Optimal Control - Homework Exercise 1

November 1, 2010

In this homework assignment, a correct solution to 2 out of 3 sub-exercises are needed to pass and the **rules** concerning the examination are given on the course website.

Exercises

A public company has in year k profits amounting to x_k SEK. The management then distributes u_k to the shareholders and invests $x_k - u_k$ in the company itself. Each SEK invested in such way will increase the company profit by θ the following year so that

$$x_{k+1} = x_k + \theta(x_k - u_k)$$

For simplicity, suppose $x_0 \geq 0$ and $0 \leq u_k \leq x_k$, so that $x_k \geq 0$ for every k. The objective of the management is to maximize the total amount distributed to the shareholders over a period of N years.

a) Formulate the problem as a discrete-time optimal control problem.

b) Apply the DP-algorithm and show that the optimal cost $J(k, x)$ can be expressed as

$$J(k, x) = \alpha_k x,$$

where α_k satisfies the recursion

$$\alpha_{k-1} = \alpha_k + \max\{\theta\alpha_k, 1\}.$$

c) Solve the backwards recursion for α_k and find the optimal control u_k^*.