EXAM IN CONTROL THEORY (TSRT09)

SAL: ISY computer rooms

TID: Friday 22nd March 2024, kl. 14.00–18.00

UTBKOD: TSRT09 Control Theory

MODUL: DAT1

INSTITUTION: ISY

ANTAL UPPGIFTER: 5 (points: 10 + 10 + 10 + 10 +10 = 50)

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BESÖKER SALEN: cirka kl. 15 och kl. 16

KURSADMINISTRATÖR: Ninna Stensgård, 013-282225, ninna.stensgard@liu.se

TILLÅTNA HJÄLPMEDEL:

1. *T. Glad & L. Ljung*: ”Reglerteori. Flervariabla och olinjära metoder”
2. *T. Glad & L. Ljung*: ”Reglerteknik. Grundläggande teori”
3. Tabeller, t.ex.:
   L. Råde & B. Westergren: ”Mathematics handbook”
   C. Nordling & J. Österman: ”Physics handbook”
   S. Söderkvist: ”Formler & tabeller”
4. Miniräknare

LANGUAGE: You can write your exam in both English (preferred) or Swedish

LÖSNINGSFÖRSLAG: The solution will be posted on the course web page at the end of the exam.

VISNING: of the exam will take place on 2024-04-10 kl 12.30-13:00 in Ljungeln, B-huset, entrance 25, A-korridoren, room 2A:514.

PRELIMINÄRA BETYGSGRÄNSER
(PRELIMINARY GRADE THRESOHLDS):

betyg 3	23 poäng
betyg 4	33 poäng
betyg 5	43 poäng

OBS! Solutions to all problems should be presented so that all steps (except trivial calculations) can be followed. Missing motivations lead to point deductions. Include your own code if useful.

Lycka till!

1. (a) What are the guaranteed stability margins in a linear quadratic regulator state feedback design for a SISO system? [2p]

(b) What type of equilibrium point is the origin for the system

$$\begin{aligned} \dot{x}_1 &= -3x_1 - 2\sin x_2 \\ \dot{x}_2 &= \tan x_1 + x_2 \end{aligned}$$

Motivate! [2p]

(c) Consider the system

$$\begin{aligned} \dot{x}_1 &= x_1^2 + x_2 \\ \dot{x}_2 &= u \\ y &= x_1 \end{aligned}$$

Find a feedback law $u = \phi(x_1, x_2, r)$ that transforms it into the following expression between reference r and output y:

$$\ddot{y} + \dot{y} + 3y = r$$

[4p]

(d) You must design a controller F_y for a stable minimum phase system, and are given the following specifications:

- $|S(i\omega)| < 0.01$ for $\omega \leq 1$ rad/s
- $|T(i\omega)| < 0.01$ for $\omega > 120$ rad/s

Translate these specifications into requirements on the loop gain $|G(i\omega)F_y(i\omega)|$. [2p]

2. Consider the following transfer function

$$G(s) = \begin{bmatrix} \frac{s+3}{(s+1)(s-1)} \\ \frac{s-2}{(s+1)(s+3)} \end{bmatrix}$$

(a) Compute poles and zeros of $G(s)$. [2p]

(b) How many singular values does this $G(s)$ have? Show them. [2p]

(c) Assume that you want to close the loop around G. What principle would you follow in designing a regulator $F_y(s)$? What would your priority be? [2p]

(d) Design a regulator according to the principle you have identified in (c) and check stability of the closed-loop system. *(Hint: even a simple P regulator could be enough.)* [4p]

3. Consider the system

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} v_1$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x + v_2$$

where v_1 and v_2 are uncorrelated white noises of spectrum $\Phi_{v_1}(\omega) = \rho_1 > 0$ and $\Phi_{v_2}(\omega) = \rho_2 > 0$.

(a) Determine the covariance Π_x of the state x. [4p]

(b) Determine the spectrum $\Phi_y(\omega)$ of the output y. [4p]

(c) Assume $\rho_1 = \rho_2 = 1$. Compute the Kalman filter of the system. [2p]

4. Consider the system

$$y = G(s)u + w, \qquad G(s) = \frac{s-3}{(s+0.1)(s+2)}$$

and the following weight functions

$$W_S = \frac{2}{s+1}, \qquad W_T = \frac{s+1}{s+4}, \qquad W_u = \frac{1}{s+1}$$

(a) Construct the extended system of Eq. (10.4) of the book

$$\begin{bmatrix} z \\ y \end{bmatrix} = G_e \begin{bmatrix} w \\ u \end{bmatrix}$$

[2p]

(b) For which of the following values of γ is the $\mathcal{H}_\infty$ problem $\|G_{ec}\|_\infty < \gamma$ feasible? (G_{ec} = closed loop transfer function)

$$\gamma = 0.5, \qquad \gamma = 1, \qquad \gamma = 5$$

[2p]

(c) If more than one of the values of γ in point (b) is feasible, which one would you use and why? [2p]

(d) For the value of γ of your choice in point (c), write down the resulting regulator F_y and plot the resulting S, T and G_{wu}. [2p]

(e) What do you expect to see in the step response of the closed loop system? [2p]

[Hint: the matlab function `hinfsyn` *relies on state space. Whenever you have a transfer function transform it to state space, using both* `minreal()` *and* `ss()`*. For example* `s=tf('s'), Wu=minreal(ss(1/(s+1)))`*.]*

5. Consider the system

$$G(s) = \frac{K}{s(s+1)(s+3)}$$

in which the output y is measured and the input u is given by the saturated proportional regulator

$$u = \begin{cases} 1 & y < -1 \\ -y & |y| \leq 1 \\ -1 & y > 1 \end{cases}$$

(a) For which values of the gain K are self-sustained oscillations possible and for which are they avoided? [6p]

(b) When self-sustained oscillations are possible, what type of oscillations do you expect? [2p]

(c) When no self-sustained oscillation is possible, what kind of behavior do you expect for the system? [2p]