EXAM IN CONTROL THEORY (TSRT09)

SAL: ISY computer rooms

TID: Friday 24th March 2023, kl. 14.00–18.00

UTBKOD: TSRT09 Control Theory

MODUL: DAT1

INSTITUTION: ISY

ANTAL UPPGIFTER: 5 (points: 10 + 10 + 10 + 10 +10 = 50)

ANSVARIG LÄRARE: Claudio Altafini, 013 281373, e-mail: claudio.altafini@liu.se

BESÖKER SALEN: cirka kl. 15 och kl. 16

KURSADMINISTRATÖR: Ninna Stensgård, 013-282225, ninna.stensgard@liu.se

TILLÅTNA HJÄLPMEDEL:

1. *T. Glad & L. Ljung*: ”Reglerteori. Flervariabla och olinjära metoder”
2. *T. Glad & L. Ljung*: ”Reglerteknik. Grundläggande teori”
3. Tabeller, t.ex.:
   L. Råde & B. Westergren: ”Mathematics handbook”
   C. Nordling & J. Österman: ”Physics handbook”
   S. Söderkvist: ”Formler & tabeller”
4. Miniräknare

LANGUAGE: You can write your exam in both English (preferred) or Swedish

LÖSNINGSFÖRSLAG: The solution will be posted on the course web page at the end of the exam.

VISNING: of the exam will take place on 2023-04-04 kl 12.30-13:00 in Ljungeln, B-huset, entrance 25, A-korridoren, room 2A:514.

PRELIMINÄRA BETYGSGRÄNSER
(PRELIMINARY GRADE THRESOHLDS):

betyg 3	23 poäng
betyg 4	33 poäng
betyg 5	43 poäng

OBS! Solutions to all problems should be presented so that all steps (except trivial calculations) can be followed. Missing motivations lead to point deductions. Include your own code if useful.

Lycka till!

1. (a) Describe briefly the type of limitation imposed on a system by a Bode integral. [2p]

   (b) Consider the system

$$\dot{x} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \\ 0 & -2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} x$$

      (i) Is the system controllable? If not, compute the controllable subspace. [2p]

      (ii) Is the system observable? If not, compute the non-observable subspace. [2p]

      (iii) Is the system stabilizable and detectable? [1p]

      (iv) Is the realization minimal? [1p]

      (v) Compute the associated transfer function $G(s)$, and poles/zeros of $G(s)$ *(Hint: do the calculation using pole and zero polynomials; do not trust matlab).* [2p]

2. Consider the first order system

$$\begin{aligned} \dot{x} =& -2x + v_1 \\ y =& x + v_2 \end{aligned} \qquad (1)$$

where v_1 and v_2 are uncorrelated white noises of variance r_1 and r_2, respectively. *(Note: since all variables are scalar, covariances become variances).*

   (a) Determine the spectrum and variance of the state $x(t)$. [2p]

   (b) The state $x(t)$ is estimated through an observer

$$\dot{\hat{x}} = -2\hat{x} + K(y - \hat{x}).$$

   Show that this observer can be written as $\hat{x} = G(s)y$ and compute $G(s)$. [2p]

   (c) Consider the estimation error $\tilde{x} = x - \hat{x}$. Show that it can be written as

$$\tilde{x} = H_1(s)v_1 + H_2(s)v_2 \qquad (2)$$

   and compute the transfer functions $H_1(s)$ and $H_2(s)$. Compute also the variances of the two parts in Eq. (2), that is, the variance of $\tilde{x}_1 = H_1(s)v_1$ and of $\tilde{x}_2 = H_2(s)v_2$. [2p]

(d) Determine the observer gain $K_{\min}$ that minimizes the total error variance

$$V(K) = E\left[\tilde{x}_1^2\right] + E\left[\tilde{x}_2^2\right].$$

[2p]

(e) Compute the Kalman filter for the state space model in Eq. (1). How does the Kalman gain K_{kalman} compare with $K_{\min}$ computed in (d)? [2p]

3. Consider the system

$$y = G(s)u + w, \qquad G(s) = \frac{s+1}{(s+0.1)(s+3)}$$

and the following weight functions

$$W_S = \frac{10}{s+1}, \qquad W_T = \frac{s+2}{s+10}, \qquad W_u = 1$$

(a) Construct the extended system of eq. (10.4) of the book

$$\begin{bmatrix} z \\ y \end{bmatrix} = G_e \begin{bmatrix} w \\ u \end{bmatrix}$$

[2p]

(b) Design an $\mathcal{H}_\infty$ controller for the system. Write down the resulting regulator F_y and the corresponding value of γ. [3p]

(c) Design an $\mathcal{H}_2$ controller for the system. Write down the resulting regulator F_y. [3p]

(d) Compare the loopshaping obtained from the two regulators, for instance comparing the resulting S, T and G_{wu}. What is the major difference between them? [2p]

[Hint: the matlab functions `h2syn/hinfsyn` *rely on state space. Whenever you have a transfer function transform it to state space, using both* `minreal()` *and* `ss()`*. For example* `s=tf('s'), WS=minreal(ss(10/(s+1)))`*.]*

4. Consider the nonlinear system in Fig. 1, where

$$G(s) = \frac{s-1}{(s+3)(s+2)} \tag{3}$$

and the static sector nonlinearity is $\phi(v) = \tanh(v)$. Assume the regulator is a P-controller: $F(s) = K$.

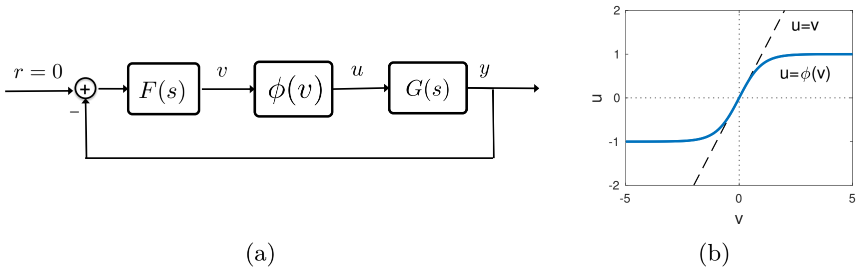

Figure 1: (a): Nonlinear system of Ex. 4. (b): Nonlinearity $\phi(v) = \tanh(v)$.

(a) Compute the maximal value of the gain K in the P controller for which the closed loop system is stable according to the small gain theorem. [3p]

(b) Compute the maximal value of the gain K in the P controller for which the closed loop system is stable according to the circle criterion. [3p]

(c) Assume that $G(s)$ is instead the following:

$$G(s) = \frac{s+1}{(s+3)(s+2)}. \tag{4}$$

What are the maximal values of K obtained from the small gain theorem and the circle criterion in this case? [3p]

(d) Can you comment on the difference between the transfer functions in Eq. (3) and (4)? [1p]

5. (a) Consider the nonlinear system

$$\dot{x}_1 = -2x_1 - 4x_1^3$$
$$\dot{x}_2 = -2x_2^3$$

Which of the following methods can be used to show that the origin is asymptotically stable?

(i) Jacobian linearization [1p]

(ii) Lyapunov function $V(x) = x_1^2 + x_2^2$. [1p]

(iii) Lyapunov function $V(x) = x_1^2 x_2^2$. [1p]

(iv) Lyapunov function $V(x) = x^T P x$, where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $P = P^T$ is a positive definite matrix. [1p]

(v) Is the asymptotic stability global? [2p]

(b) Consider instead the system

$$\dot{x}_1 = -2x_1 + x_2 - 4x_1^3$$
$$\dot{x}_2 = -2x_2 - 2x_2^3$$

What can you say on its local/global stability and phase portrait?

[4p]