Golomb and Exp-Golomb codes

For what probability distributions are Golmb and Exp-Golomb codes optimal? Let's make some calculations to find out. The following results are not mathematically strict, but gives us a good idea about the answers.

Golomb codes

The codeword lengths l_i of a Golomb code grow approximately linearly with i. Of course the lengths must be integers, but for this analysis we can ignore that.

$$l_i \approx ai + b$$

for some a and b. Ideally we would want to have $l_i = -\log p_i$, since that would mean that the mean codeword length would be the same as the entropy for the distribution. If we could choose the lengths in this way, we would have

$$l_i = ai + b = -\log p_i \;\Rightarrow\; p_i = 2^{-(ai+b)} = 2^{-b} \cdot 2^{-ai} = q \cdot p^i$$

This means that Golomb codes would be good for geometric distributions.

Exp-Golomb codes

The codeword lengths l_i of an Exp-Golomb code grow approximately logarithmically with i. Of course the lengths must be integers, but for this analysis we can ignore that.

$$l_i \approx a \cdot \log(i + c) + b$$

for some a, b and c. Ideally we would want to have $l_i = -\log p_i$, since that would mean that the mean codeword length would be the same as the entropy for the distribution. If we could choose the lengths in this way, we would have

$$l_i = a \cdot \log(i + c) + b = -\log p_i \;\Rightarrow\; p_i = 2^{-a \cdot \log(i+c) - b} = 2^{-b} \cdot (i + c)^{-a}$$

This means that Exp-Golomb codes would be good for power law distributions.