Lab in Control Theory, TSRT09

Stability analysis and regulation of nonlinear systems

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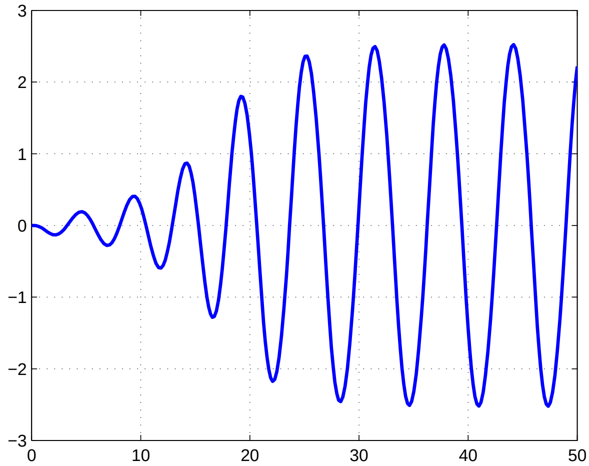

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AUTOMATIC CONTROL
LINKÖPING

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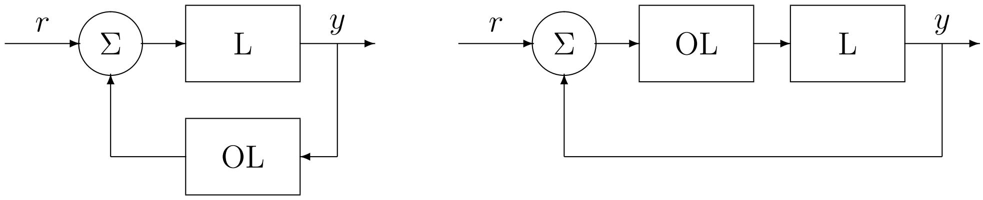
1 Introduction

This lab deals with oscillations and stability. We will concentrate on configurations such as those shown in Fig. 1, that is, feedback interconnections of a linear system and a static nonlinearity.

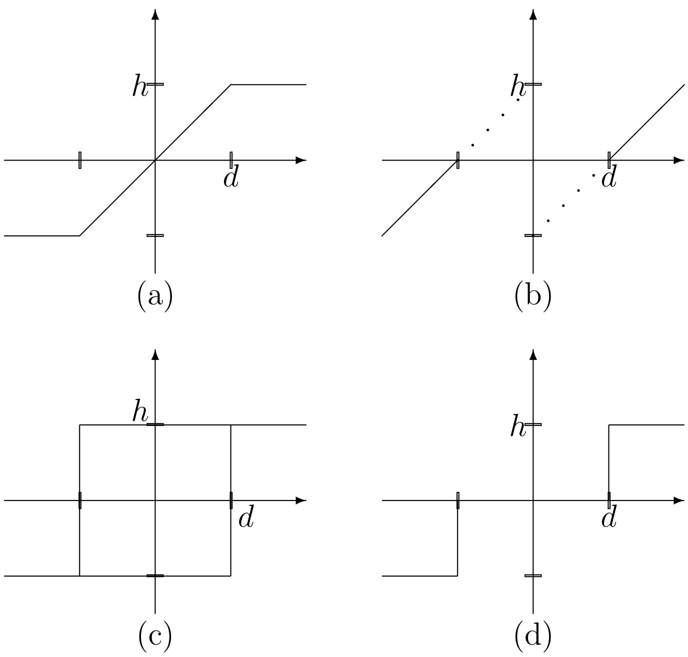


Figur 1: System structure: L = linear system, OL = nonlinear system.

We shall among other investigate the following nonlinearities: cubic nonlinearity

$$y = hu^3, \tag{1}$$

saturation, deadzone, relay with hysteresis and relay with deadzone, see Fig. 2. Tips for how to implement these nonlinearities in SIMULINK can be found at the end of these notes (page 15).



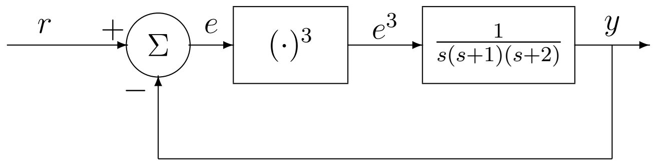
Figur 2: Static nonlinearities: saturation (a), deadzone (b), relay with hysteresis (c) and relay with deadzone (d).

To access the necessary files write the following MATLAB command

```
>> initcourse tsrt09
```

2 A few nonlinear phenomena

We begin by investigating a few nonlinear phenomena that occur in nonlinear systems but not in linear ones. Consider the system in Fig. 3. Implement it in SIMULINK.



Figur 3: System with cubic nonlinearity. The cubic function can be implemented with the block Fcn which you can find under User-Defined Functions.

Task 1 Investigate the system in Fig. 3 with e.g. a step of size 1, 2, and 3 in the reference signal r. In which way is the step response differing from that of a linear system?

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Task 2 Compare the output signals for the following reference signals, by plotting these in the same diagram:

$$r_1 = 2, \quad r_2 = 0.5\cos(100t), \quad r_3 = 2 + 0.5\cos(100t)$$

What can one say on the superposition principle?

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Task 3 Explain why the output of r_1 and r_3 differ at low frequencies even though the reference signals differ only at high frequencies.

Hint: Use the result of preparatory exercise 2.

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3 Describing function and self-sustained oscillations

We investigate self-sustained oscillations in an interconnected system with configuration as in Fig. 1. To undersand what is happening we use describing functions.

3.1 Describing functions

The most common describing functions can be found as MATLAB functions:

`dfcube`	cubic gain as in (1).
`dfsat`	saturation as in Fig. 2 (a).
`dfdeadz`	deadzone as in Fig. 2 (b).
`dfrelay`	relay with hysteresis as in Fig. 2 (c).
`dfreldz`	relay with deadzone as in Fig. 2 (d).

The value of the describing function at amplitude c with parameter value d and h (according to the definition of (1) and Fig. 2) can be obtained through

```
dfcube(c,h), dfsat(c,[d,h]), dfdeadz(c,[d,h]), ...
```

Task 4 Which of the above describing functions are real-value? Why?

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Task 5 Try to sketch below how the real-valued describing functions look like, without making any calculations. Use the interpretation that the describing function is an amplitude dependent gain. Then verify by plotting the describing functions, for example by a command sequence of the following kind,

```
c=0:0.01:10;
plot(c,dfsat(c,[2,3]))
```

which plots the describing function for the saturation with $d = 2$, $h = 3$.

Figur 4: Sketch here the real-valued describing fucntions.

3.2 Self-sustained oscillations

In order to investigate self-sustained oscillations in a system such as the one in Fig. 1, as you know you can plot in the same diagram $-1/Y_f$, where Y_f is the describing function for the nonlinearity, and the Nyquist curve for the linear system. To help you with this, there is a function

```
dfplot(dfun,p,c,g,w)
```

where `dfun` is the describing function with parameter vector `p` and `g` is a linear transfer function. The vectors `c` and `w` contain the amplitude and frequency which will be shown in the plot. `g` is represented as a LTI-object.

Example 1 If the describing function for a saturation with $d = 2$, $h = 3$ is to be plotted together with the Nyquist curve for

$$G(s) = \frac{30}{s^3 + 4s^2 + 7s + 1}$$

then one can write for instance

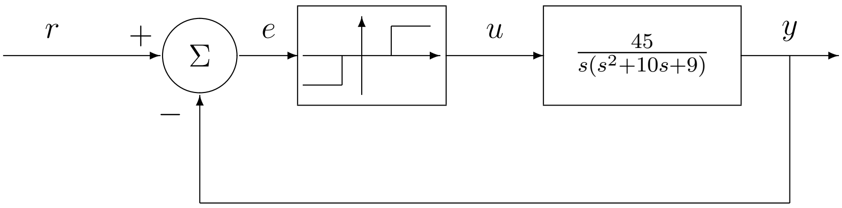
```
g=tf(30,[1 4 7 1])
c=[2,5,10]
w=[0.1,1,10]
dfplot('dfsat',[2,3],c,g,w)
```

To get a better resolution around the intersection, one can then for instance write

```
w=[1,2,5,10]
c=[2,3,4,5]
dfplot('dfsat',[2,3],c,g,w)
```

(which plots the curves for values of `c`, respectively `w`, that lie between the largest and least given value.) □

Consider now the system in Fig. 5:



Figur 5: System containing a relay with deadzone.

Task 6 Assume that $d = 0.2$, $h = 1$. Show that the describing function predicts two self-sustained oscillations with different amplitudes. What amplitudes and frequencies do you get?

Tips: Plot the describing function as you did in task 5 in order to see which amplitudes are suitable to be considered as argument `c` in `dfplot`. Use `dfplot` as in preparatory exercise 4.

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Task 7 Which of the intersections will be observed in practice? Motivates.

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Task 8 What will happen if you start a simulation in which the initial value of the output signal is 0.205, or 0.4, or 1.5? Verify through simulations.

Hint: Start from the implementation of the preparatory exercise 5.

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3.3 Regulation

We shall now study how one can alter the system in order to avoid self-sustained oscillation. According to the describing function method, one should avoid intersections between the curves $-1/Y_f$ and $G(i\omega)$. If the form of the nonlinearity of the physical implementation is available, which is commonly the case, then one has to modify $G_o(s)$. In many cases the simplest solution is to lower the gain. This however often worsen the performances of the regulation. Another choice is instead to change the properties only in a certain frequency range through a phase lead element

$$F_{\text{lead}}(s) = K\frac{\tau_D s + 1}{\beta\tau_D s + 1}$$

or a phase lag element

$$F_{\text{lag}}(s) = \frac{\tau_I s + 1}{\tau_I s + \gamma}$$

connected in series with $G(s)$ (i.e., insert $F_{\text{lead/lag}}(s)$ between the nonlinearity and $G(s)$ in Fig. 5).

Task 9 Consider again the system in Fig. 5. How does the step response looks like if the reference signal has amplitude 2? Explain why one gets the same amplitude and frequency at the oscillation as without the step.

Hint: Let the step begins after the transient has disappeared. If the initial state at the output is zero, then there is no transient.

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Task 10 Replace G with KG and chose K so that there is no intersection with $-1/Y_f$. Verify that the self-oscillation disappears from the step response. Why does the step response get a stationary error?

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Task 11 Intsert a phase lead element

$$F_{\text{lead}}(s) = 0.27\frac{1.24s + 1}{0.07 \cdot 1.24 \cdot s + 1}$$

and show that no self-sustained oscillation occurs. Verify also through simulations.

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Task 12 For the system with the phase lead element (i.e., with a "loop gain" $G_o = G \cdot F_{\text{lead}}$), how many times can the gain of the regulator be increased without any self-sustained oscillation occurring? Calculate and then verify with simulations.

Hint: It takes a while for the oscillations to disappear, so do not use a short simulation time.

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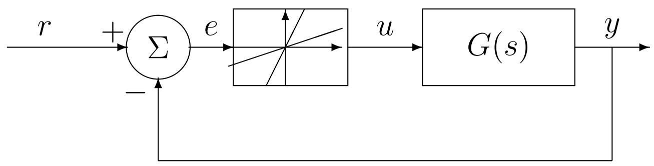
4 Circle criterion

The circle criterion deals with situations as in Fig. 6. The only thing one



Figur 6: System to which the circle criterion can be applied.

needs to know about the nonlinearity is that it is trapped between two lines of known slope, k_1 and k_2. To investigate if the criterion is satisfied, one can use the following

```
ciplot(k1,k2,G,wrange)
```

where `G` is the transfer function of the linear system and `wrange` is a vector with values of ω that should be marked on the Nyquist curve. One gets a Nyquist curve and a circle passing through $-1/k_1$, $-1/k2$, from which it is possible to check whether the criterion is satisfied or less.

We now investigate the system

$$G(s) = \frac{K}{s(s^2 + 4s + 16)}$$

with nonlinearity

$$u = \begin{cases} e & e \geq 0 \\ 0.2e & e < 0 \end{cases}$$

Task 13 The above nonlinearity is such that the describing function method cannot be applied directly. Which property of a describing function is missing?

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Task 14 For which value of K is the circle criterion fulfilled? Simulate the system for different values of K. Shows that the circle criterion is conservative, that is, that the Nyquist plot can go a bit inside the circle without compromising stability.

Hint: The nonlinearity can be implemented using the block 1D Lookup Table where *Table data* is [-0.2 0 1] and *Breakpoints 1* is [-1 0 1].

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Task 15 Use now the value of K that corresponds to the boundary of the stability, that is, that gives a self-sustained oscillation to the system (computed in task 14). Which property of the oscillation can you relate to the answer in task 13.

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5 Exact linearization

The aim of this part is to produce an exact linearization for the cross-coupled water tanks of Fig. 7 so that a step in the reference signal gives a monotone step response with a time constant of about 1 s for the output. To help you, a SIMULINK model, called `exaktlin.mdl`, is available, where you can implement your control law in the block `controller`, see Fig. 8. The step response should go from a level of 5 to a level of 6. From Fig. 7 one can obtain the following simplified model:

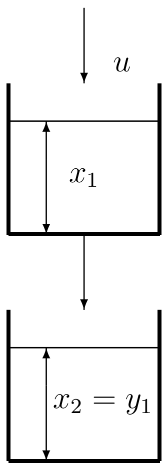
$$\dot{x}_1 = -\sqrt{x_1} + u, \quad (2a)$$

$$\dot{x}_2 = -\sqrt{x_2} + \sqrt{x_1}, \quad (2b)$$

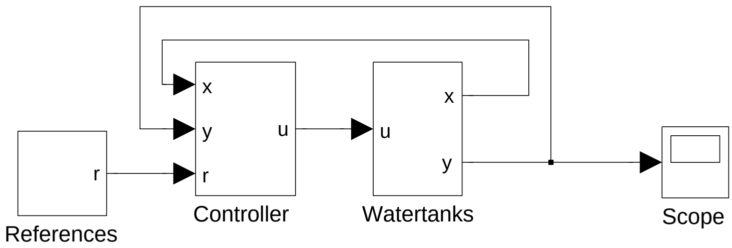
$$y_1 = x_2. \quad (2c)$$

Task 16 Implement the control law from the preparatory exercise 8 and 9 and trim it so that the step response satisfies the requirements.

Hint: How should the transfer function $G_c(s)$ (where $y = G_c(s)r$) be so that the requirements are fulfilled? How should the feedback be done in order to realize this $G_c(s)$?



Figur 7: Cross-coupled water tanks.



Figur 8: SIMULINK-modellen `exaktlin.mdl`.

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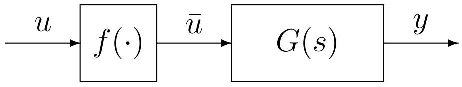

Figur 9: Linear system $G(s)$ in series with a nonlinearity $f(\cdot)$.

6 Preparatory exercises

If you want to profit from the lab and be able to complete it in the assigned time, it is very important that you complete the following preparations before the lab itself. The preparatory exercises are checked and approved by a lab assistant before you begin the lab. In order to carry out the laboratory work, all preparatory tasks must be approved.

Task 1 Read through the entire lab compendium and the associated theory in the course book. Focus specially on Chapters 11, 12.3, 14 and 17.

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Task 2 Consider the system in Fig. 9 and $u = a + b \sin \omega t$. Write an expression for $\bar{u}$ when $f = (\cdot)^3$. How is the nonlinearity influencing the frequencies in y? Compare it with the case in which f is a static linear function.

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Task 3 Map the parameters h and d into those of the SIMULINK blocks on page 15 so that each nonlinearity behaves as expected

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Task 4 Consider the system in Example 1 on page 5. What is amplitude and frequency of a self-sustained oscillation as predicted by the describing function method? Use the command `dfplot` described in Chapter 3.2.

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Task 5 Implement the system of Example 1 in SIMULINK. What amplitude and frequency do you get from the simulation with SIMULINK? Compare with the results of task 4. What happens if you start with a different amplitude in the output? Could you have predicted that from a plot of the describing function?

Hint: The initial value of the output can be computed by implementing $G(s)$ with the help of the block Transfer Fcn (with initial outputs) that you find in the library Blocksets & Toolboxes / Simulink Extras / Additional Linear.

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Task 6 Try to find a direct connection between y (or some of its derivatives) and u for the nonlinear system described in (2) at page 10.

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Task 7 What is the difference between input-output linearization and exact state space linearization? Does there exist an exact state space linearization for the system in (2) on page 10?

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Task 8 Compute a control law so that the closed-loop system becomes exactly linear.

Hint: There is no need to compute explicitly the change of variable $z_1 = y$, $z_2 = \dot{y}$, ..., $z_n = y^{n-1}$ when you compute the control law.

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Task 9 Given the system

$$\begin{aligned} \dot{z} &= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} z + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \bar{u}, \\ y &= \begin{pmatrix} 1 & 0 \end{pmatrix} z. \end{aligned}$$

Compute a state feedback $\bar{u} = L_r r - Lz$ such that the closed loop system $y = G_c r$ has a monotone step response.

Hint: Use for instance the command `place`.

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7 Model building tips

Below are suggestions on how the nonlinearities in this lab can be implemented in SIMULINK. Figure out for yourself which parameter values should be used in the SIMULINK blocks for the desired nonlinearity to be obtained.

Nonlinearity	*Implementation in* SIMULINK