EXAMINATION IN TSRT14 SENSOR FUSION

ROOM: Asgård, Egypten

TIME: 2023-08-16 at 8:00–12:00

EDUCODE: TSRT14 Sensor Fusion

MODULE: DAT1

DEPARTMENT: ISY

NUMBER OF EXERCISES: 4

RESPONSIBLE TEACHER:
Gustaf Hendeby, tel. 013-28 58 15, `gustaf.hendeby@liu.se`

VISITS: cirka 08:30, 10:00

COURSE ADMINISTRATOR:
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APPROVED TOOLS: 1. Book: *F. Gustafsson*, "Statistical Sensor Fusion", any edition.

PROVIDED MATERIAL:
1. Lecture slides; available from `/courses/TSRT14/`
2. Signal and Systems toolbox manual; available from `/courses/TSRT14/`
3. Current up to date errata for the textbook; available from `/courses/TSRT14/`

MATLAB FILES: The files that are needed for the exam are available at `/courses/TSRT14/`.

SOLUTIONS: Available at the course homepage after the exam.

The exam can be inspected and checked out 2023-09-07 at 12.30–13.00 in Gustaf Hendeby's office, room 2A:503, B-house, entrance 27, A corridor to the right.

PRELIMINARY GRADE LIMITS:

grade 3	15 points
grade 4	23 points
grade 5	30 points

NB! Solutions should include code and plots and clear cross references between these. Mark all print-outs with your AID-number, date, course code, and exam code.

Good luck!

STARTING MATLAB (Linux)
Type `matlab &` in a terminal.

PRINTING (Linux):
Printouts of regular files can be sent to a specific printer using the command

```
lp -d printername file.pdf
```

in a terminal. (Exchange `printername` for the actual printer name.) When selecting `File/Print` for a Simulink diagram, select the target printer by adding

```
-Pprintername
```

in the `Device option` box.

ADDING YOUR AID ETC TO PRINTOUTS:
Text can be added in Matlab plots with the commands `title` and `gtext`, and for scope plots in Simulink by right clicking and then change the `Axes properties`. In Simulink diagrams it is possible to double click any empty area and then simply add text by typing it.

FURTHER GUIDELINES:

- Make sure to read all exercises and prioritize before getting started. The level of difficulty is not necessarily increasing.

- Make sure to motivate every step of your solution carefully!

- Comment nontrivial steps in the code; including model choices and tuning.

- Put code for each exercise on a separate printout and keep all related paper (hand written material, code, and plots) together when you hand in your solution.

1. (a) Consider a setup of 4 sensors, similar to the one in lab 1. The sensors measure the time when a sound pulse is detected, the exact time when the pulse is broadcast is unknown. The resulting measurements (where time has been converted to distance) are assumed to be:

$$y^i = r_0 + \|p - p^i\| + e^i,$$

where r_0 handles the unknown time of the broadcast, p is the position of the sound source, p^i the position of the ith sensor, and $e^i \sim \mathcal{N}(0, R^i)$ are mutually independent. Give the measurement equation and noise characteristics for the (virtual) measurement:

$$\bar{y} = \begin{pmatrix} y^1 - y^4 & y^2 - y^4 & y^3 - y^4 \end{pmatrix}^T!$$

(Do not forget to specify the resulting measurement noise, $\bar{e}$.)　　(2p)

(b) Provide the GLR test statistics to decide between $\mathcal{H}_0$ and $\mathcal{H}_1$ given the measurement y:

$$\begin{cases} y = e & \text{under } \mathcal{H}_0 \\ y = Hx + e & \text{under } \mathcal{H}_1 \end{cases}$$

Assume that $e \sim \mathcal{N}(0, I)$ and x is *unknown*.　　(2p)

(c) Given the posterior $p(x|y)$ in Figure 1, give the minimum variance estimate, $\hat{x}^{\text{MV}}$, of x!　　(2p)

(d) Assume $x^1 \sim \mathcal{N}(\mu^1, R^1)$, $x^2 \sim \mathcal{N}(\mu^2, R^2)$, and $\text{cov}(x^1, x^2) = R^{12}$. Give the posterior distribution $p(x^1|x^2 = \hat{x})$!　　(2p)

(e) A *unscented Kalman filter* (UKF) is used to track an airplane. The result is very "laggy", and the estimates do not follow the expected movements of the plane and react noticeably late. The measurement noise covariance matrix R was derived from the data sheet provided by the sensor manufacturer.

How would you re-tune the filter parameters (R and Q) to get a more reasonable result?　　(2p)
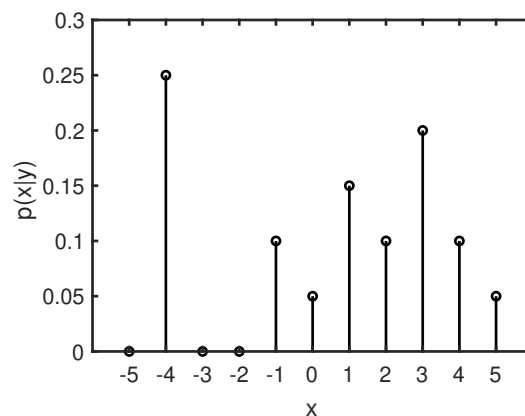

Figure 1: Posterior for assignment 1(c).

2. Assume an area with three distinct landmarks with the positions m^1, m^2, and m^3. A sensor mounted on an RC car is moving around in the area, observing the relative position of the landmarks in the global Cartesian coordinate system. At each time, and for each landmark, the following measurement is obtained

$$y_k^i = m^i - x_k + e_k^i,$$

where x_k is the position of the sensor, and the measurement noise $e_k \sim \mathcal{N}(0, R)$. The covariance matrix $R = 10^2 I$.

The file `data20230816.mat` contains the variables `ex2_x`, with an (approximate) trajectory (positions, one position per column) for the sensor, and `ex2_y`, with measurements from the sensor collected at $1\,\mathrm{Hz}$. The measurements of the landmarks are stacked, *i.e.*, the first two values in each column is y^1 according to the notation above, the next two y^2, and so on. (That is, the landmark association is assumed known.)

(a) Assume that the sensor/car positions are given, and estimate the positions of the landmarks. (The mapping problem.) Make sure to carefully motivate what you are doing. Plot the estimated landmark positions, together with the trajectory in a plot. Provide information about the uncertainty of the estimate. (4p)

(b) Now assume both landmark positions and the sensor trajectory are unknown. Repeat the task in (a) while simultaneously estimating the sensor trajectory. Illustrate the estimated trajectory, as well as the position and uncertainty of the landmarks at the end (both with $90\,\%$ covariance ellipses). (4p)

(c) Discuss the difference in the results in (a) and (b)! (2p)

3. The topic of this exercise is to explore the result of scaling the process and measurement noise covariance. Therefore, assume two different Kalman filters both using the model

$$x_{k+1} = Fx_k + w_k$$
$$y_k = Hx_k + e_k,$$

where the only difference is the scaling of the noise. Filter A uses the initial uncertainty P_0^A, the process noise covariance $\operatorname{cov} w_k = Q^A$, and the measurement noise $\operatorname{cov} e_k = R^A$. Filter B uses $P_0^B = \gamma P_0^A$, $\operatorname{cov} w_k = Q^B = \gamma Q^A$, and $\operatorname{cov} e_k = R^B = \gamma R^A$, where $\gamma > 0$. Both filters are initialized with the same state estimate $\hat{x}_{0|0}^B = \hat{x}_{0|0}^A$.

(a) Derive expressions for Filter B's filter estimate ($\hat{x}_{k|k}^B$ and $P_{k|k}^B$) in terms of Filter A's filter estimate ($\hat{x}_{k|k}^A$ and $P_{k|k}^A$), and γ.

   Hint: *One convenient way to solve this task is to use induction, and to initially study the time and measurement update steps independently, and then combine the results at the end.* (6p)

Now, substitute the linear model above for the nonlinear model

$$x_{k+1} = f(x_k) + w_k$$
$$y_k = h(x_k) + e_k,$$

everything else is the same. Now answer the following:

(b) For this model, does the result in (a) hold if you instead use a regular *extended Kalman filter* (EKF)? Motivate your answer carefully! (2p)

(c) For this model, does the result in (a) hold for the nonlinear model if you instead use a regular *unscented Kalman filter* (UKF)? Motivate your answer carefully! (2p)
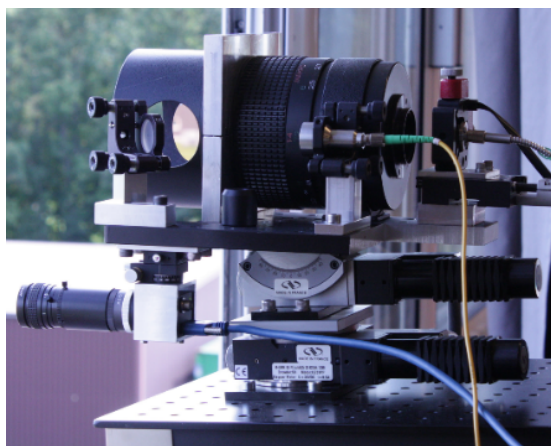
Figure 2: Experimental setup for the photon counting lidar generating the studied dataset.

4. The variable `ex4_y` in the file `data20191031.mat` contains measurements from a "photon counting" lidar (see Figure 2). In theory, the lidar works by sending out short laser pulses, and then registering when the first photon comes back, from which the distance to the reflecting surface is computed. However, in practice, many of the photons detected do not come from reflections but are simply random noise.

This yields measurements with the following properties:

- A detected photon was with probability 40% reflected in a surface, and is otherwise noise.

- A reflected photon results in the measurement $y = x + e$, where x is the distance to the surface, and $e \sim \mathcal{N}(0, 0.0043)$ is measurement noise.

- A random photon results in the measurement $y = e$, where $e \sim \mathcal{U}(250, 350)$ is uniformly distributed in the measurement volume (here between 250 and 350 meters).

(a) Derive the probability of a spurious (random) photon detection at distance y, $p(y|\mathcal{H}_0)$, given there is no reflecting surface at distance x (the $\mathcal{H}_0$ hypothesis)! (2p)

(b) Derive the probability to obtain the measurement y given that there is a reflecting surface at distance x (the $\mathcal{H}_1$ hypothesis), $p(y|x, \mathcal{H}_1)$! (4p)

(c) Derive and plot the log-likelihood-ratio $\log\big(p(\mathbf{y}|x, \mathcal{H}_1)/p(\mathbf{y}|\mathcal{H}_0)\big)$ for a reflecting surface at distance x compared to no target at all! Plot the result for x in the range 250 to 350, and determine if there are any surfaces and if so at what distances!
Hint: The individual probabilities quickly become very small, to avoid numerical issues take the logarithm of each component individually before they are combined. (4p)

Be sure to motivate all your your answers.