COLLECTION OF FORMULAS, TSIN02

1. The distortion due to uniform quantization of a signal sample is

$$D \approx \frac{\triangle^2}{12} = \frac{1}{12} \cdot (\frac{A}{M})^2$$

where $\triangle$ is the quantizer step, A is the total range of the signal and M is the number of levels in the quantizer.

2. General result for the signal-to-distortion ratio for a quantized signal sample.

$$SDR \approx 6R - const \quad \text{[dB]}$$

where R is the number of bits per sample. For uniform quantizer and Gaussian distributed signal, $const$= 7.4.

3. The entropy of a discrete random variable X with value set A, is

$$H(X) = -\sum_{x \in A} p(x) \cdot \log_2 p(x) \quad \text{bits}$$

4. For an analog (Gaussian) source with equally strong frequencies within the bandwidth B and a tolerable signal-to-distortion (S/D) ratio, the lowest number of bits per second is

$$R = B \log_2 \frac{S}{D} \approx 0.33B * SDR \qquad (SDR = S/D \text{ in dB})$$

5. Shannon theorem gives an upper bound to the capacity of a link in bits per second (bps), as a function of the available bandwidth and the signal-to-noise ratio of the link

$$C = B \cdot \log_2(1 + \frac{S}{N})$$

where B is the bandwidth, S is the average signal power and N is the average noise power.

6. The Shannon upper bound to the capacity of a binary symmetric channel (BSC)

$$C = 1 - h(p) \quad \text{[correct bits per transmitted bits]}$$

where p is the error probability of the channel and $h(p)$ is the binary entropy function.

$$h(p) = -p \cdot \log_2 p - (1-p) \cdot \log_2 (1-p)$$

7. The packet error probability for a binary symmetric channel (BSC), is

$$P = 1 - (1-p)^N \approx Np \quad (\text{if } Np << 1)$$

where N is the numbers of bits per packet and p is the bit error probability.

8. The capacity for the packet loss channel with i.i.d. losses, and packet loss probability P, is

$$C = 1 - P$$

bits/channel symbol.

9. For the first order auto-regressive (AR) model:

$$x(n) = a x_{n-1} + u_n$$

where u_n is zero-mean Gaussian with variance 1, and $a < 1$,

the average signal power:

$$\sigma^2 = \frac{1}{1-a^2}$$

the best interpolation between two neighboring samples:

$$\tilde{x}(n) = \frac{a}{1+a^2} \cdot [x(n-1) + x(n+1)]$$

the average interpolation error:

$$\text{MSE}_{\text{interp}} = \frac{1}{1+a^2}$$

the best prediction of a future sample:

$$\tilde{x}(n+k) = a^{|k|} \cdot x(n)$$

The average prediction error MSE_{pred}

$$\text{MSE}_{\text{pred}} = \frac{1 - \mathrm{a}^{2|\mathrm{k}|}}{1 - \mathrm{a}^2}$$

where k can take on all integers (also negative and 0).

10. The utility function U can be reverse engineered from the demand function D

$$U = \int D^{-1}(p)\,dp$$

if U is a smooth concave function.

10. Optical communication bands:
Original band (O-band): 1260 to 1360nm
Extended band (E-band): 1360 to 1460nm
Short band (S-band): 1460 to 1530nm
Conventional band (C-band): 1530 to 1565nm
Long band (L-band): 1565 to 1625nm
Ultralong band (U-band): 1625 to 1675nm

UDP datagram:

<table>
<tr><td>Source port number (16 bits)</td><td>Destination port number (16 bits)</td></tr>
<tr><td>UDP length (16 bits)</td><td>UDP checksum (16 bits)</td></tr>
<tr><td colspan="2">Data</td></tr>
</table>

TCP segment:

<table>
<tr><td colspan="4">Source port number (16 bits)</td><td colspan="5">Destination port number (16 bits)</td></tr>
<tr><td colspan="9">Sequence number (32 bits)</td></tr>
<tr><td colspan="9">Acknowledgment (32 bits)</td></tr>
<tr><td>Header length (4 bits)</td><td>Reserved (6 bits)</td><td>URG</td><td>ACK</td><td>PSH</td><td>RST</td><td>SYN</td><td>FIN</td><td>Window size (16 bits)</td></tr>
<tr><td colspan="4">TCP checksum (16 bits)</td><td colspan="5">Urgent pointer (16 bits)</td></tr>
<tr><td colspan="9">Options (if any)</td></tr>
<tr><td colspan="9">Data (if any)</td></tr>
</table>

IPv4 datagram:

<table>
<tr><td>Version (4 bits)</td><td>Header length (4 bits)</td><td>Type of Service (8 bits)</td><td colspan="2">Total length (16 bits)</td></tr>
<tr><td colspan="3">Identification (16 bits)</td><td>Flags (3 bits)</td><td>Fragment offset (13 bits)</td></tr>
<tr><td colspan="2">Time To Live (8 bits)</td><td>Protocol (8 bits)†</td><td colspan="2">Checksum (16 bits)</td></tr>
<tr><td colspan="5">Source IP address (32 bits)</td></tr>
<tr><td colspan="5">Destination IP address (32 bits)</td></tr>
<tr><td colspan="5">Options (if any)</td></tr>
<tr><td colspan="5">Data</td></tr>
</table>