

EXAMINATION IN TSRT14 SENSOR FUSION

ROOM: ISY's computer rooms

TIME: 2022-08-17 at 8:00–12:00

COURSE: TSRT14 Sensor Fusion

PROVKOD: DAT1

DEPARTMENT: ISY

NUMBER OF EXERCISES: 4

RESPONSIBLE TEACHER: Gustaf Hendeby, tel. 013-28 58 15,
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VISITS: cirka 09:00, 10:00, 11:00

COURSE ADMINISTRATOR: Ninna Stensgård, 013-282225,
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APPROVED TOOLS: 1. *F. Gustafsson*: “Statistical Sensor Fusion”

PROVIDED MATERIAL:

1. Lecture slides; available from `/courses/TSRT14/`
2. Signal and Systems toolbox manual; available from `/courses/TSRT14/`
3. Current up to date errata for the textbook; available from `/courses/TSRT14/`

MATLAB FILES: The files that are needed for the exam are available at `/courses/TSRT14/`.

SOLUTIONS: Available at the course homepage after the exam.

The exam can be inspected and checked out 2022-09-13 at 12.30–13.00 in Gustaf Hendeby's office, room 2A:503, B-house, entrance 27, A corridor to the right.

PRELIMINARY GRADE LIMITS: grade 3 15 points
grade 4 23 points
grade 5 30 points

NB! Solutions should include code and plots and clear cross references between these. Mark all print-outs with your AID-number, date, course code, and exam code.

Good luck!

STARTING MATLAB (Linux)

Type `matlab &` in a terminal.

PRINTING (Linux):

Printouts of regular files can be sent to a specific printer using the command

```
lp -d printername file.pdf
```

in a terminal. (Exchange `printername` for the actual printer name.) When selecting `File/Print` for a Simulink diagram, select the target printer by adding

```
-Pprintername
```

in the `Device option` box.

ADDING YOUR AID ETC TO PRINTOUTS:

Text can be added in Matlab plots with the commands `title` and `gtext`, and for scope plots in Simulink by right clicking and then change the `Axes properties`. In Simulink diagrams it is possible to double click any empty area and then simply add text by typing it.

FURTHER GUIDELINES:

- Make sure to read all exercises and prioritize before getting started. The level of difficulty is not necessarily increasing.
- Make sure to motivate every step of your solution carefully!
- Comment nontrivial steps in the code; including model choices and tuning.
- Put code for each exercise on a separate printout and keep all related paper (hand written material, code, and plots) together when you hand in your solution.

1. The following questions all require relatively short answers, a few sentences or short calculations should be enough. (Note, any incorrect statement will result in 0 p on that question.)

- (a) Consider the problem studied in lab 2 to find the orientation of a smart-phone using its builtin *inertial measurement unit* (IMU). In this setting, which statements are true?
- (i) The gyroscope is enough to solve the problem, but the solution poorly handles slow rotations.
 - (ii) An accelerometer and gyroscope can be used to solve the problem.
 - (iii) An accelerometer and a magnetometer can solve the problem, but will struggle under fast rotations.
 - (iv) The roll and pitch angles can be derived from the accelerometer on its own.
 - (v) The method developed during the lab will not work well in a roller coaster.

(2p)

- (b) You are given a batch of measurements Y , which relates to an unknown parameter x as

$$Y = Hx + e,$$

where e is Gaussian with covariance $R = \text{cov}(e)$. An estimator $\hat{x}(Y)$ is provided with the property that $\mathbb{E}(\hat{x}(Y))$ is the true parameter value. What is the best error covariance $\text{cov}(\hat{x}(Y))$ that can be achieved? (2p)

- (c) A Kalman filter has been designed to solve a state estimation problem. At time of design, the sensor specification was unknown, and filters were designed with three different noise variance values:

$$R_1 \prec R_2 \prec R_3.$$

Except for the noise covariance, the filters are otherwise identical.

Pair the three filter results (a)–(c) in Figure 1 to the measurement variances R, R_1 – R_3 , that were used! (2p)

- (d) Which of the following statements about *simultaneous localization and mapping* (SLAM) are true?
- (i) SLAM is considered an unsolved problem, without any known algorithms to solve it.
 - (ii) EKF-SLAM scales favorably with the number of landmarks in the map.
 - (iii) EKF-SLAM scales unfavorably with the number of landmarks in the map.
 - (iv) SLAM requires some map information to work.
 - (v) For SLAM to work well, it is important to consider the correlations between the positions of the different landmarks.

(2p)

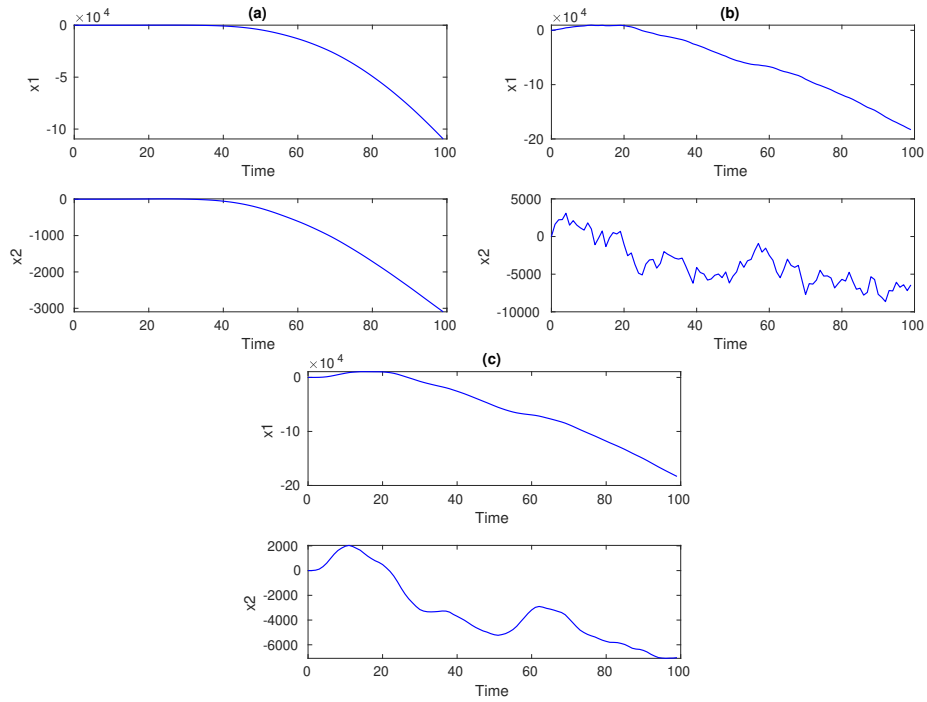


Figure 1: Filtering results for the filters designed with the different R values in Exercise 1(c).

(e) The following nonlinear system is given:

$$x_k = \begin{pmatrix} x_k^n \\ x_k^l \end{pmatrix} = \begin{pmatrix} f_k^n(x_k^n) + F_k^n x_k^l + v_k^n \\ f_k^l(x_k^n) + F_k^l x_k^l + v_k^l \end{pmatrix}$$

$$y_k = h_k(x_k^n, x_k^l) + e_k.$$

It is known that the measurements are very informative, compared to the motion model.

A standard particle filter has been designed to estimate the state x_k . It is observed that the performance is not as good as expected, suggest a suitable method to improve performance given the provided information. (2p)

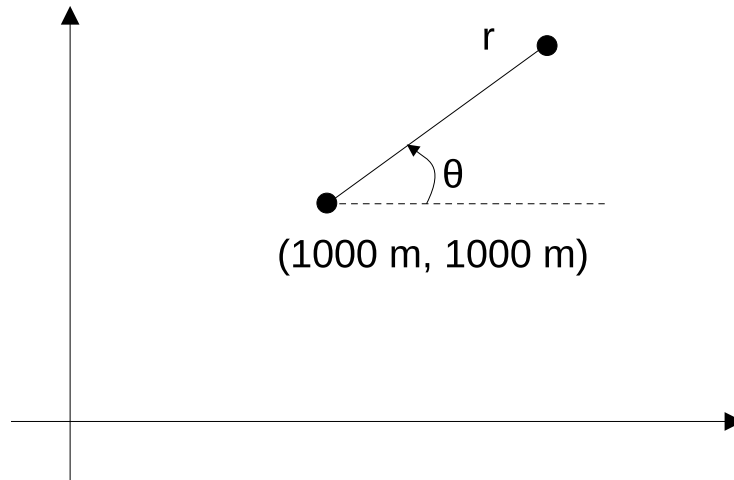


Figure 2: Radar setup considered in Exercise 2.

2. Consider the problem of tracking an aircraft flying in an area (disregard the altitude) using a range-bearing radar sensor. The radar is located at $(1000 \text{ m}, 1000 \text{ m})$, and you obtain the measurements

$$y_k = \begin{pmatrix} r_k + e_k^r \\ \theta_k + e_k^\theta \end{pmatrix}$$

using the notation given in Figure 2. The radar provides measurements with $\text{cov}(e_k^r) = 10^2 \text{ m}^2$ and $\text{cov}(e_k^\theta) = 0.1^2 \text{ rad}^2$. The measurements are available in the file `data20220817.mat`, where `ex2_t` contains timestamps, and `ex2_y` the radar measurements (one column per measurement).

Make sure to motivate any nontrivial steps, e.g., the choice of models, tuning, etc.

- (a) Construct a measurement model that describes the radar! Furthermore, plot the position of the aircraft from the measurements (in Cartesian coordinates)! Based on the plot, design a motion model to describe the targets movements! (3p)
- (b) Estimate the position of the aircraft using an UKF! Plot the estimated trajectory together with the covariance ellipses and the measurements (in Cartesian coordinates) in a new plot! (4p)
- (c) Estimate the position of the aircraft using a PF! Plot the estimated trajectory together with the covariance ellipses and the measurements (in Cartesian coordinates) in a new plot!. (3p)

3. Suppose we observe the Cartesian position of a target that is known to be located on the unit circle. The sensor model for this problem can be written

$$y = \begin{pmatrix} \cos(x) \\ \sin(x) \end{pmatrix} + e,$$
$$e \sim \mathcal{N}(0, \sigma^2 I_2).$$

- (a) Show that the maximum likelihood estimate (MLE) for the polar position (angle) is given by

$$\hat{x} = \arctan(y_2/y_1). \tag{2p}$$

- (b) Show that Gauss approximation formula (the TT1 approximation) gives

$$\text{var}(\hat{x}) = \sigma^2. \tag{3p}$$

- (c) Simulate $N = 10000$ data for $x = 0$, $\sigma^2 = 0.1$ and apply the MLE. What is the mean square error (MSE)? Compare to the approximation in (b). (5p)

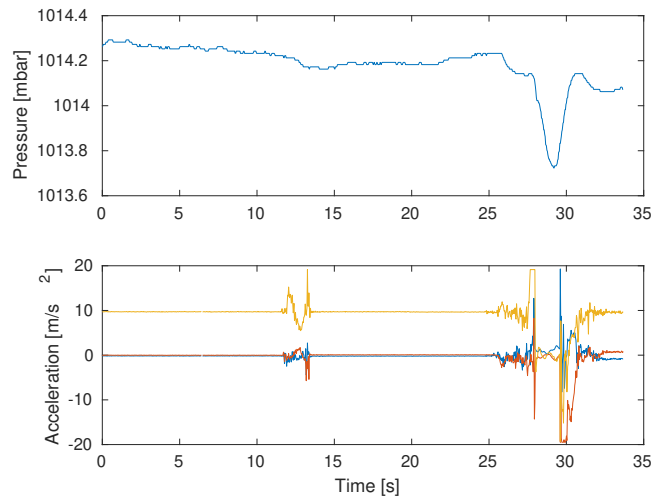


Figure 3: Smartphone data of pressure in millibar and acceleration in m/s^2

4. In a famous experiment, the examiner is throwing his smartphone four meters up in the air (catching it on the way down). Part of the experiment was a calibration procedure, during which the phone was moved from the ground to an table. In this exercise, the calibration phase will be used to estimate the height of the table. Is it possible to estimate the height based only on the dead reckoning of the accelerations?

The logged sensor signals can be seen in Figure 3. **Only** the accelerometer signal will be used in this exercise, and **only** the two intervals 6.7–11.0s (samples 200–330, the phone is stationary on a flat surface) and 11.2–13.7s (samples 335–410, the where the phone is lifted from the ground to the table). The accelerometer signal and the time stamps are found as `ex4_y` (one column per time) and `ex4_t`, respectively, in `data20220817.mat`.

To obtain full credits, the exercise should be solved with techniques inspired by the methods discussed in the course, and all steps be well motivated.

- (a) Neglect horizontal acceleration and assume that the smartphone is exactly horizontal all the time. Suggest a simple motion model, and use it to compute and plot the vertical speed and position as a function of time! You may make use of both the specified intervals to get an as good result as possible. How high is the table? (6p)
- (b) Again, neglect horizontal acceleration, but now account for that the smartphone is not exactly horizontal all the time. The vertical acceleration is with this assumption the magnitude of the measured acceleration. Modify the motion model in (a), and re-estimate the vertical speed and position as a function of time! Plot the result! What is the estimated height of the table this time? (3p)
- (c) When using the method in (b), an additional (implicit) assumption has been introduced in order to use the norm of the acceleration this way. Which one? Is it reasonable? (1p)