EXAM IN CONTROL THEORY (TSRT09)

SAL: ISY computer rooms

TID: Wednesday 22nd August 2023, kl. 14.00–18.00

UTBKOD: TSRT09 Control Theory

MODUL: DAT1

INSTITUTION: ISY

ANTAL UPPGIFTER: 5 (points: 10 + 10 + 10 + 10 +10 = 50)

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BESÖKER SALEN: cirka kl. 15 och kl. 16

KURSADMINISTRATÖR: Ninna Stensgård, 013-282225, ninna.stensgard@liu.se

TILLÅTNA HJÄLPMEDEL:

1. *T. Glad & L. Ljung*: ”Reglerteori. Flervariabla och olinjära metoder”
2. *T. Glad & L. Ljung*: ”Reglerteknik. Grundläggande teori”
3. Tabeller, t.ex.:
   L. Råde & B. Westergren: ”Mathematics handbook”
   C. Nordling & J. Österman: ”Physics handbook”
   S. Söderkvist: ”Formler & tabeller”
4. Miniräknare

LANGUAGE: You can write your exam in both English (preferred) or Swedish

LÖSNINGSFÖRSLAG: The solution will be posted on the course web page at the end of the exam.

VISNING: of the exam will take place on 2023-09-01 kl 12.30-13:00 in my office, B-huset, entrance 25, A-korridoren, room 2A:542.

PRELIMINÄRA BETYGSGRÄNSER
(PRELIMINARY GRADE THRESOHLDS):

betyg 3	23 poäng
betyg 4	33 poäng
betyg 5	43 poäng

OBS! Solutions to all problems should be presented so that all steps (except trivial calculations) can be followed. Missing motivations lead to point deductions. Include your own code if useful.

Lycka till!

1. (a) When is a system non-minimum phase? [2p]

(b) Consider the system

$$y = \frac{1}{s+1}u + \frac{2}{s+3}d$$

where y is the output, u is the control input and d is a disturbance, which is constant and lies in the interval $-1 \le d \le 3$. The input u is constrained: $|u(t)| \le 1$. Is it possible to construct a regulator which completely eliminates the influence of d on y? Motivate your answer. [2p]

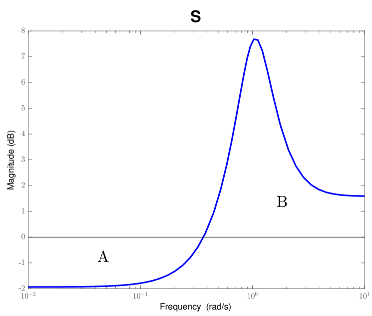
(c) Which poles (and with which multiplicity) does the following system have?

$$G(s) = \begin{bmatrix} \frac{2}{s+3} & \frac{s+2}{(s+1)(s+3)(s+5)} \end{bmatrix}$$

[2p]

(d) You have a simulator that can simulate a white noise. Describe briefly how you can use this to simulate a noise with a given spectral density. [2p]

(e) A control system has a sensitivity function like in the figure



What can you say of its loop gain if the area in the B region is larger than that of the A region? [2p]

2. In order to regulate the system

$$G(s) = \begin{bmatrix} \frac{1}{s+2} & \frac{5}{s+1} \\ \frac{4}{s+3} & \frac{3}{s+4} \end{bmatrix}$$

two possible P-regulators are available:

$$u_1 = K_1(r_1 - y_1), \quad u_2 = K_2(r_2 - y_2)$$

and

$$u_1 = K_1(r_2 - y_2), \quad u_2 = K_2(r_1 - y_1)$$

(a) Use RGA to choose one of the two. [3p]

(b) Compute the poles of the closed loop system (with gains $K_1 = K_2 = 2$) for the two cases and check if they confirm/contradict the choice that you made at point (a) (and why). [3p]

(c) For the regulator that you have chosen at point (a), plot the singular values of the sensitivity function. How well is the closed loop system dealing with constant disturbances? In what frequency region are disturbances attenuated? [4p]

3. Consider the system

$$y = G(s)u + w, \qquad G(s) = \frac{s+2}{(s+0.2)(s+1)}$$

and the following weight functions

$$W_S = \frac{K(s+3)}{(s+1)(s+5)}, \qquad W_T = \frac{s+1}{s+5}, \qquad W_u = \frac{10}{s+1}$$

where K is a parameter that can take the values $K = \{1, 10\}$.

(a) Construct the extended system of eq. (10.4) of the book

$$\begin{bmatrix} z \\ y \end{bmatrix} = G_e \begin{bmatrix} w \\ u \end{bmatrix}$$

[2p]

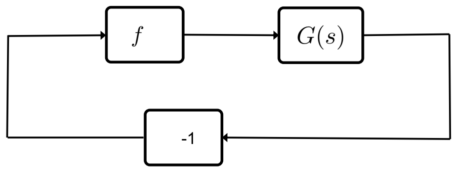
(b) Which of the two values of K gives an $\mathcal{H}_2$-regulator such that the closed loop system has the least sensitivity to low frequency disturbances w? [2p]

(c) For the controller that you have chosen at point (b), write down the resulting regulator F_y and plot the resulting S, T and G_{wu}. [3p]

(d) Compare the loopshaping obtained from the two $\mathcal{H}_2$-regulators obtained for $K = 1$ and $K = 10$. In what do they differ significantly? [3p]

[Hint: the matlab function `h2syn` *relies on state space. Whenever you have a transfer function transform it to state space, using both* `minreal()` *and* `ss()`*. For example* `s=tf('s')`, `Wu=minreal(ss(10/(s+1)))`*.]*

4. Consider the system given by



where f is the cubic nonlinearity $f(u) = u + u^3$. Which of the following 3 transfer functions is such that the closed loop system satisfies the circle criterion? And for what values of the gain K?

$$
\begin{aligned}
G_1(s) &= \frac{K(s-2)}{(s+3)(s+1)(s+4)} \\
G_2(s) &= \frac{K(s+2)}{(s+1)(s+4)} \\
G_3(s) &= \frac{K(s+2)}{s(s+1)(s+4)}
\end{aligned}
$$

[10p]

5. Consider the nonlinear system

$$
\begin{aligned}
\dot{x}_1 &= x_1^3 + x_2 \\
\dot{x}_2 &= u \\
\dot{x}_3 &= x_3 + \sin x_1
\end{aligned}
$$

(a) When $u = 0$, show that $x_{\text{eq}} = 0$ is an equilibrium point, and compute its stability character. [3p]

(b) Can you stabilize the system by linear state feedback $u = -Lx$ using the Jacobian linearization method? If yes, find one L accomplishing the task. [3p]

(c) Assume that you want to use the feedback linearization method. Only one of the following two output equations

$$y = h_1(x) = x_1, \qquad y = h_2(x) = x_3$$

allows for an easy feedback linearization design. Which one and why? [4p]