EXAM IN OPTIMAL CONTROL (TSRT08)

ROOM: A37

TIME: Tuesday, August 27, 2024, 8–12

COURSE: TSRT08 OPTIMAL CONTROL

CODE: TEN1

DEPARTMENT: ISY

NUMBER OF EXERCISES: 4

NUMBER OF PAGES (including cover page): 3

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VISITS: 9 and 11

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APPROVED TOOLS: Collections of formulas and tables printed by a publishing house, calculator. No other books are allowed.

SOLUTIONS: Linked from the course home page after the exam.

The exam can be inspected and checked out 2024-09-19, Room 2A:573, B-building, entrance 25, A-corridor, to the right.

PRELIMINARY GRADING:		
	grade 3	15 points
	grade 4	23 points
	grade 5	30 points

All solutions should be well motivated. Writing should be neat and clean.

Good Luck!

1. (a) Find the optimal solution to the problem

$$\begin{aligned} \underset{u(\cdot)}{\text{minimize}} \quad & \int_0^{t_f} \left((x(t) - \cos t)^2 + u^2(t)\right) dt \\ \text{subject to} \quad & \dot{x}(t) = u(t), \\ & x(0) = 0, \end{aligned}$$

expressed as a system of ordinary differential equations. (5p)

(b) Consider the optimal control problem

$$\begin{aligned} \underset{u(\cdot)}{\text{minimize}} \quad & \frac{\gamma}{2} x(t_f)^2 + \frac{1}{2} \int_{t_i}^{t_f} u^2(t)\, dt \\ \text{subject to} \quad & \dot{x}(t) = u(t) \\ & x(t_i) = x_i. \end{aligned}$$

Derive an optimal *feedback* policy using PMP. (5p)

2. Among all curves of length l in the upper half plane passing through the points $(-a, 0)$ and $(a, 0)$ find the one which encloses the largest area in the interval $[-a, a]$, i.e., solve

$$\begin{aligned} \text{maximize} \quad & \int_{-a}^{a} x(t)\, dt \\ \text{subject to} \quad & x(-a) = 0, \\ & x(a) = 0, \\ & \int_{-a}^{a} \sqrt{1 + \dot{x}(t)^2}\, dt = l. \end{aligned}$$

with variable x. *Hint:* Consider defining an additional state given by $z(t) = \int_{-a}^{t} \sqrt{1 + \dot{x}(s)^2} ds$. (10p)

3. (a) You are able to carry out experiments on a dynamical system such that if you let the vector of inputs $u = (u(0), \ldots, u(N-1))$ for times 0 to $N-1$ be applied to the system you can measure the output vector $y = (y(1), \ldots, y(N))$ for times 1 to N. You want to find an input u such that the output y is as close as possible to a desired reference value $r = (r(1), \ldots, r(N))$. Explain how you can do this using iterative learning control with root-finding. (5p)

(b) Assume that $y = H(u)$ for some function H. What are sufficient conditions for root-finding to converge to $y = r$? (5p)

4. Consider the system

$$x_{k+1} = x_k + u_k, \quad k = 0, 1, 2, 3,$$

with the initial state $x_0 = 5$ and the cost function

$$\sum_{k=0}^{3} (x_k^2 + u_k^2).$$

Apply the dynamic programming algorithm to the problem when the control constraints are

$$U(k, x_k) = \{u \mid 0 \leq x_k + u \leq 5, \ u \in \mathbb{Z}\}.$$

(10p)