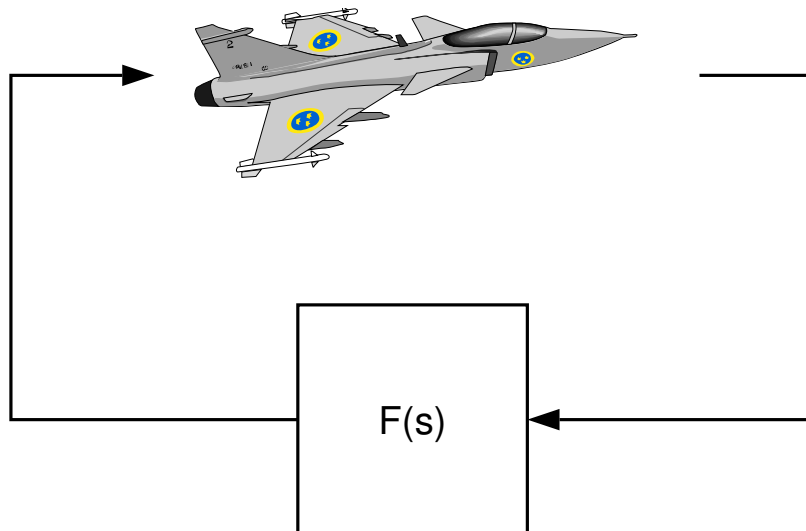


Control Theory, TSRT09

Robust controller design of JAS 39 Gripen

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1 Introduction

This lab deals with the control design of the JAS 39 Gripen using optimal \mathcal{H}_∞ methodology. The goal of the laboratory is to develop a controller that meets typical requirements on reference tracking, noise suppression and robustness.

The control system of an aircraft is intended to help the pilot to fly the plane by converting the pilot's commands to the appropriate rudders so that the aircraft moves according to the pilot's wishes. By measuring and feeding back the aircraft's movements, the control system can also automatically mitigate the impact of wind disturbances. To replace the mechanical coupling between joystick and rudders with electric wires this way is called "fly-by-wire" technology.

The aircraft's dynamics depend on the height and speed of the plane because the air resistance varies with these parameters. The most common approach is to divide the altitude and speed range you want to fly in (flight envelope) in a number of different operating points. Around each operating point a linear model of the dynamics is calculated for which one linear controller is constructed. The control signal is then computed given the aircraft's height and speed by interpolating the control laws from nearby operating points. This method is called gain scheduling and is used today in Gripen's control system.

It is also common to treat the aircraft's movements in height, pitch and lateral separately when the control system is constructed. In this laboration we only consider the lateral dynamics.

2 Model

The aircraft model used in the laboratory describes the lateral dynamics of the Gripen at 500 m height and at speed Mach 0.6 (730 km/h). The model is given on state space form, has two control signals, three outputs and five states:

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + w\end{aligned}$$



The control inputs u_1 and u_2 are the commanded skew- and side-rudder angles. A skew-rudder command is realized on Gripen by having the right and left hind rudders move in different directions, see Figure 1. The hind rudders are also used (together with the nose wings) to realize high altitude commands while more traditional aircrafts have separate rudders for that.

The outputs (measurements) are sideslip-angle β , roll angular velocity p and turning angular velocity r , see figure 2. This model is described in example 2.3 in the course book (p. 46) where also the course-angle and roll-angle are modeled with states. In the lab these have been left out since the control of these variables normally is left to the pilot.

The process noise w represents the influence of wind disturbances on the plane. In the laboration, Drydens wind model, which is a standard model, is used. Using this model, wind disturbances are simulated by having white noise pass through a linear filter. The influence of these disturbances on the plane are described in example 5.4 in the course book (p. 136). In total we have

$$w = H(p)e$$

where e is the white noise sources in turning- and roll- direction. The singular values for H are shown in figure 3.

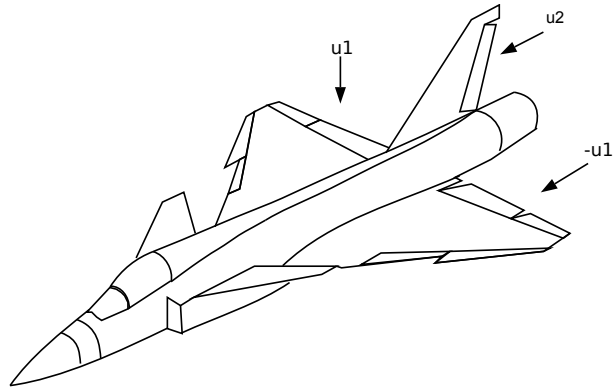


Figure 1: Rudders used in the lab.

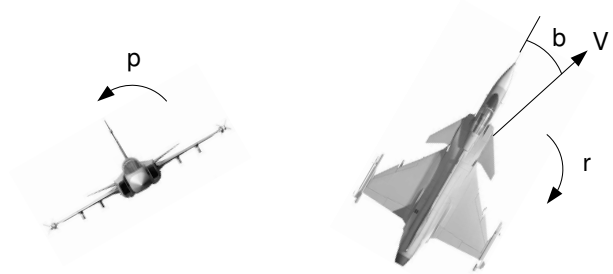


Figure 2: Illustration of roll angular velocity p , sideslip angle β and turning angular velocity r . V represents the wind direction.

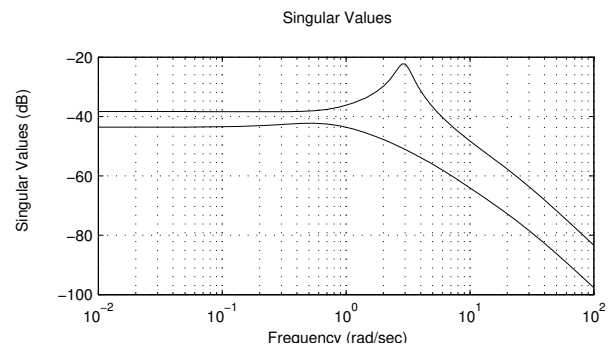


Figure 3: Singular values for the transfer function of the wind disturbance $H(s)$.

3 Design objectives

Your task is to design a control system for the lateral dynamics of JAS 39 Gripen based on \mathcal{H}_∞ control and with a structure as in figure 4. Note that this structure differs somewhat from the structure that is used in chapter 6 in the course book. Choice of controller structure is discussed in section 8.2 in the book. The control system takes reference values for sideslip angle ¹ β and roll angular velocity p and meet the following requirements:

3.1 Reference tracking

- The step response in roll angular velocity p shall look like a first order system with time constant 0.3 s.
- The step response for a β -command shall lie within the bounds in figure 5.
- Cross couplings between the controlled variables should be kept to a minimum.

3.2 Noise suppression

- The influence of the wind on the plane should be made acceptable. This can be quantified by putting frequency domain constraints on $S \cdot H$, where S is the sensitivity function and H is the transfer function for the wind disturbance. For H in figure 3, the influence of the wind is considered acceptable for instance when

$$|SH| < -40\text{dB}, \omega < 1 \text{ rad/s.}$$

$$|SH| < -27\text{dB}, \omega = 3 \text{ rad/s.}$$

¹Normally $\beta = 0$ is wanted so that the engines air intake is in the direction of the wind and so that the pilot is not subject to unwanted side-forces due to that the plane "slide" through the air.

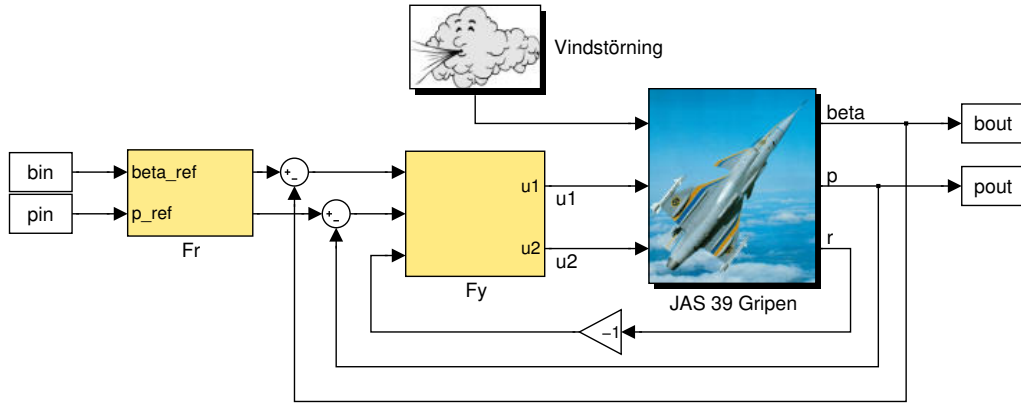


Figure 4: Block diagram over the closed system.

3.3 Robustness

- In order not to excite flexibilities in the body of the airplane (structural modes) the amplification of the feedback signals in the controller may not exceed 3.0 for frequencies above 4 Hz.
- To achieve suitable phase- and amplitude-margins the controlled system shall manage a 30 % relative modelling error on all frequencies.
- The influence of an extra pole in the rudder-dynamics (common to both rudders) located above 40 rad/s may not lead to instability. The true system is given by

$$G_0 = G \begin{pmatrix} \frac{1}{s/p_0+1} & 0 \\ 0 & \frac{1}{s/p_0+1} \end{pmatrix}, \quad p_0 \geq 40$$

4 Optimal \mathcal{H}_∞ -control with Matlab

The Control System Toolbox supports classical design methods such as pole-placement and root locus and even LQG-design. There is however no support for robust control design. To this end there is instead a Robust Control Toolbox that has commands for e.g. LTR (Loop Transfer Recovery) and optimal \mathcal{H}_2 - and \mathcal{H}_∞ -control.

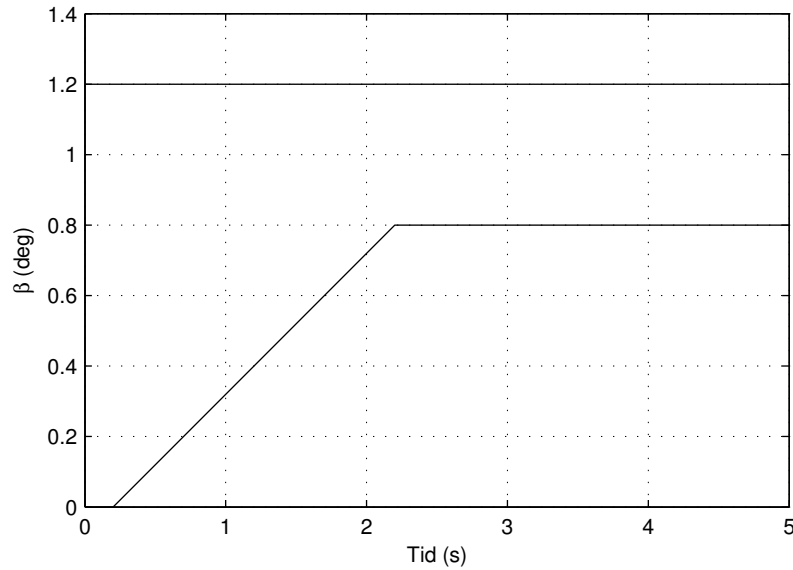


Figure 5: Bounds for a step response on the sideslip angle β .

`plot([0 0.2 2.2 5],[0 0 0.8 0.8 ; 1.2 1.2 1.2 1.2])`
generates the figure.

In the lab only one command from the Robust Control Toolbox is used which is `hinfsyn` for \mathcal{H}_∞ -design. With this method the design requirements are described with weight function for the sensitivity functions and the control signal. According to section 10.2 in the course book the norm for the extended transfer-function G_e from w to z minimized by a controller $u = -F_y y$,

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ y \end{bmatrix} = \begin{bmatrix} W_u u \\ W_T G u \\ W_S (G u + w) \\ G u + w \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & W_u \\ 0 & W_T G \\ W_S & W_S G \\ I & G \end{bmatrix}}_{G_e} \begin{bmatrix} w \\ u \end{bmatrix}$$

For a system with two inputs and three outputs, the extended system is constructed as a LTI-object in Matlab with

```
» Ge = [zeros(2,3) WU;zeros(3,3) WT*G; WS WS*G;eye(3) G];
» Ge = minreal(Ge);
```


The function `minreal` constructs a minimal realisation of \mathbf{G}_e . This is necessary since several copies of e.g. \mathbf{G} otherwise will exist in \mathbf{G}_e .

As described in the course book, the upper bound γ is iterated when doing \mathcal{H}_∞ -optimization until a minimal value of γ is obtained. In Robust Control Toolbox this is done by the command `hinfsvn`. This command takes as parameters the extended system \mathbf{G}_e , the number of in- and outputs of the model as well as intervals and precision on γ . For a (3×2) -system, extended as above, the \mathcal{H}_∞ -solution is computed with for example the command

```
» [Fy, cl, gamma, info] =  
    hinfsvn(Ge,3,2,'GMIN',.1,'GMAX',100,'TOLGAM',.01);  
» Fy = -Fy;
```

In this case γ -values between 0.1 and 100 are tested and a precision of 0.01 is obtained. Run `help hinfsvn` to investigate the command. The change in sign of the controller F_y above comes from the convention of Robust Control Toolbox to use positive feedback.

5 Execution

The lab has two tasks. In the first task the open system shall be analyzed in order to sort out why a control system is needed at all – obviously it is possible to control many airplanes without electronic control systems, i.e. gliders. In task 2 a control system should be designed based on optimal \mathcal{H}_∞ -control according to the design objectives in section 3.

The lab is done with Matlab. Necessary files are available on the shared drive for the course.

Task 1: Analysis of the open system

The objective with this task is to answer the question: Why is a control system needed?

The following questions can be used as a basis for discussion: How does the open system relate to the design objectives of the closed loop system? What requirements need feedback? Is the system stable? How coupled is the system? What control input and system output are most coupled? How much is the plane influenced by the wind disturbances?

Tips for execution:

- Download the models for Gripen (**G**) and the wind disturbances (**H**):
 » `load jasmodell`
- Type `jas` to open the Simulink-model for Gripen, see figure 6.
- Double click on the switches to turn on and off the wind disturbances as well as to generate rudder movements.
- After a simulation the flight trajectory can be visualized by typing `jastraj`.
- Compute the system poles and zeros and its singular values and relate these to the simulations.
- The system matrices for G are obtained from **G.A**, **G.B**, **G.C** and **G.D**.
- Useful Matlab-commands: `step`, `sigma`, `norm(G,inf)`, `eig`, `pole`, `zero`.

Task 2: Design of a \mathcal{H}_∞ -controller

The objective with this task is to design a \mathcal{H}_∞ -controller that meets the design objectives in section 3.

Tips for execution:

- Type `jasstyr` to open the Simulink-model for the closed loop system, see figure 7.
- The controller is defined by the LTI-objects `Fr` and `Fy`.
- Write your Matlab-code in a m-file that
 - defines the weight functions (see preparatory exercise 7)
 - computes the controller (see section 4 and preparatory exercise 8)
- Strive to find weight functions that gives $\gamma \approx 1$ a solution.
- One possibility to influence the response of the pilot commands is to introduce dynamics in the prefilter F_r .

Evaluation

The controller can be evaluated by

- plotting the singular values for S , T , etc. (see preparatory exercise 6).
- plot step responses (via Simulink or with the `step`-command).
- simulate the influence of the wind disturbances with and without a controller.
- step responses that show that the system is decoupled.

At the evaluation you should have figures that clearly show that the requirements are met. Figures of the singular values should also include the bounds/requirements that exist. To verify that the step responses meet the requirements, the lines in figure 5 should be drawn or the step response for a first order system with time constant 0.3 s.

To facilitate the evaluation of the sensitivity function you should investigate the frequency components of the wind's influence on the output according to the suggestion given in Section 3.2.

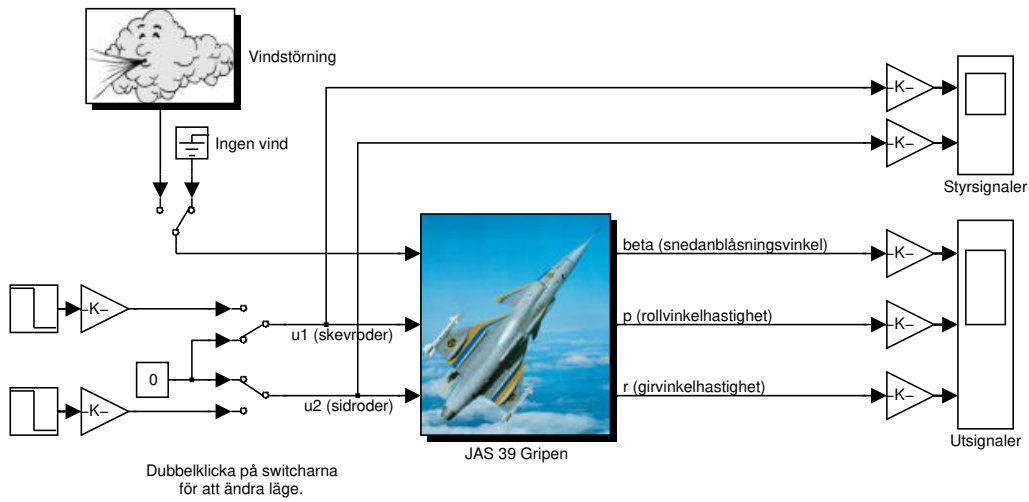


Figure 6: Simulink scheme in task 1.

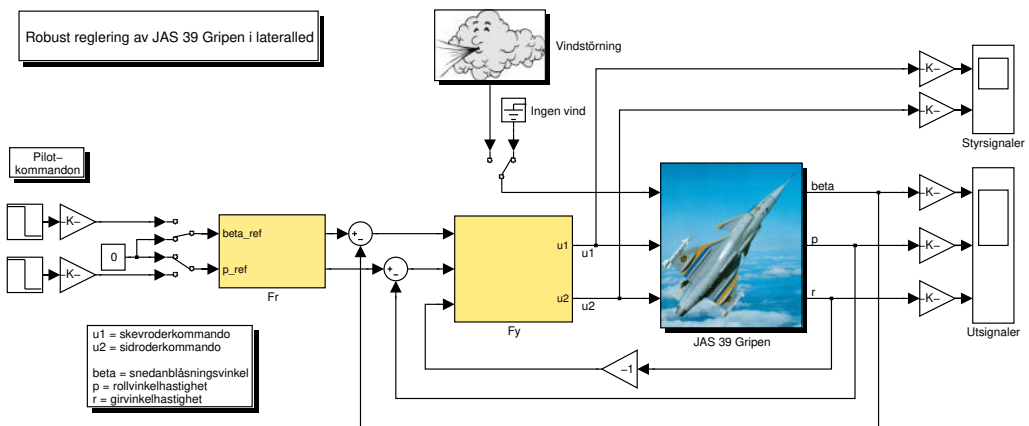


Figure 7: Simulink scheme in task 2.

6 Preparatory exercises

For the lab to be rewarding (and fun) it is of great importance that you have completed the following preparatory exercises before the lab. The preparatory exercises must be approved by the course assistant at a pre-booked time, before starting the lab. A skeleton for the matlab code is available on the course web page (`code_skeleton.m`).

1. Read the complete lab pm and study sections 10.2 and 10.4 in the course book.
2. Draw the transfer function for a first order system with time constant 0.3 and static gain 1.
3. What do the following functions describe
 - the sensitivity function, S
 - the complementary sensitivity function, T
 - the transfer function from process noise to control input, G_{wu}

Also, how are they defined and how can the command `feedback` in MATLAB be used to compute them!

Tips

- The call `feedback(A,B)` gives the transfer matrix $(I + AB)^{-1}A$. Pay attention to what A and B are in the 3 functions above.

4. Formulate the requirements on noise suppression and robustness as requirements on S , on T and on the controller F_y . For S , see Section 3.2, for T and F_y , see Section 3.3.

Tips

- The two conditions of Sections 3.2 can be roughly summarized by imposing that $|S(i\omega)|$ is as small as possible for frequencies up to around 6 rad/s.
- According to (6.29) in the book, robustness can be translated into requirements on T by quantifying ΔG . Two of the items of Section 3.3 admit such a formulation. Both lead to requirements on $|T(i\omega)|$.
- The true system can be rewritten as

$$G_0 = \frac{1}{s/p_0 + 1} I_{3 \times 3} G = (I_{3 \times 3} + \Delta G)G$$

with a suitable choice of ΔG . Why?

5. a) Sketch the amplitude curve for the transfer function

$$W(s) = k \frac{s/a + c}{s/b + 1}.$$

Let $c = 1$, what happens with the amplitude curve in the cases $a > b$ and $a < b$ respectively. What does it look like if $a = \infty$? What does it look like if $c = 0$?

Tips

- See section 4.3 in the basic control theory book (Glad och Ljung, *Reglerteknik: Grundläggande teori*).

- b) Construct a bandpass filter centered around 3 Hz as $H(s) = W_1(s)W_2(s)$ with suitable choices of parameters a_1, b_1, c_1, k_1 for $W_1(s)$, and a_2, b_2, c_2, k_2 for $W_2(s)$.

6. Study the following MIMO-system

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{1}{(s+1)(s+4)} \\ \frac{1}{s+3} & \frac{1}{(s+1)(s+5)} \end{bmatrix}$$

- a) Assume that we have the controller $F_y = I$. Compute the transfer matrices S , T and G_{wu} for the closed loop system (with the same principal structure as in figure 4). Plot the singular values for the transfer matrices.

b) Let the weight matrices be given by

$$W_T(s) = \begin{bmatrix} \frac{s}{0.3s+1} & 0 \\ 0 & \frac{s}{0.3s+1} \end{bmatrix}$$

$$W_S(s) = \begin{bmatrix} \frac{1}{2s+1} & 0 \\ 0 & \frac{1}{2s+1} \end{bmatrix}$$

$$W_u(s) = \begin{bmatrix} \frac{s}{3s+1} & 0 \\ 0 & \frac{s}{3s+1} \end{bmatrix}$$

Now design a controller $F_y(s)$ based on H_∞ -methodology according to the instructions in section 4. Describe how the singular values for T , S and G_{wu} change with this controller. Motivate! Plot the singular values and compare with what you predicted. Is the result as expected? Comment the results!

c) Now have W_T defined as

$$W_T(s) = \begin{bmatrix} \frac{10(s+1)}{0.3s+1} & 0 \\ 0 & \frac{10(s+1)}{0.3s+1} \end{bmatrix}$$

How does the singular values differ when this weight matrix is used compared to 6b? Design a new controller $F_y(s)$ according to section 4. How does this controller change the complementary sensitivity function T ? Is the sensitivity function S influenced? Explain your observations!

Tips

- Once F_y has been chosen, S , T and G_{wu} are completely determined.

7. Suggest weight functions W_S , W_T and W_u that are reasonable given the requirements in prep. exercise 4. Plot them in Matlab.

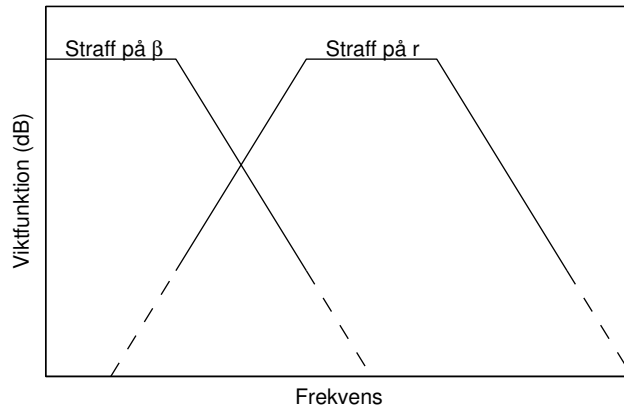


Figure 8: Suggested penalties (i.e, weight functions) on the wind disturbance influence on β resp. r . The fact that the influence on β should be penalized for low frequencies is motivated by e.g. that a constant side-wind should not give a too large sideslip angle.

Tips

- Start with diagonal weight functions.
- Think as in preparatory exercise 5a. During the lab you might need to adjust the weight functions, then it is good if they are parameterized as in preparatory exercise 5a.
- With only two control inputs it is not possible to obtain good disturbance cancellation on all three outputs simultaneously. One solution is to formulate requirements on the disturbance cancellation in β and r in different frequency ranges and use weight functions for S as in figure 8.
- The weight function for p shall look like the one for β (in figure 8) but with a different bandwidth.
- Also for T there are 2 different requirements, but one matters only for frequency higher than 40 rad/s.
- The requirement on the amplification of F_y can approximately be replaced with a requirement on the transfer function from process disturbance to control signal, G_{wu} .

8. Given a 3-by-2 transfer matrix G , and a 2-by-3 transfer matrix F_y , as in Figure 4, write Matlab-code for how the static gain of the prefilter $F_r(0)$ should be selected to obtain decoupling (from reference to outputs) at stationarity for β and p in closed loop.

Tips

- What would the expression be if the turning angular velocity r was not an output and the prefilter was $F_r = I_2$?
- The transfer function from the m first inputs to the p first outputs of a system G is obtained with $G(1:p, 1:m)$.
- The command `dcgain(G)` returns $G(0)$.

9. Write Matlab-code for computing the closed loop system G_c from reference signals to β and p , given F_r and F_y .