisp
135
    tnextdisp = t + 1/DISPLAY_FREQ; % Next vizualization update
    setOrientation(ownView, x(1:4));
    title(ownView, 'OWN', 'FontSize', 16);
    if ~any(isnan(orientation))
        if isempty(googleView)
140
            subplot(1, 2, 2);
            % Used for visualization.
            googleView = OrientationView('Google filter', gca);
        end
        setOrientation(googleView, orientation);
145
        title(googleView, 'GOOGLE', 'FontSize', 16);
    end
end

% Save estimates
150
xhat.x(:, end+1) = x;
xhat.P(:, :, end+1) = P;
xhat.t(end+1) = t - t0;

meas.t(end+1) = t - t0;
155
meas.acc(:, end+1) = acc;
meas.gyr(:, end+1) = gyr;
meas.mag(:, end+1) = mag;
meas.orient(:, end+1) = orientation;
end
160 end

```

---

## A.2 Utility Functions

### A.2.1 Normalize quaternion

---

```

1 function [x, P] = mu_normalizeQ(x, P)
% MU_NORMALIZEQ Normalize the quaternion

    x(1:4) = x(1:4) / norm(x(1:4));
5   x = x*sign(x(1));
end

```

---



### A.2.2 Convert quaternions to Euler angles

---

```

1 function euler = q2euler(q)
  % Q2EULER Convert quaternions to Euler angles
  % euler = q2euler(q)
  % q is a quaternion in columns (4xN)
5 % euler = [yaw(z) ; pitch(x) ; roll(y)]
  q = [q(1,:);q(3,:);q(2,:);q(4,:)];
  euler = zeros(3, size(q, 2));

  xzpy = q(2, :).*q(4, :) + q(1, :).*q(3, :);

10  IN = xzpy+sqrt(eps)>0.5; % Handle the north pole
  euler(1, IN) = 2*atan2(q(2, IN), q(1, IN));
  IS = xzpy-sqrt(eps)<-0.5; % Handle the south pole
  euler(1, IS) = -2*atan2(q(2, IS), q(1, IS));

15  I = ~(IN | IS); % Handle the default case

  euler(1, I) = atan2(2*(q(2, I).*q(3, I) - q(1, I).*q(4, I)), ...
    1-2*(q(3, I).^2 + q(4, I).^2));

20  euler(3, I) = atan2(2*(q(3, I).*q(4, I) - q(1, I).*q(2, I)), ...
    1-2*(q(2, I).^2 + q(3, I).^2));

25  euler(2, :) = -asin(2*xzpy);

  euler = mod(euler+pi, 2*pi) - pi;
end

```

---

### A.2.3 $Q(q)$

---

```

1 function Q=Qq(q)
  % The matrix Q(q) defined in (13.16)
  q0=q(1); q1=q(2); q2=q(3); q3=q(4);
  Q = [2*(q0^2+q1^2) - 1  2*(q1*q2-q0*q3)  2*(q1*q3+q0*q2);
5      2*(q1*q2+q0*q3)  2*(q0^2+q2^2) - 1  2*(q2*q3-q0*q1);
      2*(q1*q3-q0*q2)  2*(q2*q3+q0*q1)  2*(q0^2+q3^2) - 1];

end

```

---

### A.2.4 $\bar{S}(q)$

---

```

1 function S=Sq(q)
  % The matrix S(q) defined in (13.11c)
  q0=q(1); q1=q(2); q2=q(3); q3=q(4);
  S=[-q1 -q2 -q3;
5     q0 -q3 q2;
     q3 q0 -q1;
    -q2 q1 q0];

end

```

---

### A.2.5 $S(\omega)$

---

```
1 function S=Somega(w)
% The matrix S(omega) defined in (13.11b)
    wx=w(1);    wy=w(2);    wz=w(3);
    S=[ 0  -wx  -wy  -wz;
5      wx   0   wz  -wy;
        wy  -wz   0   wx;
        wz   wy  -wx   0];
end
```

---

### A.2.6 $dQ(q)/dq$

---

```
1 function [Q0, Q1, Q2, Q3] = dQq(q)
% The derivative of Q(q) wrt qi, i={0,1,2,3}, as implicitly defined
% in (13.20d)
    q0=q(1);    q1=q(2);    q2=q(3);    q3=q(4);
5    Q0 = 2* [2*q0  -q3    q2;
              q3   2*q0  -q1;
              -q2   q1   2*q0];
    Q1 = 2* [2*q1    q2    q3;
              q2     0   -q0;
10             q3    q0    0];
    Q2 = 2* [ 0    q1    q0;
              q1   2*q2   q3;
              -q0   q3    0];
15    Q3 = 2* [ 0   -q0   q1;
              q0    0   q2;
              q1   q2   2*q3];
end
```

---

### A.2.7 OrientationView

The class orientation view provides a way to easily visualize orientations. The class is used in the filter template.

**self = OrientationView(figname)** Create a OrientationView figure with a given figure name.

**setOrientation(self, q, P)** Set the orientation to display, if P is given it is used to visualize the uncertainty in the orientation.

**title(self, str)** Set the title of the view to str.

**setStandStill(self, flag)** Set the stand still indicator on or off.

**setAccDist(self, flag)** Set the acceleration disturbance indicator on or off.

**setMagDist(self, flag)** Set the magnetometer disturbance indicator on or off.

### A.2.8 showIP()

showIP provides easy access to the IP the computer is using.

### A.3 sensordata.jar

The Java package `sensordata.jar` provides the functionality needed to communicate with the Sensor Fusion app and to integrate the measurements in real time into for instance Matlab. The documentation of the public interface can be found here: <https://goo.gl/uNKkL3>.



### A.4 labreport.m

The script `labreport.m` is provided as help for you to collect the material needed to present your work to the lab assistant. The report template highlights important aspects of the lab, and ensures that you have the appropriate plots etc. available.