Examination TSTE25

Date:
Time:
Tentajour:
Permitted aids: A sheet of paper with formulae and a scientific calculator.

Exercise 1. A power transmission company has been tasked with building a 600 kV 5000 MW HVDC link between Stavanger, Norway to Fraserburgh, Scotland. A key component in the HVDC link is the power converter. What is the type and rating of semiconductor switches should they consider for the converter? Motivate. (4 point)

Solution
Semiconductor switch type: A modular multilevel converter with several submodules with IGBTs.

Minimum Voltage rating:

$$\frac{600\,\text{kV}}{\text{Total number of submodules}}$$

Exercise 2. Consider all components to be ideal in a step-down converter used in a small portable PV phone charger and let the average output voltage (V_o) be held constant at 5 V. If input voltage (V_i) is 10-40 V, output power, $P_o \geq 15$ W, and switching frequency, $f_s = 50$ kHz, calculate the following:

a) The switch duty ratio (D) that is required to maintain the average output voltage of 5 V. (1 point)

Solution
Duty cycle,

$$D = \frac{V_o}{V_{in}}.$$

$D = 50\%$ when $V_{in} = 10$ V and $D = 12.5\%$ when $V_{in} = 40$ V.

b) The minimum inductance (L) required to keep the converter operation in a continuous-conduction mode under all conditions. (4 point)

Solution
The average output current I_o is

$$I_o = \frac{P_o}{V_o} = \frac{15}{5} = 3\,\text{A}.$$

For the converter operating at the boundary between continuous and discontinuous conduction mode, the peak-to-peak current ripple ($\Delta I_{L(pp)}$) is

$$\Delta I_{L(pp)} = 2\, I_o = 6\,\text{A}.$$

The inductance at the boundary condition is

$$L = \frac{V_o}{\Delta I_{L(pp)}\, f_{sw}}\,(1 - D)\,.$$

$L = 8.3\,\mu$H at $D = 50\%$ when $V_{in} = 10$ V and $L = 14.58\,\mu$H at $D = 12.5\%$ when $V_{in} = 40$ V.

If $L = 8.3\,\mu$H at $D = 12.5\%$ when $V_{in} = 40$ V. Then the peak-to-peak current ripple ($\Delta I'_{L(pp)}$) is

$$\Delta I'_{L(pp)} = \frac{V_o}{L\, f_{sw}}\,(1 - D) = 10.5\,\text{A} > 6\,\text{A (i.e., } 2\, I_o).$$

Since $\Delta I'_{L(pp)} > 2\, I_o$, the converter is in discontinuous conduction mode. If $L = 14.58\,\mu$H at $D = 50\%$ when $V_{in} = 10$ V. Then the peak-to-peak current ripple ($\Delta I''_{L(pp)}$) is

$$\Delta I''_{L(pp)} = \frac{V_o}{L\, f_{sw}}\,(1 - D) = 3.43\,\text{A} < 6\,\text{A (i.e., } 2\, I_o).$$

Since $\Delta I''_{L(pp)} < 2\, I_o$, the converter is in continuous conduction mode.

Therefore, $L = 14.58\,\mu$H is the minimum inductance required to keep the converter operation in a continuous-conduction mode under all conditions.

c) The minimum capacitance (C) required to keep the output peak-to-peak ripple to be 10% of the average output voltage. (2 point)

Solution

The output peak-to-peak ripple ($\Delta V_{o(pp)}$) is 10% of the average output voltage V_o, i.e., $\Delta V_{o(pp)} = 0.1\, V_o = 0.5\,\text{V}$.

The capacitance at the to keep the output peak-to-peak ripple to be 10% of V_o is

$$C = \frac{(1 - D)\; V_o}{8\, f_{sw}^2\, L\, \Delta V_{o(pp)}}.$$

$C = 17.15\,\mu\text{F}$ at $D = 50\%$ when $V_{in} = 10\,\text{V}$ and $C = 30\,\mu\text{F}$ at $D = 12.5\%$ when $V_{in} = 40\,\text{V}$. Since $\Delta I'_{L(pp)} > 2\, I_o$, the converter is in discontinuous conduction mode. If $C = 30\,\mu\text{F}$ at $D = 50\%$ when $V_{in} = 10\,\text{V}$. Then the peak-to-peak voltage ripple ($\Delta V''_{o(pp)}$) is

$$\Delta V'_{o(pp)} = \frac{(1 - D)\; V_o}{8\, f_{sw}^2\, L\, C} = 0.3\,\text{V} < 0.5\,\text{V (i.e., } 0.1\, V_o).$$

With $C = 30\,\mu\text{F}$, the output peak-to-peak ripple is lower than 10% of V_o. If $C = 17.5\,\mu\text{F}$ at $D = 12.5\%$ when $V_{in} = 40\,\text{V}$. Then the peak-to-peak voltage ripple ($\Delta V'_{o(pp)}$) is

$$\Delta V'_{o(pp)} = \frac{(1 - D)\; V_o}{8\, f_{sw}^2\, L\, C} = 0.88\,\text{V} > 0.5\,\text{V (i.e., } 0.1\, V_o).$$

With $C = 17.5\,\mu\text{F}$, the output peak-to-peak ripple is larger than 10% of V_o.

Therefore, $C = 30\,\mu\text{F}$ is the minimum capacitance required to keep the converter output voltage below 10% of V_o.

d) The minimum efficiency of the DC-DC converter. IRF540 MOSFETs are employed (datasheet found at the end of the document) and the voltage and current switching transient times are (4 point)

Solution

$$t_{ri} = 19\,\text{ns}, \qquad t_{fv} = 34\,\text{ns}, \qquad t_{rv} = 12\,\text{ns}, \qquad t_{fi} = 16\,\text{ns}.$$

The current through the MOSFET is assumed to be constant to simplify the calculations. Therefore, the RMS current through the MOSFET ($I_{v(\text{rms})}$) is

$$I_{v(\text{rms})} = I_o.$$

From the MOSFET datasheets, the on-state resistance of the MOSFET ($r_{ds(\text{on})}$) is

$$r_{ds(\text{on})} = 0.077\,\Omega.$$

The conduction losses of the MOSFET (P_c^l) is

$$P_c^l = D\, I_{v(\text{rms})}^2\, r_{ds(\text{on})}.$$

$P_c^l = 0.35\,\text{W}$ at $D = 50\%$ when $V_{in} = 10\,\text{V}$ and $P_c^l = 0.087\,\text{W}$ at $D = 12.5\%$ when $V_{in} = 40\,\text{V}$. The switching losses of the MOSFET (P_s^l) is

$$P_s^l = \frac{1}{2}\, V_{in}\, I_o\, t_{sw}\, f_{sw} = \frac{1}{2}\, V_{in}\, I_o\, (t_{ri} + t_{fv} + t_{rv} + t_{fi})\, f_{sw}.$$

$P_s^l = 0.061\,\text{W}$ at $D = 50\%$ when $V_{in} = 10\,\text{V}$ and $P_s^l = 0.24\,\text{W}$ at $D = 12.5\%$ when $V_{in} = 40\,\text{V}$.

Note that the switching losses increase when D is lower. This is because $P_s^l \propto V_{in}\, I_o$ but $P_s^l \propto I_o^2$.

The total losses in the MOSET (P^l) is

$$P^l = P_c^l + P_s^l.$$

$P^l = 0.41\,\text{W}$ at $D = 50\%$ when $V_{in} = 10\,\text{V}$ and $P^l = 0.33\,\text{W}$ at $D = 12.5\%$ when $V_{in} = 40\,\text{V}$.

When P^l is high the efficiency is lower, i.e.,

$$\eta = \frac{P_o}{P_o + P^l}$$

the minimum efficiency (η) is 97.35% at $D = 50\%$ when $V_{in} = 10\,\text{V}$.

Exercise 3. Consider all components to be ideal in a step-up converter used in a power management system in a vehicle and let the average output voltage (V_o) be held constant at 5 V. If battery voltage on the input side (V_i) is 2.9-4.2 V, $P_o \geq 200$ W, and due to cost limitations, the only available passive components are an inductor, $L = 10\,\mu$H, and several capacitors of 100 μF. Calculate the following:

a) The switch duty ratio (D) that is required to maintain the average output voltage of 5 V. (1 point)

Solution
Duty cycle,

$$D = 1 - \frac{V_{in}}{V_o}.$$

$D = 42\%$ when $V_{in} = 2.9$ V and $D = 16\%$ when $V_{in} = 4.2$ V.

b) The minimum switching frequency (f_s) required to keep the battery current peak-to-peak ripple below 10% of the average battery current. (4 point)

Solution
In the boost converter, the input current is the current through the inductor. Assuming a loss-less DC-DC converter (i.e., power balance), the average inductor current (I_L) is The average output current I_o is

$$I_L = \frac{P_o}{V_{in}}.$$

$I_L = 68.97$ A when $V_{in} = 2.9$ V and $I_L = 47.62$ A when $V_{in} = 4.2$ V.

The peak-to-peak current ripple ($\Delta I_{L(pp)}$) is

$$\Delta I_{L(pp)} = 0.1\, I_L.$$

$\Delta I_{L(pp)} = 6.9$ A when $V_{in} = 2.9$ V and $\Delta I_{L(pp)} = 4.8$ A when $V_{in} = 4.2$ V.

The switching frequency required to achieve a desired peak-to-peak current ripple using an inductor with inductance (L) is

$$f_{sw} = \frac{D\, V_o}{\Delta I_{L(pp)}\, L} \left(1 - D\right).$$

$f_{sw} = 17.66$ kHz at $D = 42\%$ when $V_{in} = 2.9$ V and $f_{sw} = 14.11$ kHz at $D = 16\%$ when $V_{in} = 4.2$ V.

If $f_{sw} = 17.66$ kHz at $D = 16\%$ when $V_{in} = 4.2$ V. Then the peak-to-peak current ripple ($\Delta I'_{L(pp)}$) is

$$\Delta I'_{L(pp)} = \frac{D\, V_o}{L\, f_{sw}} \left(1 - D\right) = 3.81 \text{ A} < 4.8 \text{ A (i.e., } 0.1\, I_L).$$

Since $\Delta I'_{L(pp)} < 0.1\, I_L$, the peak-to-peak ripple is lesser than the desired value.

If $f_{sw} = 14.11$ kHz at $D = 42\%$ when $V_{in} = 2.9$ V. Then the peak-to-peak current ripple ($\Delta I''_{L(pp)}$) is

$$\Delta I''_{L(pp)} = \frac{D\, V_o}{L\, f_{sw}} \left(1 - D\right) = 8.63 \text{ A} > 6.9 \text{ A (i.e., } 0.1\, I_L).$$

Since $\Delta I''_{L(pp)} > 0.1\, I_L$, the peak-to-peak ripple is greater than the desired value.

Therefore, $f_{sw} = 17.66$ kHz is the minimum switching frequency required to keep the converter inductor (input) peak-to-peak converter ripple current less than 10% of the average inductor (input) current.

c) The minimum number of capacitors required to keep the output peak-to-peak voltage ripple below 10% of the average output voltage. (3 point)

Solution
The average output current (I_o) is

$$I_o = \frac{P_o}{V_o} = 40\,\text{A}.$$

The output voltage peak-to-peak ripple ($\Delta V_{o(pp)}$) is

$$\Delta V_{o(pp)} = 0.1\,V_o = 0.5\,\text{V}$$

The capacitance (C) required to keep the required peak-to-peak output voltage ripple is

$$C = \frac{D\,I_o}{f_{sw}\,\Delta V_{o(pp)}}$$

$C = 1902.5\,\mu\text{F}$ at $D = 42\%$ when $V_{in} = 2.9\,\text{V}$ and $C = 725\,\mu\text{F}$ at $D = 16\%$ when $V_{in} = 4.2\,\text{V}$.

If $C = 725\,\mu\text{F}$ at $D = 42\%$ when $V_{in} = 2.9\,\text{V}$, the peak-to-peak output voltage ripple ($\Delta V'_{o(pp)}$) is

$$\Delta V'_{o(pp)} = \frac{D\,I_o}{f_{sw}\,C} = 1.31\,\text{V} > 0.5\,\text{V (i.e., } 0.1\,V_o).$$

Since $\Delta V'_{o(pp)} > 0.1\,V_o$, the peak-to-peak ripple is greater than the desired value.

If $C = 1902.5\,\mu\text{F}$ at $D = 16\%$ when $V_{in} = 4.2\,\text{V}$, the peak-to-peak output voltage ripple ($\Delta V''_{o(pp)}$) is

$$\Delta V''_{o(pp)} = \frac{D\,I_o}{f_{sw}\,C} = 0.19\,\text{V} < 0.5\,\text{V (i.e., } 0.1\,V_o).$$

Since $\Delta V''_{o(pp)} < 0.1\,V_o$, the peak-to-peak ripple is lesser than the desired value.

Therefore, $C = 1902.5\,\mu\text{F}$ is the minimum capacitance required to keep the output voltage peak-to-peak ripple less than 10% of the average output voltage.

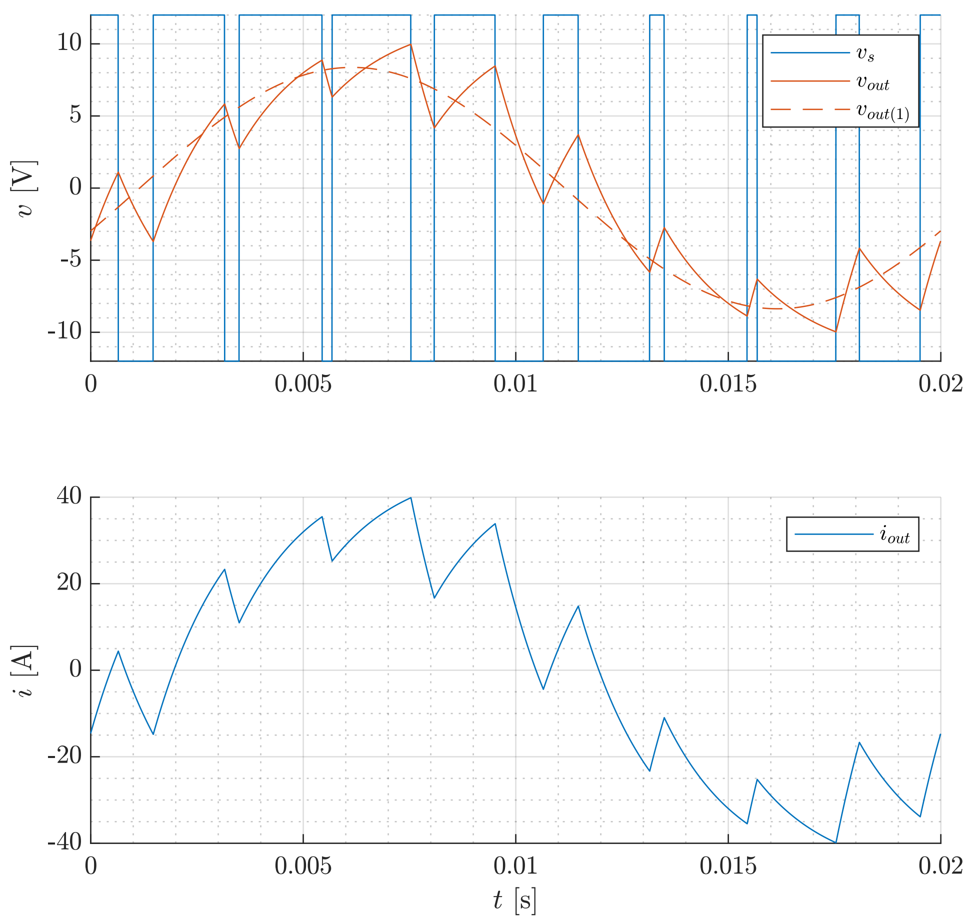
Since only $100\,\mu\text{F}$ capacitors are available, the minimum number of capacitors ($N_{p(cap)}$) required is by connecting them in parallel. i.e.,

$$N_{p(cap)} = \left\lceil \frac{C}{100\,\mu} \right\rceil = 20.$$

Exercise 4. For a half-bridge inverter, assuming all ideal components, the output waveforms are presented in Figure 1, where

$$\begin{aligned} \text{peak inverter side voltage} \quad & \hat{v}_s = 12\,\text{V}, \\ \text{peak inverter side voltage (fundamental)} \quad & \hat{v}_{s(1)} = 9.61\,\text{V}, \\ \text{peak output current} \quad & \hat{i}_{out} = 40.1\,\text{A}, \\ \text{peak output voltage} \quad & \hat{v}_{out} = 11\,\text{V}, \\ \text{peak output voltage (fundamental)} \quad & \hat{v}_{out(1)} = 8.37\,\text{V}. \end{aligned}$$

Determine:



Figur 1: half-bridge inverter output waveforms.

a) The switching frequency used (2 point)

Solution
Counting the number of positive/negative pulses (or, rising/falling edges) for v_s in Figure 1, gives the pulse number (m_f) as

$$m_f = 9.$$

Since m_f is defined as

$$m_f = \frac{f_{sw}}{f_1} \qquad \Longrightarrow f_{sw} = m_f\, f_1,$$

where f_1 is the fundamental frequency, and from Figure 1

$$f_1 = \frac{1}{0.02\,\mathrm{s}} = 50\,\mathrm{Hz}.$$

Therefore,

$$f_{sw} = m_f\, f_1 = 450\,\mathrm{Hz}.$$

b) The inductance (3 point)

Solution
From Figure 1, during the time interval $t \in [0.008, 0.095]\,\mathrm{s}$, the $v_v = 12\,\mathrm{V}$ and for simplicity the voltage after the inductor v_{out} is assumed to be 6 V. Then the voltage drop across the inductor is

$$V_L = (v_{out}(t) - v_v(t))\Big|_{t\in[0.008,0.095]} = L\frac{di(t)}{t}\Bigg|_{t\in[0.008,0.095]} \qquad \Longrightarrow L = \frac{v_{out}(t) - v_v(t)}{\frac{di}{dt}}\Bigg|_{t\in[0.008,0.095]}.$$

$dt = 0.0095\,\mathrm{s} - 0.008\,\mathrm{s}$, and from Figure 1, $di = 35\,\mathrm{A} - 15\,\mathrm{A}$. Therefore the inductance, L is

$$L = \frac{12-6}{\frac{20}{0.0015}} = 0.45\,\mathrm{mH}.$$

c) The peak fundamental current (2 point)

Solution
The peak fundamental output current occurs at $0.0057\,\mathrm{s} \leq t \leq 0.0075\,\mathrm{s}$. The average current in this interval is the peak Fundamental output current ($\hat{i}_{out(1)}$)

$$\hat{i}_{out(1)} = \frac{40.1 + 25}{2} = 32.55\,\mathrm{A}.$$

d) The pole-to-pole DC-link voltage (V_d) and modulation index (m_a) (2 point)

Solution
In the half-bridge inverter, the pole-to-pole DC-link voltage (V_d) is

$$V_d = 2\,\hat{v}_s = 24\,\mathrm{V}.$$

The modulation index (m_a) is

$$m_a = \frac{\hat{v}_{s(1)}}{V_d/2} = 0.8.$$

e) The active power on the load at the fundamental frequency. (1 point)

Solution
The active power on the load (P_{out}) at the fundamental frequency is

$$P_{out} = \frac{\hat{v}_{out(1)}\,\hat{i}_{out(1)}}{2} = 136.22\,\text{W}.$$

f) The phase angle of the fundamental current with respect to the inverter side voltage. (2 point)

Solution
At time $t = 0\,\text{s}$ the reference signal (fundamental converter output voltage) is $0\,\text{V}$ thus the converter output voltage (v_s) is also assumed to be $0\,\text{V}$. However, the fundamental load current (or voltage) is $0\,\text{A}$ (or $0\,\text{V}$) at $t = 0.0012\,\text{s}$. The time delay (δt) is

$$\delta t = 0.0012\,\text{s}.$$

If time $t = T = 0.02\,\text{s}$ is 2π, then time $t = \delta t$, i.e., phase angle (ϕ) is

$$\phi = \frac{\delta t}{T}\,2\,\pi = 21.6°.$$

g) The active and reactive power on the converter at the fundamental frequency. (2 point)

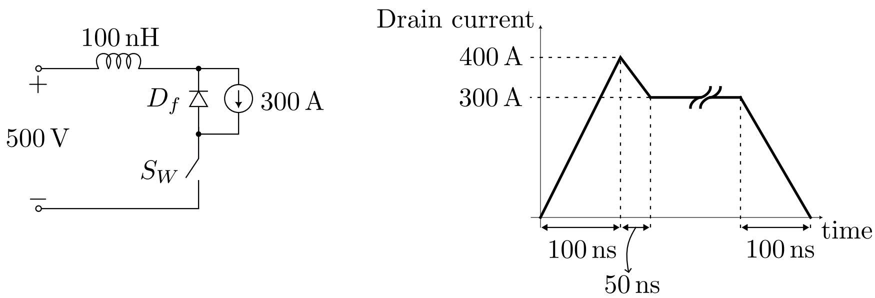
Solution
The active power (P_s) on the converter side is

$$P_s = \frac{\hat{v}_{s(1)}\,\hat{i}_{out(1)}}{2}\,\cos(\phi) = 145.42\,\text{W}.$$

The reactive power (Q_s) on the converter side is

$$Q_s = \frac{\hat{v}_{s(1)}\,\hat{i}_{out(1)}}{2}\,\sin(\phi) = 57.58\,\text{Var}.$$

Exercise 5. Consider the switched step-down converter shown in Figure 2. The drain current of the MOSFET as a function through one complete turn-on and turn-off sequence is shown in Figure 2. The switch (S_W) and diode (D_f) data are provided in Table 1.



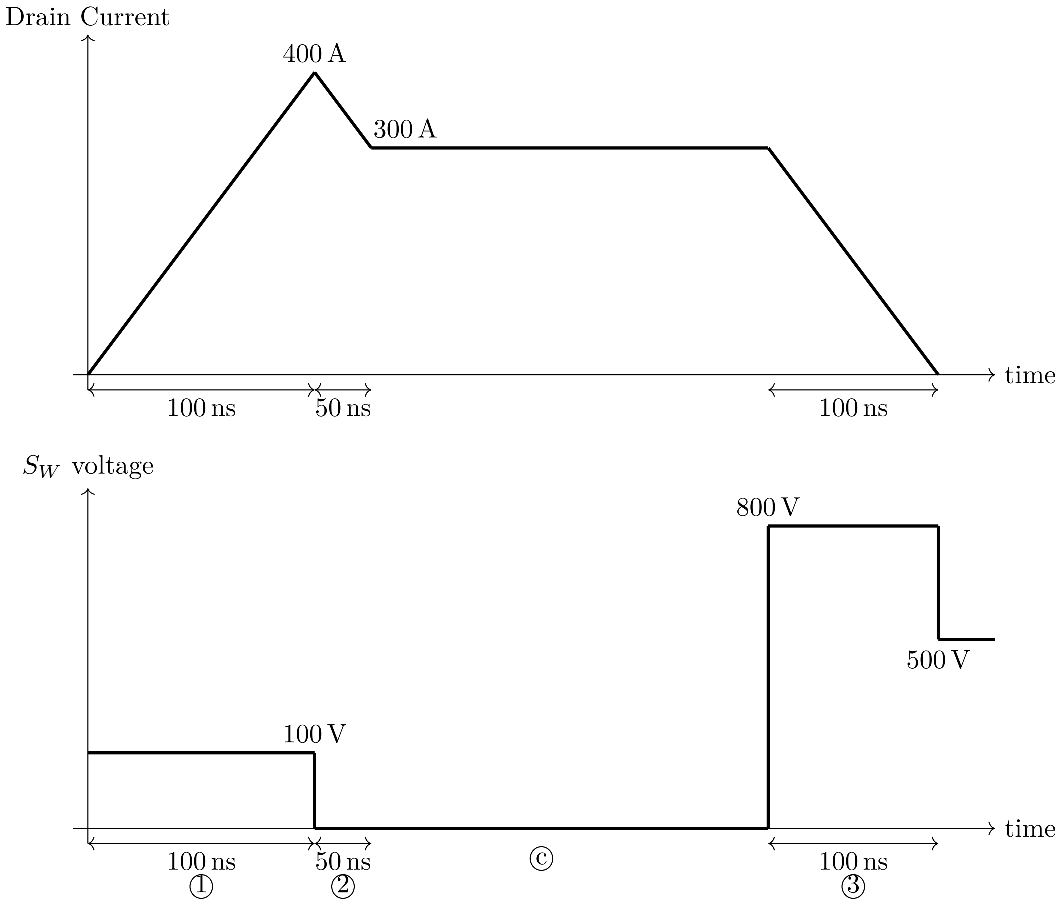
Figur 2: Switched step-down converter and drain current transients.

Tabell 1: switch (S_W) and diode (D_f) data.

S_W data		D_f data	
$V_{ds(\text{max})}$	700 V	V_{rm}	800 V
$I_{d(\text{max})}$	400 V	I_{max}	400 A
$T_{j(\text{max})}$	150°C	$T_{j(\text{max})}$	150°C
$R_{\theta(\text{ja})}$	0.1°C/W	$R_{\theta(\text{ja})}$	1°C/W
R_{on}	0.01 Ω	R_{on}	0.1 Ω
		V_{on}	0.7 V

a) Sketch and dimension the drain-source voltage of S_W as a function of time. Assume that the voltage across the switch can change instantaneously and is only limited by the external circuit. (4 point)

Solution



Applying KVL to the circuit in Figure 2,

$$V_d = V_L + V_{D_f} + V_{S_W},$$

where $V_d = 500\,\mathrm{V}$.

In region ①, the diode is in conduction, i.e., $V_{D_F} = 0$, and V_{S_W} is

$$V_{S_W} = V_d - V_L = 500 - \frac{400 - 0}{100\,\mathrm{n}} 100\,\mathrm{n} = 100\,V.$$

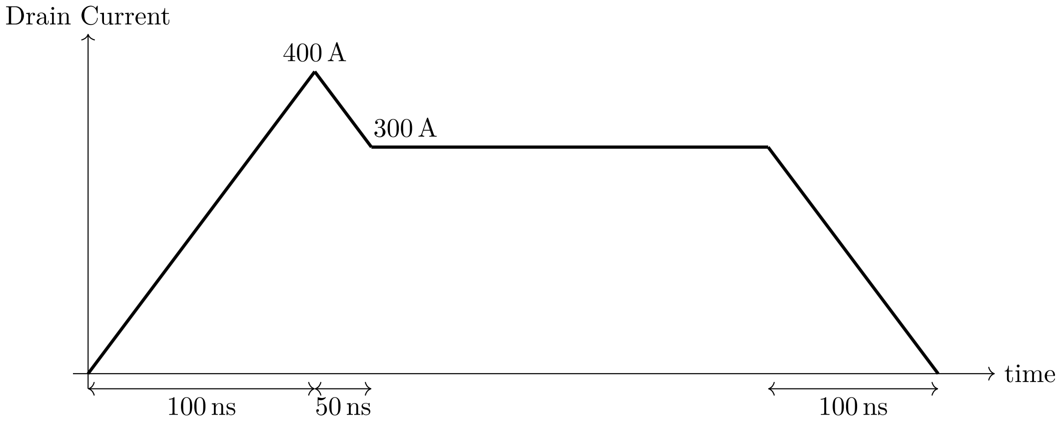
When the diode current changes direction, i.e., becomes negative in region ②, the diode starts to block the voltage, i.e.,

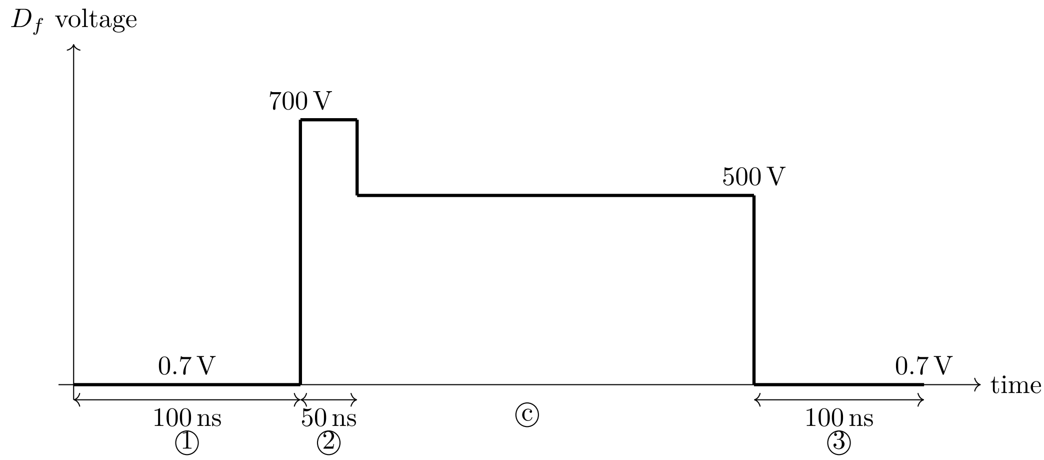
$$V_{D_f} = V_d - V_L = 500 - \frac{300 - 400}{50\,\mathrm{n}} 100\,\mathrm{n} = 700\,V. \qquad \text{and} \quad V_{S_W} = 0\,\mathrm{V}.$$

In region ③, the diode starts conduction, i.e., $V_{D_F} = 0$, and V_{S_W} is

$$V_{S_W} = V_d - V_L = 500 - \frac{0 - 300}{100\,\mathrm{n}} 100\,\mathrm{n} = 800\,V.$$

b) Sketch and dimension the diode voltage as a function of time. (3 point)





c) Are either the diode or the switch overstressed with respect to voltage? If so, specify by how much. (2 point)

Solution

The diode is not overstressed because the breakdown voltage of the diode is 800 V and the maximum voltage across the diode is 700 V in region ②. However, the switch S_W is overstressed because in region ③, the voltage across the switch is 800 V but the rated voltage of the switch is 700 V.

d) Determine the junction temperature of the switch and the diode. Do not forget the losses during the switching transients, the diode reverse recovery, a duty cycle of 90%, and assuming a switching frequency of 10 kHz at an ambient temperature of 25°C . (6 point)

Solution

The losses in the switch during conduction ($P^l_{S_W(c)}$) in region ⓒ is

$$P^l_{S_W(c)} = D\, I_d^2\, R_{(on)} = 0.9\, 300^2\, 0.01 = 810\,\text{W}.$$

The losses in the switch during thee switching transients ($P^l_{S_W(sw)}$) in regions ① and in region ③ is

$$P^l_{S_W(sw)} = \frac{1}{2} I_d\, V_{S_W}\, t_{on}\, f_{sw} + \frac{1}{2} I_d\, V_{S_W}\, t_{off}\, f_{sw} = \frac{1}{2} 400\, 100\, 100\text{n}\, 10\text{k} + \frac{1}{2} 300\, 800\, 100\text{n}\, 10\text{k} = 140\,\text{W}.$$

The total losses in the switch ($P^l_{S_W}$) is

$$P^l_{S_W} = P^l_{S_W(c)} + P^l_{S_W(sw)} = 950\,\text{W}.$$

The junction temperature ($T_{j(S_W)}$) of the switch is

$$T_{j(S_W)} = P^l_{S_W}\, R_{\theta ja} + T_a = 950 \times 0.1 + 25 = 120°\text{C}.$$

If the rate of change of current is $\mathcal{M}_i$, the RMS current through the diode in region ① ($I_{D_f ①}$) is

$$\begin{aligned} I_{D_f ①} &= \sqrt{\frac{1}{T_{sw}} \int_0^{t_①} (\mathcal{M}_i\, t)^2\, dt} = \sqrt{\frac{1}{T_{sw}} \int_0^{t_①} (\mathcal{M}_i)^2\, t^2\, dt} = \mathcal{M}_i \sqrt{\frac{1}{T_{sw}} \int_0^{t_①} t^2\, dt} \\ &= \mathcal{M}_i \sqrt{\frac{1}{T_{sw}} \left[t^3\right]_0^{t_①}} = \mathcal{M}_i \sqrt{\frac{1}{T_{sw}} \left(t^3_①\right)} = \mathcal{M}_i\, t_① \sqrt{f_{sw}\, t_①} \\ &= \frac{400}{100 \times 10^{-9}}\, 100 \times 10^{-9} \sqrt{100 \times 10^{-9} \times 10^4} = 12.64\,\text{A}. \end{aligned}$$

Similarly, the RMS current through the diode in region ③ ($I_{D_f ③}$) is

$$I_{D_f ③} = 300\, \sqrt{100 \times 10^{-9} \times 10^4} = 9.5\,\text{A}.$$

The losses in the diode during conduction ($P^l_{D_f(c)}$) in region ① and ③ is

$$P^l_{D_f(c)} = I^2_{D_f ①}\, R_{(on)} + \frac{1}{2}\, I_d\, V_{(on)}\, t_①\, f_{sw} + I^2_{D_f ③}\, R_{on} + \frac{1}{2}\, I_d\, V_{(on)}\, t_③\, f_{sw} = 25.25\,\text{W}.$$

The losses in the diode during switching transients, i.e., reverse recovery, in ($P^l_{D_f(rr)}$) in ② is

$$P^l_{D_f(rr)} = \frac{1}{2}\, I_d\, V_{D_f}\, t_②\, f_{sw} = \frac{1}{2}\, (400 - 300)\, 700\, 50\text{n}\, 10\text{k} = 17.5\,\text{W}.$$

The total losses in the diode ($P^l_{D_f}$) is

$$P^l_{D_f} = P^l_{D_f(c)} + P^l_{D_f(rr)} = 42.75\,\text{W}.$$

The junction temperature ($T_{j(D_f)}$) of the diode is

$$T_{j(D_f)} = P^l_{D_f}\, R_{\theta ja} + T_a = 42.75 \times 1 + 25 = 67.75°\text{C}.$$

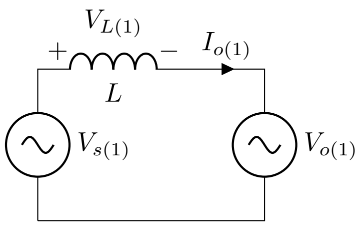
Exercise 6. Consider the problem of ripple in the output current of a single-phase full-bridge inverter. Assume $V_{o(1)} = 200\,\text{V}$ and $I_{o(1)} = 10\,\text{A}$ at a frequency of 50 Hz and an induction motor load with the inductance of L = 10 mH. Calculate the peak value of the inverter ripple current if the converter is operating in a sinusoidal unipolar PWM mode, with $m_f = 21$ and $m_a = 0.8$. (8 point)

Tabell 2: Generalized harmonics of a half-bridge inverter output voltage for a large m_f.

$h \downarrow \quad m_a \rightarrow$	0.2	0.4	0.6	0.8	1
1 Fundamental	0.2	0.4	0.6	0.8	1
m_f	1.242	1.15	1.006	0.818	0.6023
$m_f \pm 2$	0.061	0.061	0.131	0.22	0.318
$m_f \pm 4$					0.018
$2m_f \pm 1$	0.19	0.326	0.37	0.314	0.181
$2m_f \pm 3$		0.024	0.071	0.139	0.212
$2m_f \pm 5$				0.013	0.033
$3m_f$	0.335	0.123	0.083	0.171	0.133
$3m_f \pm 2$	0.044	0.139	0.203	0.176	0.062
$3m_f \pm 4$		0.012	0.047	0.104	0.157
$3m_f \pm 6$				0.016	0.044
$4m_f \pm 1$	0.163	0.157	0.088	0.105	0.068
$4m_f \pm 3$	0.012	0.070	0.132	0.115	0.009
$4m_f \pm 5$			0.034	0.084	0.119
$4m_f \pm 7$				0.017	0.05

Note: output voltage ($\hat{V}_o$) is $\hat{V}_o = m_a\, V_d/2$.

Solution
The equivalent circuit at the fundamental frequency is given as follows



From the figure,

$$V_{L(1)} = \omega_1\, L\, I_{o(1)} = 2\pi\, 50\, 10 \times 10^{-3}\, 10 = 31.42\,\text{V}.$$

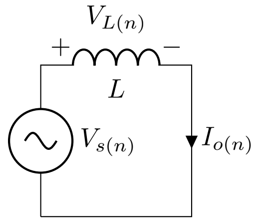
Also from the figure,

$$V_{s(1)} = V_{L(1)} + V_{o(1)} = 31.46 + 200 = 231.42\,\text{V}.$$

If the modulation index, $m_a = 0.8$, then the DC-link pole-to-pole voltage (V_d) is

$$V_d = \frac{V_{s(1)}\,\sqrt{2}}{m_a} = 409.1\,\text{V}.$$

The equivalent circuit at the n^{th} harmonic, where $n \neq 1$ is given as follows



From the figure,

$$V_{s(n)} = V_{L(n)} = \omega_n \, L \, I_{o(n)} = 2\pi \, n \, f_1 \, L \, I_{o(n)}.$$

or,

$$I_{o(n)} = \frac{V_{s(n)}}{2\pi \, n \, f_1 \, L} = \frac{k \, V_d}{2\pi \, n \, f_1 \, L}.$$

where k is presented in Table 2.

from Table 2, $V_{s(n)}$ and $I_{o(n)}$ for different values of n is presented as follows

n	k	$V_{s(n)}$ [V]	$I_{o(n)}$ [A]	Comment
m_f	0.818	0	0	Unipolar modulation odd carrier multiples are zero.
$m_f \pm 2$	0.22	0	0	Unipolar modulation odd carrier multiples are zero.
$2\,m_f - 5$	0.013	5.32	0.05	
$2\,m_f - 3$	0.139	56.87	0.46	
$2\,m_f - 1$	0.314	128.46	1	
$2\,m_f + 1$	0.314	128.46	0.95	
$2\,m_f + 3$	0.139	56.87	0.42	
$2\,m_f + 5$	0.013	5.32	0.04	
$3m_f$	0.818	0	0	Unipolar modulation odd carrier multiples are zero.
$3m_f \pm 2$	0.22	0	0	Unipolar modulation odd carrier multiples are zero.
$4\,m_f - 5$	0.084	34.35	0.14	
$4\,m_f - 3$	0.115	47.05	0.19	
$4\,m_f - 1$	0.105	43	0.18	
$4\,m_f + 1$	0.105	43	0.18	
$4\,m_f + 3$	0.115	47.05	0.17	
$4\,m_f + 5$	0.084	34.35	0.12	
		$\sqrt{\sum_n I_{o(n)}^2}$	4 A	Total RMS

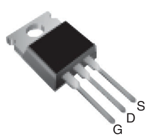
The peak ripple current is about $1\,\sqrt{2} = 1.414\,\mathrm{A}$ at 2050 Hz ($2\,m_f - 1$).

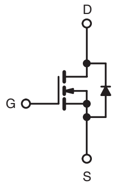
The total RMS ripple current is 4 A.

Power MOSFET

PRODUCT SUMMARY		
V_{DS} (V)	100	
$R_{DS(on)}$ (Ω)	V_{GS} = 10 V	0.077
Q_g (Max.) (nC)	72	
Q_{gs} (nC)	11	
Q_{gd} (nC)	32	
Configuration	Single	

TO-220





N-Channel MOSFET

FEATURES

- Dynamic dV/dt Rating
- Repetitive Avalanche Rated
- 175 °C Operating Temperature
- Fast Switching
- Ease of Paralleling
- Simple Drive Requirements
- Lead (Pb)-free Available

Pb Available
RoHS*
COMPLIANT

DESCRIPTION

Third generation Power MOSFETs from Vishay provide the designer with the best combination of fast switching, ruggedized device design, low on-resistance and cost-effectiveness.
The TO-220 package is universally preferred for all commercial-industrial applications at power dissipation levels to approximately 50 W. The low thermal resistance and low package cost of the TO-220 contribute to its wide acceptance throughout the industry.

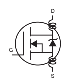
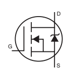
ORDERING INFORMATION	
Package	TO-220
Lead (Pb)-free	IRF540PbF
	SiHF540-E3
SnPb	IRF540
	SiHF540

ABSOLUTE MAXIMUM RATINGS T_C = 25 °C, unless otherwise noted					
PARAMETER			**SYMBOL**	**LIMIT**	**UNIT**
Drain-Source Voltage			V_{DS}	100	V
Gate-Source Voltage			V_{GS}	± 20	
Continuous Drain Current	V_{GS} at 10 V	T_C = 25 °C	I_D	28	A
		T_C = 100 °C		20	
Pulsed Drain Current[a]			I_{DM}	110	
Linear Derating Factor				1.0	W/°C
Single Pulse Avalanche Energy[b]			E_{AS}	230	mJ
Repetitive Avalanche Current[a]			I_{AR}	28	A
Repetitive Avalanche Energy[a]			E_{AR}	15	mJ
Maximum Power Dissipation	T_C = 25 °C		P_D	150	W
Peak Diode Recovery dV/dt[c]			dV/dt	5.5	V/ns
Operating Junction and Storage Temperature Range			T_J, T_{stg}	- 55 to + 175	°C
Soldering Recommendations (Peak Temperature)	for 10 s			300[d]	
Mounting Torque	6-32 or M3 screw			10	lbf · in
				1.1	N · m

Notes
a. Repetitive rating; pulse width limited by maximum junction temperature (see fig. 11).
b. V_{DD} = 25 V, starting T_J = 25 °C, L = 440 µH, R_G = 25 Ω, I_{AS} = 28 A (see fig. 12).
c. $I_{SD} \le 28$ A, dI/dt ≤ 170 A/µs, $V_{DD} \le V_{DS}$, $T_J \le 175$ °C.
d. 1.6 mm from case.

* Pb containing terminations are not RoHS compliant, exemptions may apply

THERMAL RESISTANCE RATINGS				
PARAMETER	**SYMBOL**	**TYP.**	**MAX.**	**UNIT**
Maximum Junction-to-Ambient	R_{thJA}	-	62	°C/W
Case-to-Sink, Flat, Greased Surface	R_{thCS}	0.50	-	
Maximum Junction-to-Case (Drain)	R_{thJC}	-	1.0	

SPECIFICATIONS T_J = 25 °C, unless otherwise noted							
PARAMETER	**SYMBOL**	**TEST CONDITIONS**		**MIN.**	**TYP.**	**MAX.**	**UNIT**
Static							
Drain-Source Breakdown Voltage	V_{DS}	V_{GS} = 0 V, I_D = 250 μA		100	-	-	V
V_{DS} Temperature Coefficient	$\Delta V_{DS}/T_J$	Reference to 25 °C, I_D = 1 mA		-	0.13	-	V/°C
Gate-Source Threshold Voltage	$V_{GS(th)}$	V_{DS} = V_{GS}, I_D = 250 μA		2.0	-	4.0	V
Gate-Source Leakage	I_{GSS}	V_{GS} = ± 20 V		-	-	± 100	nA
Zero Gate Voltage Drain Current	I_{DSS}	V_{DS} = 100 V, V_{GS} = 0 V		-	-	25	μA
		V_{DS} = 80 V, V_{GS} = 0 V, T_J = 150 °C		-	-	250	
Drain-Source On-State Resistance	$R_{DS(on)}$	V_{GS} = 10 V	I_D = 17 A[b]	-	-	0.077	Ω
Forward Transconductance	g_{fs}	V_{DS} = 50 V, I_D = 17 A[b]		8.7	-	-	S
Dynamic							
Input Capacitance	C_{iss}	V_{GS} = 0 V, V_{DS} = 25 V, f = 1.0 MHz, see fig. 5		-	1700	-	pF
Output Capacitance	C_{oss}			-	560	-	
Reverse Transfer Capacitance	C_{rss}			-	120	-	
Total Gate Charge	Q_g	V_{GS} = 10 V	I_D = 17 A, V_{DS} = 80 V, see fig. 6 and 13[b]	-	-	72	nC
Gate-Source Charge	Q_{gs}			-	-	11	
Gate-Drain Charge	Q_{gd}			-	-	32	
Turn-On Delay Time	$t_{d(on)}$	V_{DD} = 50 V, I_D = 17 A, R_G = 9.1 Ω, R_D = 2.9 Ω, see fig. 10[b]		-	11	-	ns
Rise Time	t_r			-	44	-	
Turn-Off Delay Time	$t_{d(off)}$			-	53	-	
Fall Time	t_f			-	43	-	
Internal Drain Inductance	L_D	Between lead, 6 mm (0.25") from package and center of die contact		-	4.5	-	nH
Internal Source Inductance	L_S			-	7.5	-	
Drain-Source Body Diode Characteristics							
Continuous Source-Drain Diode Current	I_S	MOSFET symbol showing the integral reverse p - n junction diode		-	-	28	A
Pulsed Diode Forward Current[a]	I_{SM}			-	-	110	
Body Diode Voltage	V_{SD}	T_J = 25 °C, I_S = 28 A, V_{GS} = 0 V[b]		-	-	2.5	V
Body Diode Reverse Recovery Time	t_{rr}	T_J = 25 °C, I_F = 17 A, dI/dt = 100 A/μs[b]		-	180	360	ns
Body Diode Reverse Recovery Charge	Q_{rr}			-	1.3	2.8	μC
Forward Turn-On Time	t_{on}	Intrinsic turn-on time is negligible (turn-on is dominated by L_S and L_D)					

Notes

a. Repetitive rating; pulse width limited by maximum junction temperature (see fig. 11).
b. Pulse width ≤ 300 μs; duty cycle ≤ 2 %.