## TSIN02 Internetworking

## Exercise class 4 solutions

Exercise 1:
a) 25 SEK .
b) -200 SEK.
c) Under flat rate, the customers are unhappy since the net utility is negative. On the other hand, with usage-based pricing, the users are happy since the net utility is positive. This follows from that the underlying utility function is based on a user poll where the users tell how much they are willing to pay for different amounts of data.

Exercise 2: The net user utility before service upgrade is 200 SEK, and after service upgrade is - 800 SEK. The service upgrade should thus not be done while applying this new pricing scheme.

Exercise 3:
a) See Fig. 1 .
b) See Fig. 2 .
c) Both groups of customers have positive net utility for usage-based pricing.


Figure 1: Demand $D(p)$ as a function of price $p$ in solution 3.a.


Figure 2: Demand $D(p)$ as a function of price $p$ in solution 3.b.

## Exercise 4:

a) 500 SEK
b) -150 SEK.

Exercise 5: The cost of exceeding capacity at day is

$$
c_{d a y}=\max \left[100\left(d_{A, d a y}\left(1-q_{A}(p)\right)+d_{B, \text { day }}\left(1-q_{B}(p)\right)-12\right), 0\right] .
$$

The cost of exceeding capacity at night is

$$
c_{n i g h t}=\max \left[100\left(d_{A, n i g h t}+d_{A, d a y} q_{A}(p)+d_{B, n i g h t}+d_{B, \text { day }} q_{B}(p)-12\right), 0\right]
$$

The rewards given out is

$$
c_{\text {reward }}=p\left(d_{A, \text { day }} q_{A}(p)+d_{B, \text { day }} q_{B}(p)\right) .
$$

Therefore the objective function is to minimize $c_{d a y}+c_{n i g h t}+c_{\text {reward }}$ with the constraint $p \in[0,100]$.
The optimal $p$ can be found by a numerical search over $p$ from 0 to 100 , and find the one with lowest objective function value.

