EXAMINATION IN TSRT14 SENSOR FUSION

ROOM: Olympen, Asgård

TIME: 2024-01-02 at 8:00–12:00

EDUCODE: TSRT14 Sensor Fusion

MODULE: DAT1

DEPARTMENT: ISY

NUMBER OF EXERCISES: 4

RESPONSIBLE TEACHER:
Gustaf Hendeby, tel. 013-28 58 15, `gustaf.hendeby@liu.se`

VISITS: cirka 09:00, 10:00, 11:00

COURSE ADMINISTRATOR:
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APPROVED TOOLS: 1. Book: *F. Gustafsson*, “Statistical Sensor Fusion”, any edition.

PROVIDED MATERIAL:

1. Lecture slides; available from `/courses/TSRT14/`
2. Signal and Systems toolbox manual; available from `/courses/TSRT14/`
3. Current up to date errata for the textbook; available from `/courses/TSRT14/`

MATLAB FILES: The files that are needed for the exam are available at `/courses/TSRT14/`.

SOLUTIONS: Available at the course homepage after the exam.

The exam can be inspected and checked out 2024-01-24 at 12.30–13.00 in Gustaf Hendeby’s office, room 2A:503, B-house, entrance 27, A corridor to the right.

PRELIMINARY GRADE LIMITS:		
	grade 3	15 points
	grade 4	23 points
	grade 5	30 points

NB! Solutions should include code and plots and clear cross references between these. Mark all print-outs with your AID-number, date, course code, and exam code.

Good luck!

STARTING MATLAB (Linux)
Type `matlab &` in a terminal.

PRINTING (Linux):
Printouts of regular files can be sent to a specific printer using the command

```
lp -d printername file.pdf
```

in a terminal. (Exchange `printername` for the actual printer name.) When selecting `File/Print` for a Simulink diagram, select the target printer by adding

```
-Pprintername
```

in the `Device option` box.

ADDING YOUR AID ETC TO PRINTOUTS:
Text can be added in Matlab plots with the commands `title` and `gtext`, and for scope plots in Simulink by right clicking and then change the `Axes properties`. In Simulink diagrams it is possible to double click any empty area and then simply add text by typing it.

FURTHER GUIDELINES:

- Make sure to read all exercises and prioritize before getting started. The level of difficulty is not necessarily increasing.
- Make sure to motivate every step of your solution carefully!
- Comment nontrivial steps in the code; including model choices and tuning.
- Put code for each exercise on a separate printout and keep all related paper (hand written material, code, and plots) together when you hand in your solution.

1. The following questions all require relatively short answers, a few sentences or short calculations should be enough. (Note, an incorrect statement will result in 0 p on that subexercise.)

   (a) Consider the phone (senors) that you used in lab 2. Assume, it lies horizontal on top a laptop keyboard. The x-axis points right, the y-axis forward, and the z-axis up. What does the accelerometer, gyroscope, and magnetometer measure. Give the values (including unit) the (noise and bias free) sensors would measure, or motivate why you cannot tell. (2p)

   (b) Consider the *receiver operation characteristic* (ROC) curves in Figure 1. Which of these statements are true?

      (i) Detector A can never be realized in practice.
      (ii) Detector B can be obtained using random guessing.
      (iii) Detector C is obtained using a *Kalman filter* (KF)
      (iv) Detector C is the best of the provided detectors.
      (v) Detector C is the most powerful detector.
      (vi) Detector D is better than random guessing.
      (vii) Detector D is worse than random guessing.

   (2p)

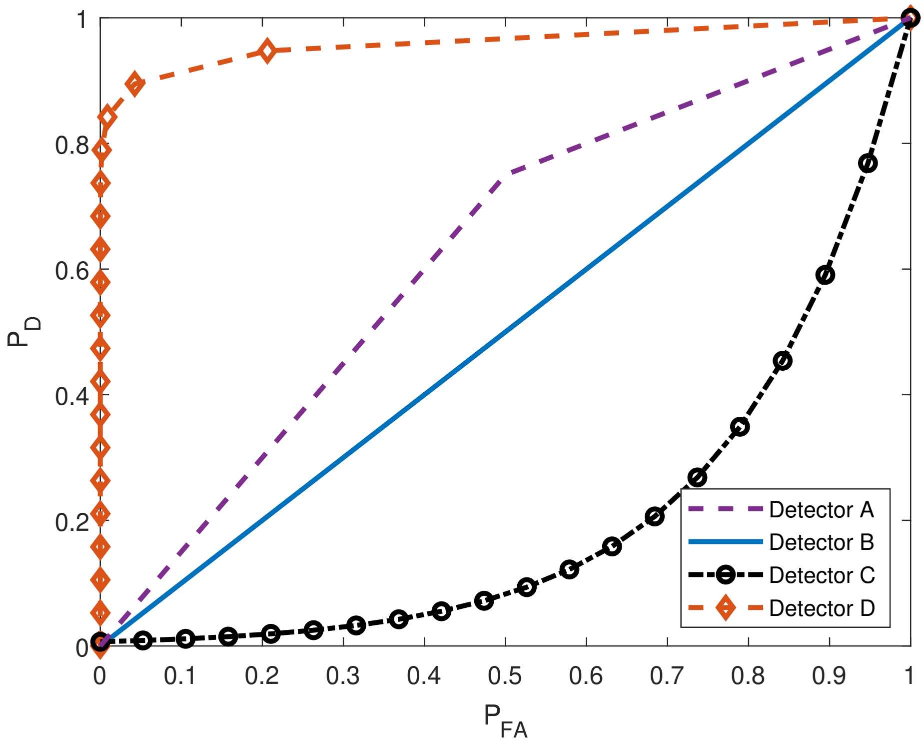

Figure 1: ROC curve for three different detectors.

(c) Provide an expression for the *weighted least squares* (WLS) estimate of the parameter x given the independent measurements $y_i = H_i x + e_i$, $\mathsf{E}(e_i) = \mu_i$, and $\text{cov}(e_i) = R_i$ for $i = 1, \ldots, N$. (2p)

(d) A stationary Kalman filter is used to estimate the level of water in a tank. In an attempt to improve the estimates, Q and R are both scaled with a factor k. How does this change the output $\hat{x}$ and P from the filter? (2p)

(e) Which of the following statements about different filter algorithms are correct?

(i) The *extended Kalman filter* (EKF) requires that the measurement model is linear.

(ii) The *extended Kalman filter* (EKF) provides identical results to the Kalman filter if the problem is linear.

(iii) The *particle filter* (PF) can, contrary to the *extended* and *unscented Kalman filter* (EKF, UKF), represent multi-modal posterior distributions.

(iv) The *particle filter* (PF) provides identical results to the Kalman filter if the problem is linear.

(v) The *particle filter* (PF) provides slightly different results each time it is run due to its random component.

(vi) The *unscented Kalmanfilter* (UKF) is guaranteed to always reach the *Cramér-Rao lower bound* (CRLB).

(2p)

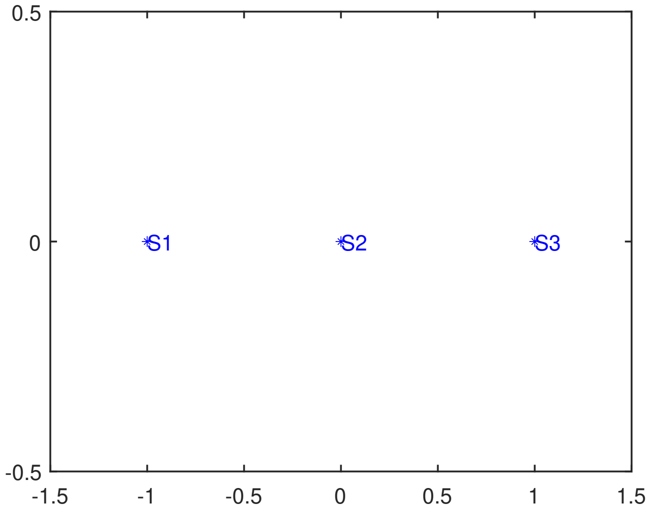

Figure 2: Sensor network for Exercise 2.

2. Consider the sensor network scenario in Figure 2.

   (a) Define a sensor network in Matlab according to Figure 2, and assume that each sensor measures the distance from the target to the sensor. The measurement noise is Gaussian with variance $R = 0.001$. Estimate the first target position from the first measurement (column) in `ex2_y` found in `data20240102`. The first row corresponds to the measurement from the first sensor, and so on. Present a plot of the network with a 90 % confidence ellipsoid for the target position. (5p)

   (b) Using a constant velocity mode with process noise $Q = \left(\begin{smallmatrix} 0.01 & 0 \\ 0 & 0.1 \end{smallmatrix}\right)$, apply an EKF and a PF to estimate the position of the target observed with the measurements in `ex2_y`. Motivate your choice of number of particles, `Np`! Plot the result from the filters in the same figure with 90 % confidence ellipsoids. (5p)

3. Consider the model

$$y_k = Hx + e_k, \qquad e_k \sim \mathcal{N}(0, \lambda),$$

where y_k, H, x and e_k are all scalar.

(a) Derive the *maximum likelihood* (ML) estimate for λ given y_1 and x. (4p)

(b) Compute the *Cramér-Rao lower bound* (CRLB) for estimating λ from y_1 given x. (4p)

(c) Compute the CRLB for estimating λ from $y_{1:N}$ given x. (2p)

4. Consider a heading estimation problem, where we have access to a compass signal y^h with poor resolution and a yaw rate sensor signal y^ω with good accuracy but perturbed with a constant bias. The models for these signals are

$$\begin{aligned} y_k^h &= h_k + e_k^h, & e_k^h &\in \mathcal{N}(0, R^h), \\ y_k^\omega &= \omega_k + b + e_k^\omega, & e_k^\omega &\in \mathcal{N}(0, R^\omega), \end{aligned}$$

where h is the heading, $\omega = \dot{h}$ is the yaw rate and b is a constant bias. Consider a scenario with $R^h = 10^2$ and $R^\omega = 1^2$, where both sensors sample with sample time of $T_s = 1$. Measurements from this setup is available in `data20240102.mat`. Measured values of the signals y^h is available as `ex4_yh` and y^ω as `ex4_yw`. The true states h, ω and b called `ex4_h`, `ex4_w`, and `ex4_b`, respectively, are also available for evaluation purposes only.

(a) Suggest a simple linear state-space model for this problem with $x_k = \begin{pmatrix} h_k & b \end{pmatrix}^T$. Apply a *Kalman filter* (KF) and present separate plots illustrating how the filter tracks the heading h_k and estimates the bias b, respectively. Plot the ground truth, the estimates, and the uncertainty, in the same plot. (6p)

(b) Suggest a simple linear state-space model for this problem with $x_k = \begin{pmatrix} h_k & \omega_k & b \end{pmatrix}^T$. Apply a KF and present separate plots illustrating how the filter tracks the heading h_k and estimates the bias b, respectively. (4p)