EXAM IN CONTROL THEORY (TSRT09)

SAL: ISY computer rooms

TID: Wednesday 7th June 2023, kl. 8.00–12.00

UTBKOD: TSRT09 Control Theory

MODUL: DAT1

INSTITUTION: ISY

ANTAL UPPGIFTER: 5 (points: 10 + 10 + 10 + 10 +10 = 50)

ANSVARIG LÄRARE: Claudio Altafini, 013 281373, e-mail: claudio.altafini@liu.se

BESÖKER SALEN: cirka kl. 9 och kl. 10

KURSADMINISTRATÖR: Ninna Stensgård, 013-282225, ninna.stensgard@liu.se

TILLÅTNA HJÄLPMEDEL:

1. *T. Glad & L. Ljung*: ”Reglerteori. Flervariabla och olinjära metoder”
2. *T. Glad & L. Ljung*: ”Reglerteknik. Grundläggande teori”
3. Tabeller, t.ex.:
   L. Råde & B. Westergren: ”Mathematics handbook”
   C. Nordling & J. Österman: ”Physics handbook”
   S. Söderkvist: ”Formler & tabeller”
4. Miniräknare

LANGUAGE: You can write your exam in both English (preferred) or Swedish

LÖSNINGSFÖRSLAG: The solution will be posted on the course web page at the end of the exam.

VISNING: of the exam will take place on 2023-06-21 kl 12.30-13:00 in my office, B-huset, entrance 25, A-korridoren, room 2A:542.

PRELIMINÄRA BETYGSGRÄNSER
(PRELIMINARY GRADE THRESOHLDS):

betyg 3	23 poäng
betyg 4	33 poäng
betyg 5	43 poäng

OBS! Solutions to all problems should be presented so that all steps (except trivial calculations) can be followed. Missing motivations lead to point deductions. Include your own code if useful.

Lycka till!

1. (a) What is a Lyapunov equation? [2p]

   (b) Consider the system

$$\dot{x} = -2x + v$$

   where v is a white noise of zero mean and variance r. What is the variance of x at stationarity? [2p]

   (c) For which values of a and b is the system

$$\dot{x} = \begin{bmatrix} a & b \\ 3 & 2 \end{bmatrix} x + \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

   observable? [2p]

   (d) Is it possible to stabilize the following system?

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

   [2p]

   (e) Choose the parameter b so that the system

$$\begin{aligned} \dot{x}_1 &= \sin x_2 + bu \\ \dot{x}_2 &= u \\ y &= x_1 + x_2 \end{aligned}$$

   has relative degree 2. [2p]

2. Consider the system

$$G(s) = \begin{bmatrix} \frac{2}{s+2} & \frac{1}{s+1} \\ \frac{4}{s+2} & \frac{3}{s+2} \end{bmatrix}$$

   (a) How many poles in $s = -2$ does the system have? Motivate from the definition (no matlab). [2p]

   (b) What is RGA for $G(0)$? Which input-output pairing does it suggest? [3p]

   (c) Assume that output y_2 and input u_2 are paired together through a P-regulator:

$$u_2 = -Ky_2$$

   Compute the transfer function from u_1 to y_1 which one gets with this P-regulator. In particular, compute the static gain that you get as a function of K. [4p]

   (d) What is the link between the results in (b) and (c)? [1p]

3. Consider the SISO system

$$G(s) = \frac{s+2}{(s+1)(s+0.2)}$$

(a) Compute the minimal state space realization of $G(s)$ in controller canonical form, see Example 2.4 on page 49 for the Swedish book (page 36 for the English book). [1p]

(b) Design a LQ regulator for $G(s)$, assuming that all weights and noise covariance matrices are equal to the identity (of suitable dimension). Measurement noise and system disturbance are also uncorrelated. Report the Kalman gain and the regulator gain. [2p]

(c) Assume that you want to have a faster step response for the resulting closed-loop system. Which of the weight matrices or noise covariance matrices would you modify and how? Report the new matrices, the corresponding Kalman/regulator gains and the plots of the step response in the two cases (you can use the matlab function `step`). [4p]

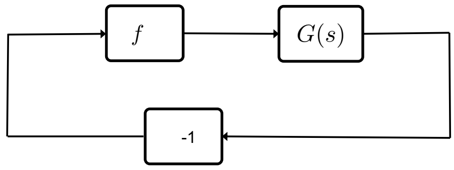
(d) Compute the sensitivity and complementary sensitivity function of the two regulators above. How do they differ? [2p]

(e) Assume that instead of $G(s)$ you have the following:

$$\tilde{G}(s) = \frac{s-2}{(s+1)(s+0.2)}$$

what do you expect to change in the step response? [1p]

4. A nonlinear system is given by



where f is a "soft saturation", of describing function

$$Y_f(C) = \frac{1}{1+C^2}$$

(where C is the amplitude), and G is given by

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

(a) For which values of K can we expect the system to have self-sustained oscillations? [4p]

(b) What will be their (approximate) amplitude as a function of K? [4p]

(c) Will the self-sustaining oscillations be stable in amplitude? [2p]

5. Consider the nonlinear system

$$\dot{x}_1 = 2x_2 - x_1 x_2^2 + u$$
$$\dot{x}_2 = -x_1$$

Assume $u = 0$.

(a) Show that $x_{\text{eq}} = 0$ is an equilibrium point. Is there any other equilibrium point for the system? [2p]

(b) Investigate the stability character of $x_{\text{eq}} = 0$ [2p]

Assume now that u is a control input and that you want to design a state feedback for the system so that $x_{\text{eq}} = 0$ is asymptotically stable.

(c) Can you use Jacobian linearization? Motivate your answer. [2p]

(d) Can you use (some) nonlinear control design? Motivate your answer. [2p]

(e) Produce a controller using any of the methods above (or any other method of your knowledge). [2p]