EXAM IN OPTIMAL CONTROL (TSRT08)

ROOM: ??

TIME: Wednesday, January 10, 2024, 14.00–18.00

COURSE: TSRT08 OPTIMAL CONTROL

CODE: TEN1

DEPARTMENT: ISY

NUMBER OF EXERCISES: 4

NUMBER OF PAGES (including cover page): ????

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VISITS: 15 and 17

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APPROVED TOOLS: Collections of formulas and tables printed by a publishing house, calculator. No other books are allowed.

SOLUTIONS: Linked from the course home page after the exam.

The exam can be inspected and checked out 2024-01-31, Room 2A:573, B-building, entrance 25, A-corridor, to the right.

PRELIMINARY GRADING:	grade 3	15 points
	grade 4	23 points
	grade 5	30 points

All solutions should be well motivated. Writing should be neat and clean.

Good Luck!

1. Find the extremals of the following functionals

   (a) $J = \int_0^1 (y^2 + \dot{y}^2 - 2y \sin t)\, dt,$ (3p)

   (b) $J = \int_0^1 \frac{\dot{y}^2}{t^3}\, dt,$ (3p)

   (c) $J = \int_0^1 (y^2 + \dot{y}^2 + 2ye^t)\, dt.$ (4p)

   where $y(0) = 0$.

2. A certain material is passed through a sequence of two ovens. Denote by

   x_0: the initial temperature of the material,

   $x_k,\ k = 1, 2$: the temperature of the material at the exit of oven k,

   $u_{k-1},\ k = 1, 2$: the prevailing temperature in oven k.

   The ovens are modeled as

   $$x_{k+1} = (1 - a)x_k + au_k, \quad k = 0, 1,$$

   where a is a known scalar from the interval $(0, 1)$. The objective is to get the final temperature x_2 close to a given target T, while expending relatively little energy. This is expressed by a cost function of the form

   $$r(x_2 - T)^2 + u_0^2 + u_1^2,$$

   where $r > 0$ is a given scalar. For simplicity, we assume no constraints on u_k.

   (a) Formulate the problem as an optimization problem. (3p)

   (b) Solve the problem using the dynamic programming recursion when $a = 1/2$, $T = 0$ and $r = 1$. (3p)

   (c) Solve the problem for all feasible a, T and r. (4p)

3. (a) What is the reason for using the Q-function in reinforcement learning instead of using the value function as in dynamic programming? (3p)

   (b) Suggest an alternative to reinforcement learning when no model for the system you want to control is known, but you can perform experiments on the system. (3p)

   (c) What is the key assumption required in order to use Iterative Learning Control (ILC) which does not apply to general control problems? (4p)

4. A producer with production rate $x(t)$ at time t may allocate a portion $u(t)$ of his/her production rate to reinvestments in a factory (thus increasing the production rate) and use the rest $(1 - u(t))$ to store goods in a warehouse. Thus $x(t)$ evolves according to

$$\dot{x}(t) = \alpha u(t) x(t), \tag{1}$$

where α is a given constant. The producer wants to maximize the total amount of goods stored summed with the capacity of the factory at final time. This gives us the following problem:

$$\begin{array}{ll} \text{maximize} & x(T) + \int_0^T (1 - u(t))x(t)\,dt \\ \text{subject to} & \dot{x}(t) = \alpha u(t)x(t), \; 0 < \alpha < 1 \\ & x(0) = x_0 > 0, \\ & 0 \leq u(t) \leq 1, \; \forall t \in [0, T]. \end{array}$$

with variables u and x. Find an analytical solution to the problem above using the Pontryagin maximum principle. (10p)