Optimal Control, Lecture 1: Introduction and Shortest Path

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Contents

Applications

Applications

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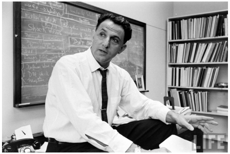
Applications

- Routing of airplanes
- Logistics
- Economical systems
- many more

History

- Roots in calculus of variations (Bernoulli, Euler, Lagrange, Weirstrass,...)
- Optimal control emerged in the 1950s during the space race
- Dynamic programming (Richard Bellman)
- Maximum principle (Lev Semenovich Pontryagin)
- Linear quadratic control (Rudolph Kalman)



Organization

- Examiner and lecturer: Anders Hansson, e-mail: anders.g.hansson@liu.se
- Course assistant: Reza Jafari, e-mail: seyyed.reza.jafari@liu.se (responsible for homeworks and exercise sessions)
- Course home page: http://www.control.isy.liu.se/student/tsrt08/

Course Evaluation and Course Development

- 26 registered students
- Evaliuate answer frequency 30.77%
- Overall evaluation of the course 3.25
- Basis for changes: Increased relevance of reinforcement learning
- Changes carried out: New course material based on a new book

Optimal Control

- Discrete time optimal control
- Continuous time optimal control

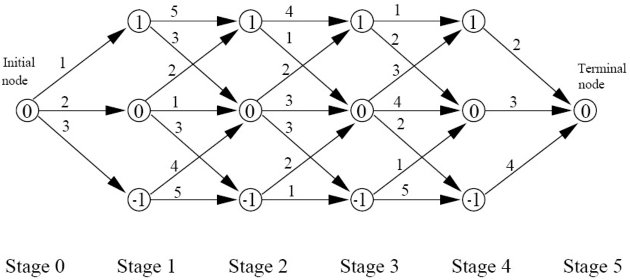
Discrete Time Optimal Control

$$\begin{array}{ll} \text{minimize} & \phi(x_N) + \sum_{k=0}^{N-1} f_k(x_k, u_k) \\ \text{subject to} & x_{k+1} = F_k(, x_k, u_k) \\ & x_0 \text{ given } , x_k \in X_k \\ & u_k \in U(k, x_k) \end{array}$$

Sequential decision problem which include as special cases

- Graph search problem
- Combinatorial optimization
- Discrete time optimal control

Shortest Path Problem



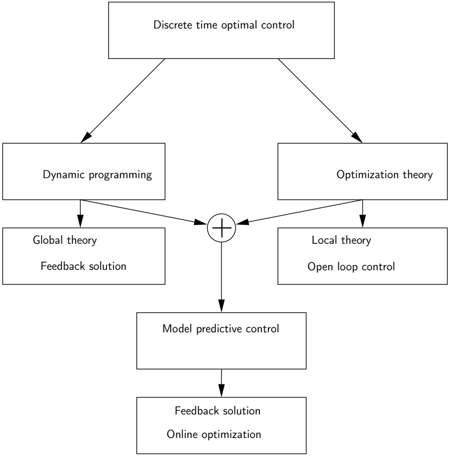
Find the shortest path from the initial node at stage 0 to the terminal node at stage 5.

Knapsack Problem

maximize $\quad \sum_{j=1}^{n} p_j x_j$

subject to $\quad \sum_{j=1}^{n} w_j x_j \leq c, x_j = 0$ or $1,\ j = 1, \ldots, n$

Discrete Time Optimal Control



Reinforcement Learning

Is about solving the optimal control problem *without knowing* the dynamical model of the system to control.

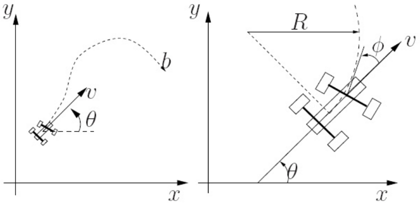
Based on carrying out experiments.

Continuous Time Optimal Control

$$\begin{array}{ll} \text{minimize} & \phi(t_1, x(t_1)) + \int_{t_0}^{t_1} f(t, x(t), u(t))dt \\ \text{subject to} & \dot{x}(t) = F(t, x(t), u(t)) \\ & x(t_0) \in S_0 \\ & x(t_1) \in S_1 \\ & u(t) \in U(x(t)) \end{array}$$

S_0 and S_1 manifolds (intersections of surfaces)

Optimal Control of Car



Problem: shortest path (here t_1 is a free variable)

$$
\begin{array}{ll}
\text{minimize} & \int_0^{t_1} 1 dt \\
\text{subject to} & \dot{x}(t) = v\cos(\theta(t)),\ x(0) = 0,\ x(t_1) = \bar{x} \\
& \dot{y}(t) = v\sin(\theta(t)),\ y(0) = 0,\ y(t_1) = \bar{y} \\
& \dot{\theta}(t) = \omega(t),\ \theta(0) = 0,\ \theta(t_1) = \bar{\theta} \\
& |\omega(t)| \leq v/R,\ t_1 \geq 0
\end{array}
$$

Resource Allocation

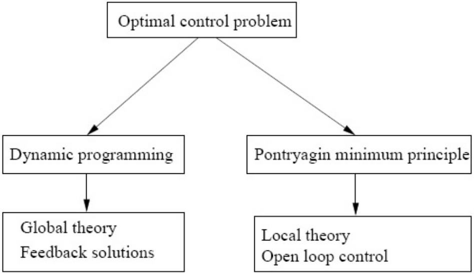
$$\begin{array}{ll} \text{maximize} & x(t_1) + \int_0^{t_1} (1-u(t))x(t)dt \\ \text{subject to} & \dot{x}(t) = \alpha u(t)x(t),\ 0 < \alpha < 1 \\ & x(0) = x_0 > 0 \\ & u(t) \in [0,1],\ \forall t \end{array}$$

The portion $u(t)$ of the production rate $x(t)$ is invested in the factory (and increases the production capacity)

The rest $(1-u(t))x(t)$ is stored in a warehouse.

Maximize the sum of the total amount of goods stored and the final capacity.

Continuous Optimal Control



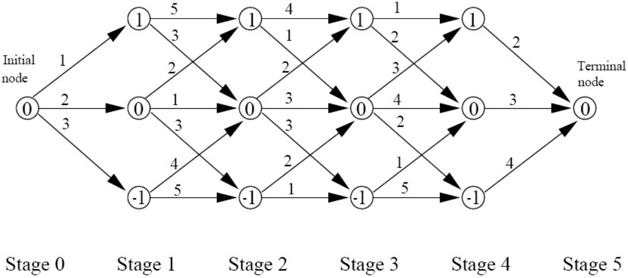
- Feedback control $u(t) = \psi(t, x(t))$ (depends on state)
- Open loop control $u(t) = \psi(t, x(t_0))$

Note: we will not do dynamic programming in continuous time.

Discrete Time Optimal Control

- Optimal control problems in discrete time
  - Discrete state space
  - Continuous state space
- In discrete time it is easier to understand
  - The principle of optimality
  - The dynamic programming equation
- Discrete version of the Pontryagin Maximum Principle (PMP)
- Optimal control on infinite time horizon
- Feedback versus open loop control
- Approximate solutions
- Model Predictive Control (MPC)
- Reinforcement learning

Shortest Path Problem



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