EXAM IN CONTROL THEORY (TSRT09)

SAL: ISY computer rooms

TID: Tuesday 23rd August 2022, kl. 14.00–18.00

KURS: TSRT09 Control Theory

PROVKOD: DAT1

INSTITUTION: ISY

ANTAL UPPGIFTER: 5 (points: 10 + 10 + 10 + 10 +10 = 50)

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BESÖKER SALEN: cirka kl. 15 och kl. 16

KURSADMINISTRATÖR: Ninna Stensgård, 013-282225, ninna.stensgard@liu.se

TILLÅTNA HJÄLPMEDEL:

1. *T. Glad & L. Ljung*: "Reglerteori. Flervariabla och olinjära metoder"
2. *T. Glad & L. Ljung*: "Reglerteknik. Grundläggande teori"
3. Tabeller, t.ex.:
   L. Råde & B. Westergren: "Mathematics handbook"
   C. Nordling & J. Österman: "Physics handbook"
   S. Söderkvist: "Formler & tabeller"
4. Miniräknare

LANGUAGE: You can write your exam in both English (preferred) or Swedish

LÖSNINGSFÖRSLAG: The solution will be posted on the course web page at the end of the exam.

VISNING: of the exam will take place on 2022-09-02 kl 12.30-13:00 in Ljungeln, B-huset, entrance 25, A-korridoren, room 2A:514.

PRELIMINÄRA BETYGSGRÄNSER
(PRELIMINARY GRADE THRESOHLDS):

betyg 3	23 poäng
betyg 4	33 poäng
betyg 5	43 poäng

OBS! Solutions to all problems should be presented so that all steps (except trivial calculations) can be followed. Missing motivations lead to point deductions. Include your own code if useful.

Lycka till!

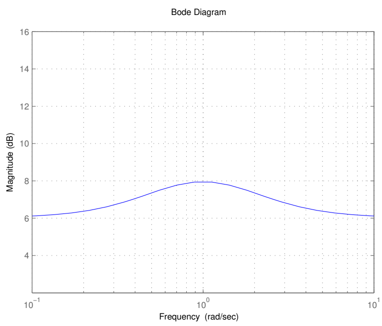
1. (a) Why is it difficult to control a system with both a zero and a pole in the right half of the complex plane? How is the position of the zero relative to that of the pole (both in the RHP) influencing this difficulty? [2p]

   (b) What is the spectrum of the output of the transfer function

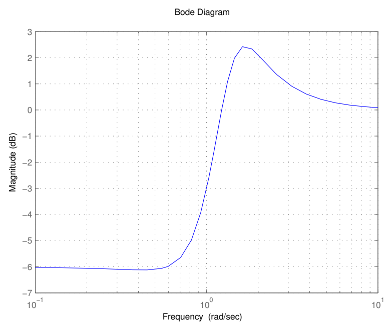
$$\frac{3}{s+5}$$

   when the input is a white noise? [2p]

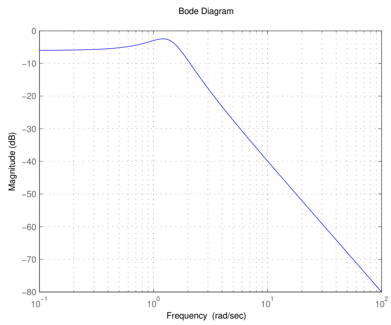
   (c) A linear SISO system G is feedback regulated by a linear regulator F_y so that the closed loop is stable. Furthermore, the loop gain GF_y does not have any pole in the right half of the complex plane and it is such that $|GF_y| \leq \text{const.}/|s|^2$ for large $|s|$. Which of the following Bode diagrams *cannot* represent the amplitude of the sensitivity function of the closed loop system? Motivate your answer. [3p]



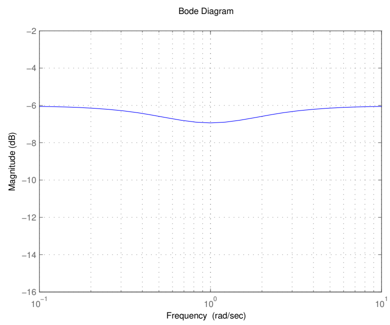
(a)



(b)



(c)



(d)

(d) Compute a feedback law $u = \phi(x_1, x_2, r)$ for the system

$$\begin{aligned} \dot{x}_1 &= x_2^3 + u \\ \dot{x}_2 &= -x_2 + u \\ y &= x_1 \end{aligned}$$

so that the resulting relation between the reference signal r and y is given by

$$\dot{y} = -y + r$$

What is the associated zero dynamics? [3p]

2. The MIMO system

$$G = \begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+2} \\ \frac{2s+4}{s^2+4s+3} & \frac{1}{s+3} \end{bmatrix}$$

is feedback controlled by

$$u = -F_y y.$$

(a) Which of the following two regulators

$$F_{y,1} = \begin{bmatrix} \frac{1}{s+4} & 0 \\ 0 & \frac{1}{s+5} \end{bmatrix}, \qquad F_{y,2} = \begin{bmatrix} 0 & \frac{1}{s+4} \\ \frac{1}{s+5} & 0 \end{bmatrix},$$

would you use and why? Motivate your answer. [3p]

(b) For your choice of regulator, consider the sensitivity function S and the input sensitivity function S_u. Plot the corresponding singular values. [2p]

(c) Show that at the frequency $\omega = 0.1\,\text{rad/sec}$ the product of the singular values of S is identical to the product of the singular values of S_u. [2p]

(d) Show that when G and F_y are square matrices, the condition of (c) is valid for all frequencies in which $\det(G) \neq 0$. *(Hint: use the equality $SG = GS_u$).* [3p]

3. A DC-motor is described by the following system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_2 + v$$
$$y = x_1$$

where $y = x_1$ is the angle, x_2 the angular velocity, and v the applied voltage. The motor is driven by a nonlinear amplifier of static equation

$$v = u + u^3$$

where u is the control input. The input is generated by a P-regulator

$$u = K(r - y)$$

For the sake of simplicity we assume here that $r = 0$.

(a) What are the equilibrium points of the closed-loop system? Investigate their properties (stability/instability, node/focus/saddlepoint, etc.) as you vary K. [6p]

(b) Use the Lyapunov function

$$V = ax_1^2 + bx_1^4 + cx_2^2$$

with suitable values of the parameters a, b and c to show that the system is globally asymptotically stable for all $K > 0$. [4p]

4. Consider the system

$$\dot{x} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} u$$

Compute a state feedback $u = -Lx$ so that the evolution of the closed loop system from the initial condition

$$x(0) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

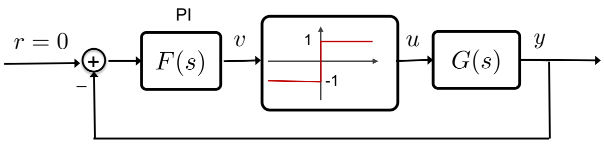
satisfies the following constraints on the state

$$|x_1(t)| \leq 0.1; \quad |x_2(t)| \leq 0.2; \quad |x_3(t)| \leq 0.3; \qquad \text{for all } t \geq 1.5$$

and the following one on the input

$$|u(t)| \leq 15; \quad \text{for all } t \qquad [10p]$$

5. The system



with reference $r = 0$ and

$$G(s) = \frac{1}{(s+2)(s+3)}$$

is controlled by means of the PI regulator

$$F(s) = K\left(1 + \frac{1}{\tau s}\right)$$

where $\tau = 0.1$. The signal v at the output of the PI passes through an ideal relay:

$$u(t) = \begin{cases} 1 & \text{if } v > 0 \\ -1 & \text{if } v < 0 \end{cases}$$

(a) Are self-sustained oscillations possible for the system, and for which values of K can they occur? [4p]

(b) What is the frequency of the oscillations? Is it changing with K? [2p]

(c) Are the oscillations stable in amplitude? [2p]

(d) What values of K (if any) guarantee that the oscillations have amplitude ≤ 2? [2p]