TSTE25 Power Electronics

- Lecture 5
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- ISY/EKS

Outline

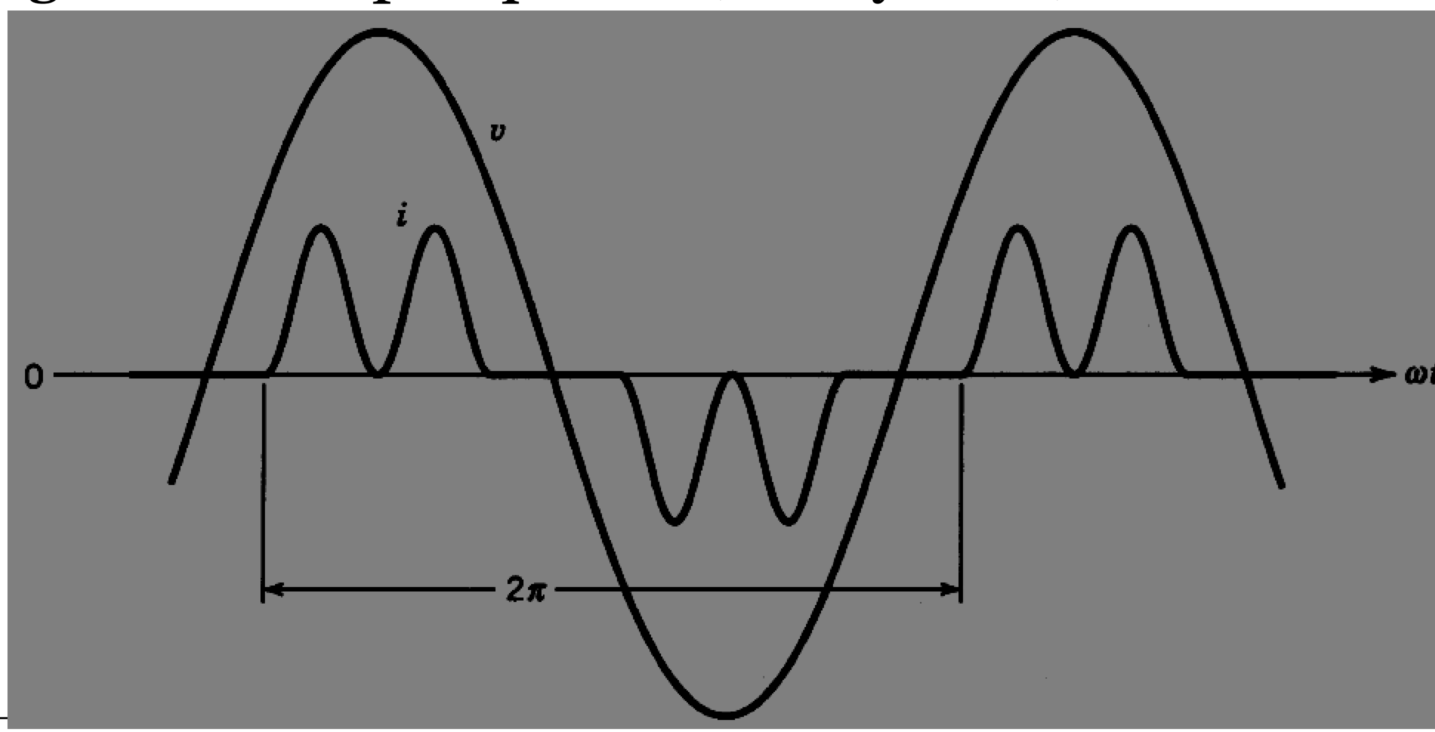
- Harmonics
- DC-AC switching inverters 2
  - Harmonics
- Gate drive supply – Boot strapping

Lecture 5

Harmonics

Steady state voltages and currents

- Assume repeating waveform
- Ignore startup sequence (steady state)



Fourier series

Non-sinusoidal repeated signal with angular frequency omega, ω_1

$$f(t) = F_0 + \sum_{h=1}^{\infty} f_h(t) =$$

$$= \frac{1}{2} a_0 + \sum_{h=1}^{\infty} \{a_h \cos(h\omega_1 t) + b_h \sin(h\omega_1 t)\}$$

$$a_h = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(h\omega_1 t) d(\omega_1 t), h = 0, \ldots, \infty$$

$$b_h = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(h\omega_1 t) d(\omega_1 t), h = 1, \ldots, \infty$$

Joseph Fourier

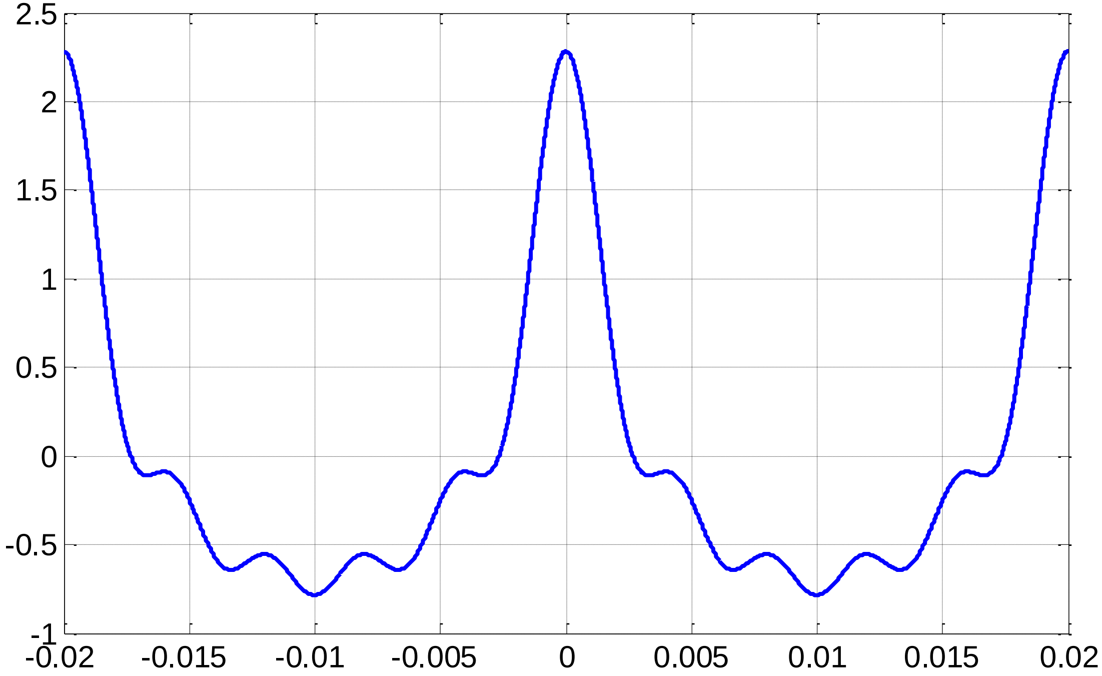
Jean-Baptiste Joseph Fourier

Born	21 March 1768 Auxerre, Burgundy, Kingdom of France (now in Yonne, France)
Died	16 May 1830 (aged 62) Paris, Kingdom of France

(see list)
Fourier number
Fourier series
Fourier transform
Fourier's law of conduction
Fourier–Motzkin elimination
Greenhouse effect

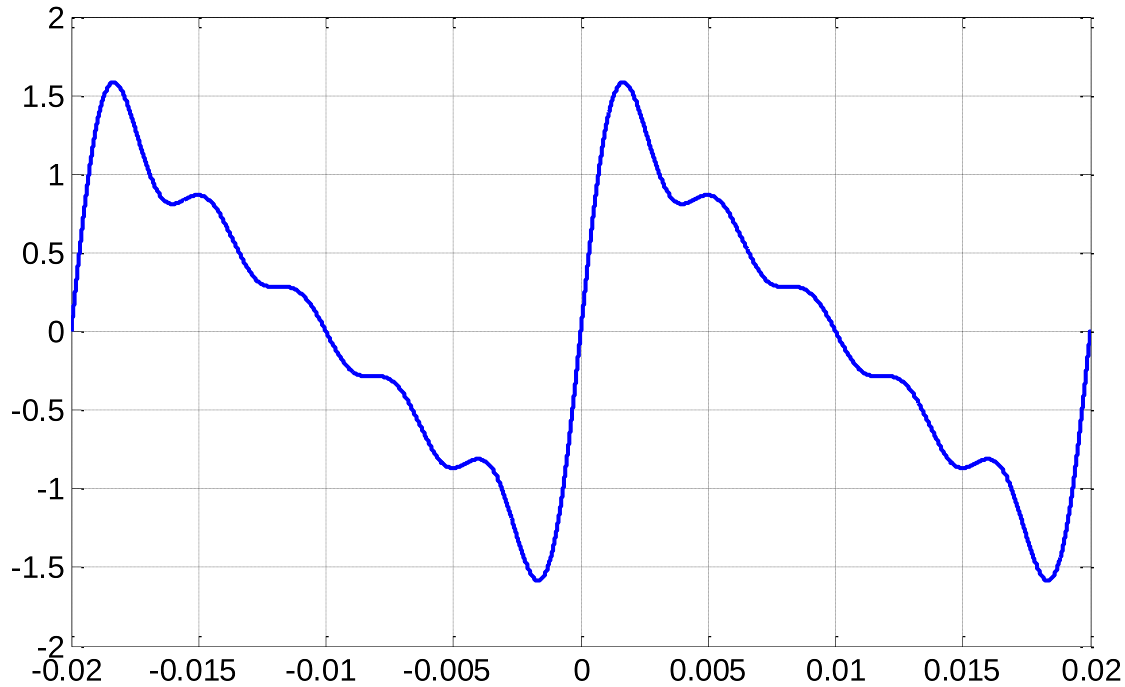
Even function $f(-t) = f(t)$

$$\cos(\omega t) + \frac{1}{2}\cos(2\omega t) + \frac{1}{3}\cos(3\omega t) + \frac{1}{4}\cos(4\omega t) + \frac{1}{5}\cos(5\omega t)$$



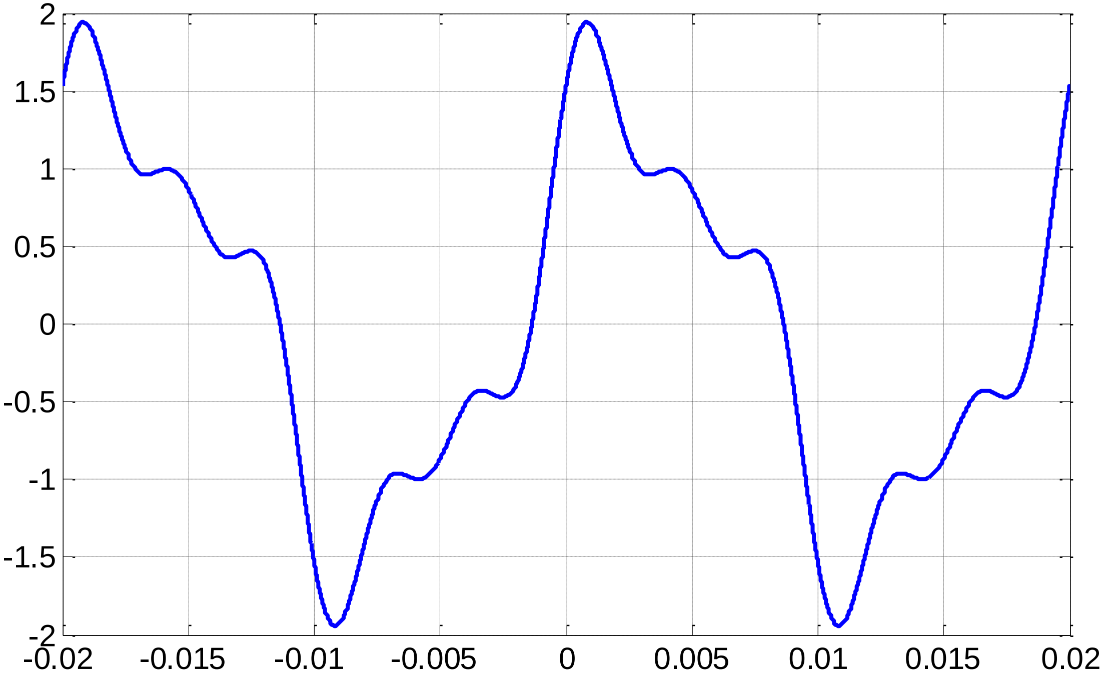
Odd function $f(-t) = -f(t)$

$$\sin(\omega t) + \frac{1}{2} sin(2\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{4}\sin(4\omega t) + \frac{1}{5}\sin(5\omega t)$$



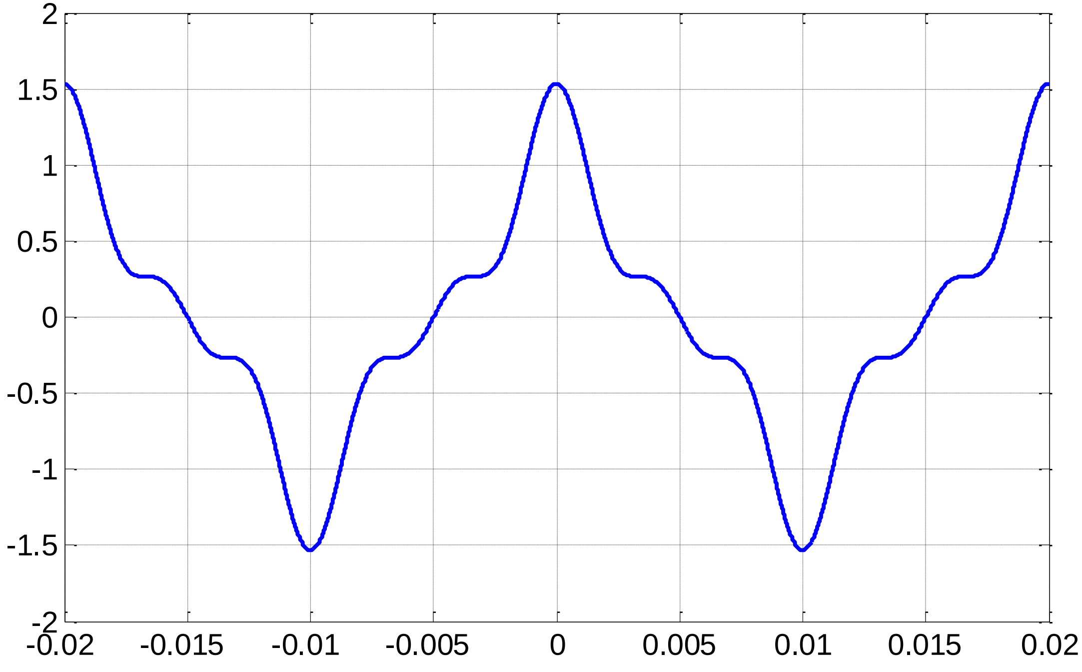
Half-wave symmetry $f(t) = -f(t + \frac{1}{2}T)$

$$\sin(\omega t) + \cos(\omega t) + \frac{1}{3} sin(3\omega t) + \frac{1}{3}\cos(3\omega t) + \frac{1}{5}\sin(5\omega t) + \frac{1}{5}\cos(5\omega t)$$



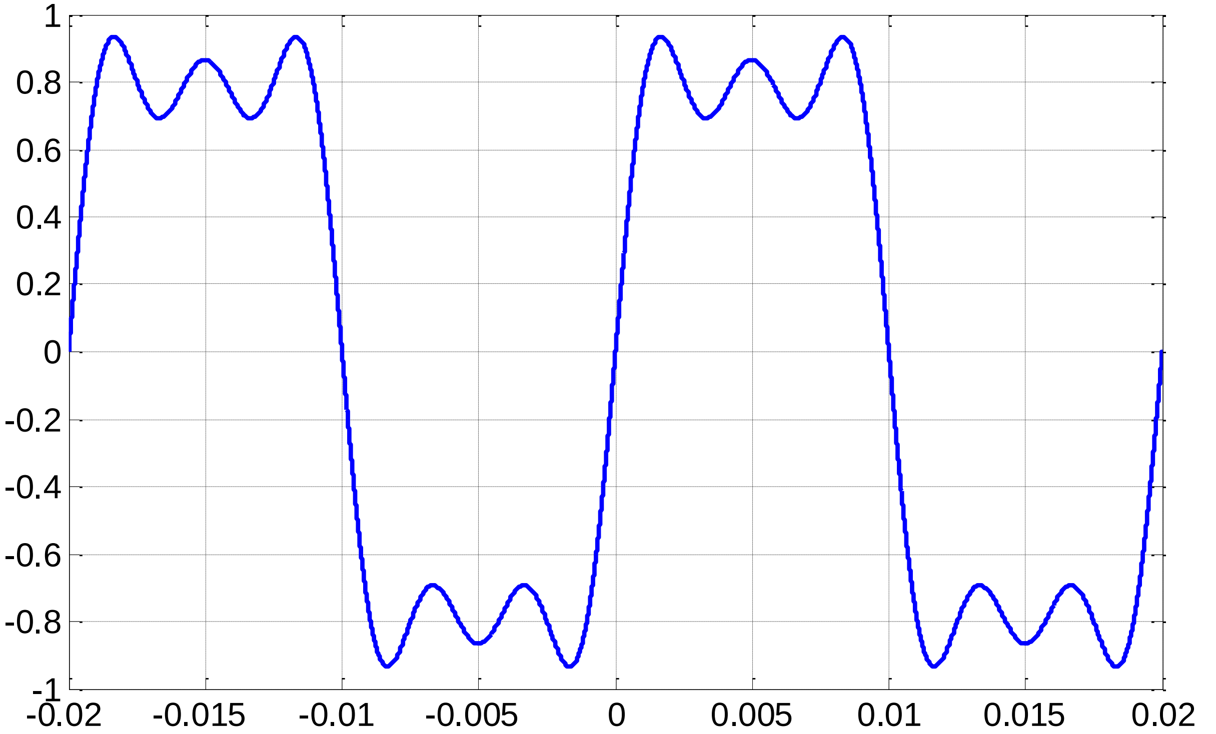
Even function, half-wave symmetry = Even quarter-wave

$$\cos(\omega t) + \frac{1}{3}\cos(3\omega t) + \frac{1}{5}\cos(5\omega t)$$



Odd function, half-wave symmetry = Odd quarter-wave

$$\sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t)$$

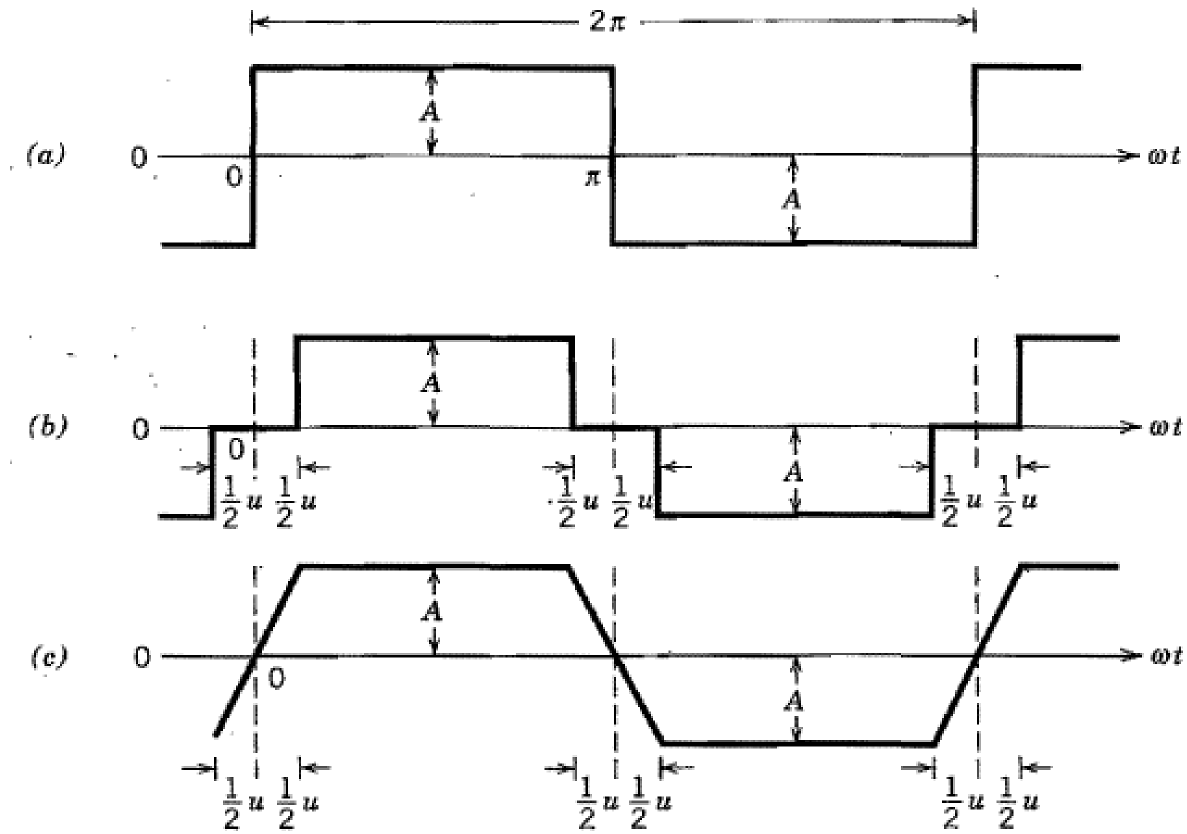


$$f(t) = \frac{1}{2}a_0 + \sum_{h=1}^{\infty}\{a_h\cos(h\omega_1 t) + b_h\sin(h\omega_1 t)\}$$

Table 3-1 Use of Symmetry in Fourier Analysis

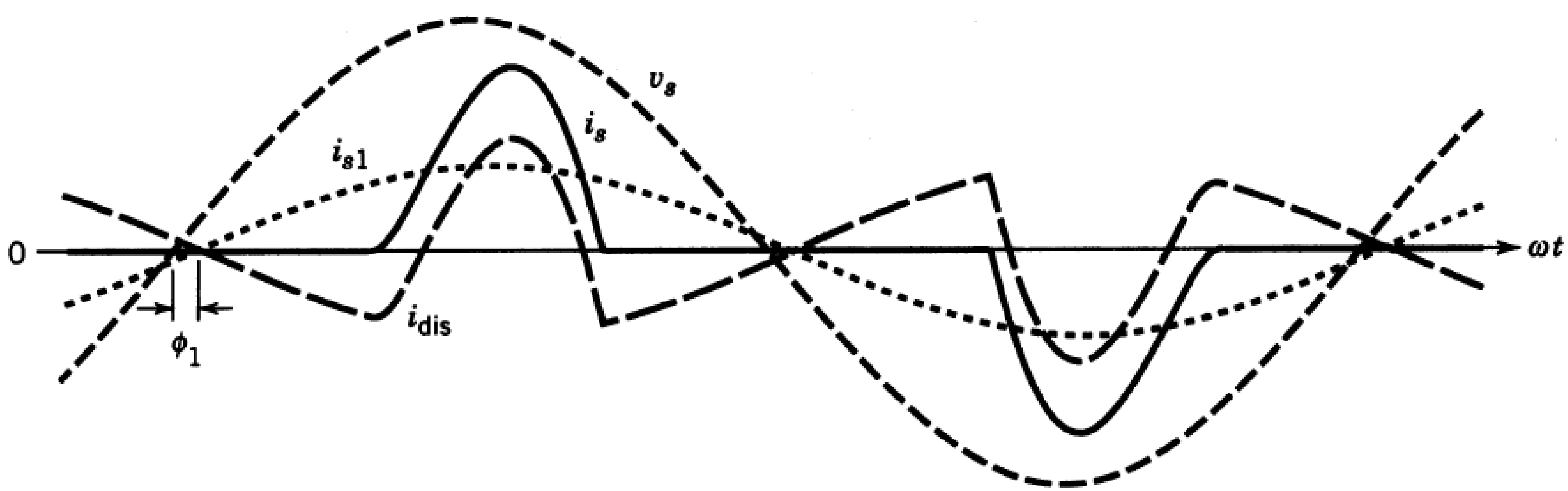
Symmetry	*Condition Required*	a_h and b_h
Even	$f(-t) = f(t)$	$b_h = 0 \quad a_h = \frac{2}{\pi}\int_0^{\pi} f(t)\cos(h\omega t)\, d(\omega t)$
Odd	$f(-t) = -f(t)$	$a_h = 0 \quad b_h = \frac{2}{\pi}\int_0^{\pi} f(t)\sin(h\omega t)\, d(\omega t)$
Half-wave	$f(t) = -f(t + \frac{1}{2}T)$	$a_h = b_h = 0$ for even h $a_h = \frac{2}{\pi}\int_0^{\pi} f(t)\cos(h\omega t)\, d(\omega t)$ for odd h $b_h = \frac{2}{\pi}\int_0^{\pi} f(t)\sin(h\omega t)\, d(\omega t)$ for odd h
Even quarter-wave	Even and half-wave	$b_h = 0$ for all h $a_h = \begin{cases} \frac{4}{\pi}\int_0^{\pi/2} f(t)\cos(h\omega t)\, d(\omega t) & \text{for odd } h \\ 0 & \text{for even } h \end{cases}$
Odd quarter-wave	Odd and half-wave	$a_h = 0$ for all h $b_h = \begin{cases} \frac{4}{\pi}\int_0^{\pi/2} f(t)\sin(h\omega t)\, d(\omega t) & \text{for odd } h \\ 0 & \text{for even } h \end{cases}$

3-3 For the waveforms in Fig. P3-3, calculate their RMS values of the fundamental and the harmonic frequency components.



Total RMS incl harmonics

- $i_s(t) = i_{s1}(t) + \sum_{h=2}^{n} i_{sh}(t)$
- $I_s = \sqrt{\frac{1}{T_1}\int_0^{T_1} i_s^2(t)dt} = \sqrt{I_{s1}^2 + \sum_{h=2}^{n} I_{sh}^2} = \sqrt{I_{s1}^2 + I_{s3}^2 + I_{s5}^2 \ldots\ldots}$
- (All cross-product terms, $i_{s1} \cdot i_{s2}, i_{s1} \cdot i_{s3}$=0)

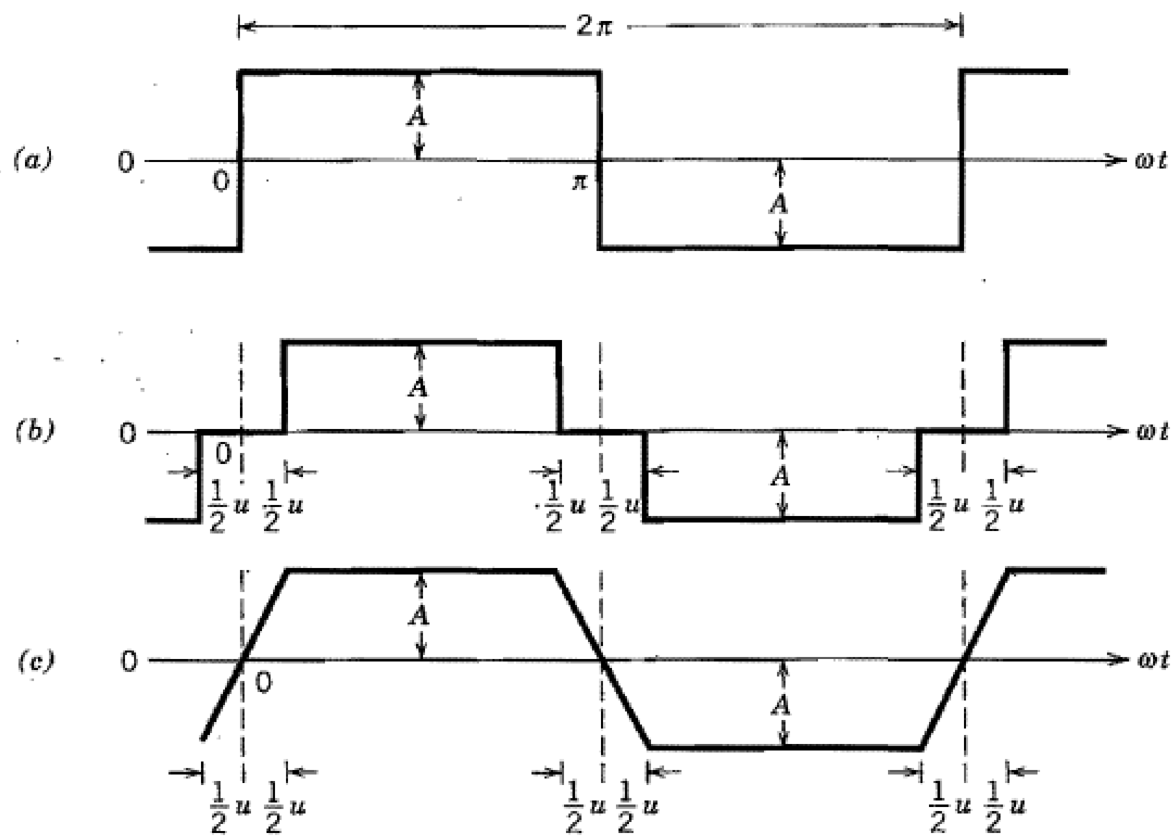


3-4

In the waveforms of Fig. P3-3 of Problem 3-3, A = 10 and u = 20° ($u_1 = u_2 = u/2$), where applicable. Calculate their total rms values as follows:

a) By using the results of Problem 3-3 in Eq. 3-28.

$$I_s = \sqrt{I_{s1}^2 + \sum_{h=2}^{n} I_{sh}^2}$$



Line current distortion

- Non-sinusoidal currents may give distortion on utility-supply voltage.
- Assume purely sinusoidal current at fundamental frequency {grundton}
- Input current is sum of a fundamental plus harmonics {övertoner}
- $$i_s(t) = i_{s1}(t) + \sum_{h \neq 1} i_{sh}(t)$$
- Distortion part is the harmonics (excluding fundamental). In RMS form

$$I_{dis} = \sqrt{\left(\sum_{h \neq 1} I_{sh}^2\right)}$$

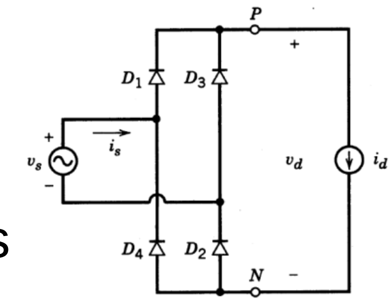
THD, Total Harmonic Distortion

- Distortion on a current waveform

$$THD_i = \frac{I_{dis}}{I_{s1}} = \frac{\sqrt{\sum_{h \neq 1} (I_{sh})^2}}{I_{s1}}$$

- Energy in the harmonics compared to the fundamental
- THD can be larger than 1. (> 100%)

Single phase rectifier, input current

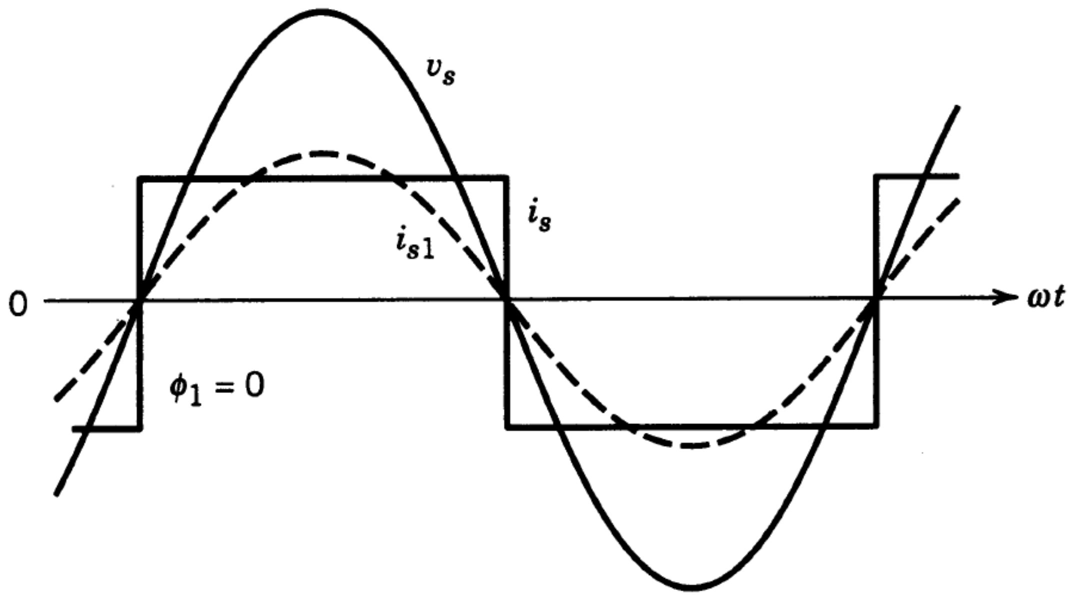


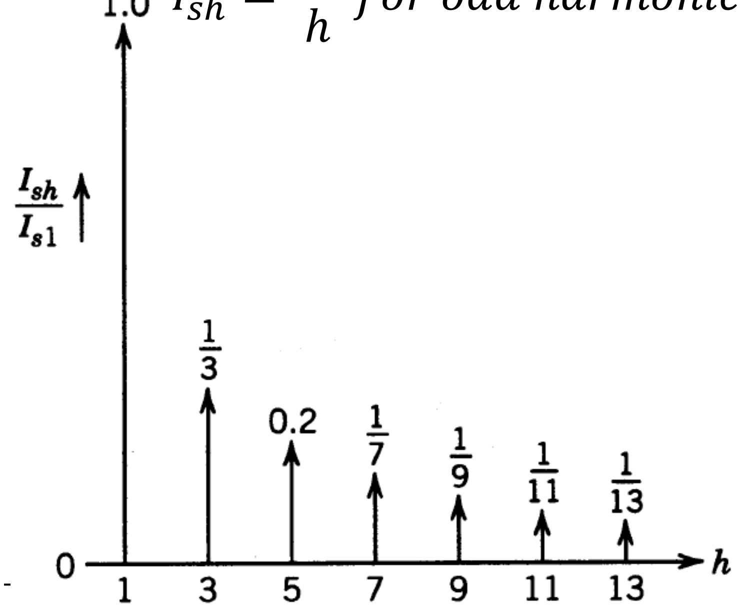
- Fourier analysis gives additional harmonic components
  - Remember calculation uses RMS of I_s, I_{s1} and I_d

$$I_{s1} = \frac{2}{\pi}\sqrt{2}I_d = 0.9I_d$$

$$I_{sh} = 0 \; for\ even\ harmonics$$

$$I_{sh} = \frac{I_{s1}}{h} \; for\ odd\ harmonics$$



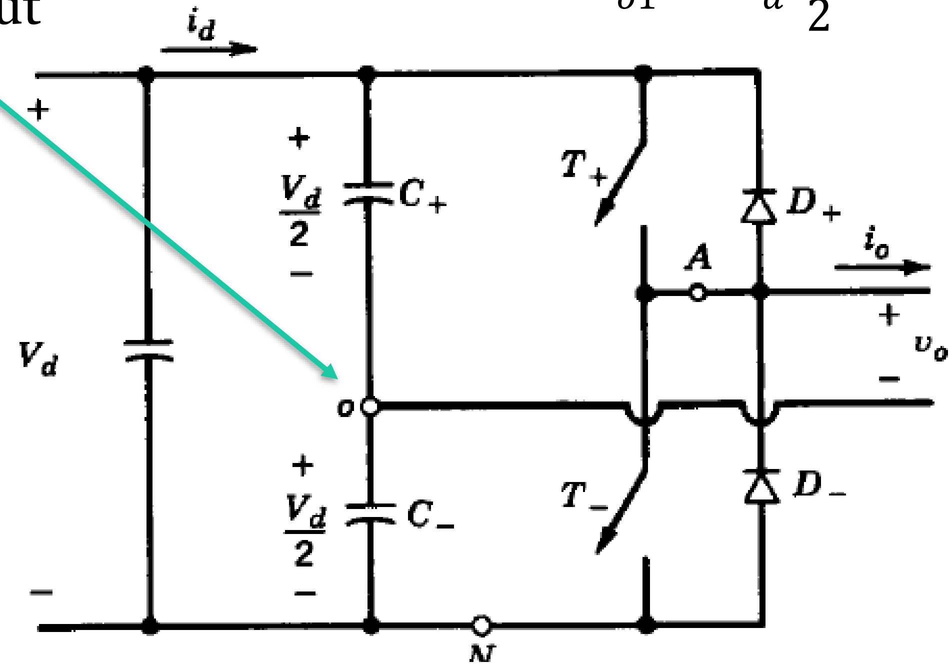


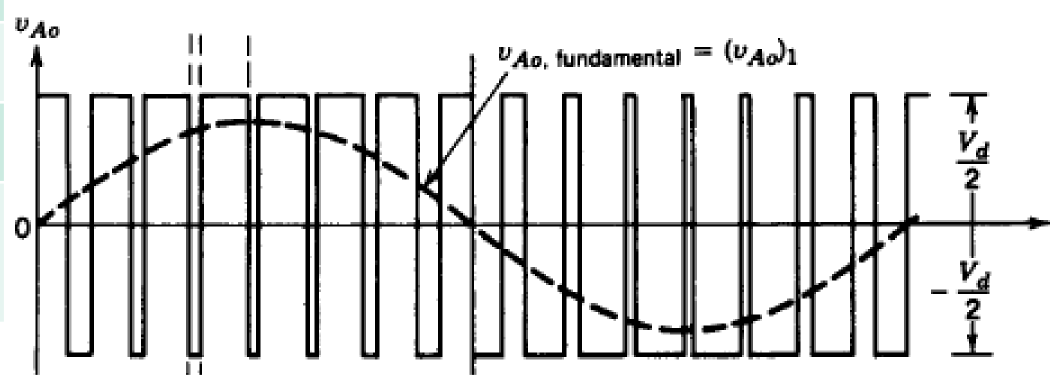
Half-bridge (2-level) converter

- DC-side midpoint 'o'
  reference point for ac-output
- Output voltage, v_{A0}, switched between $+\frac{V_d}{2}$ and $-\frac{V_d}{2}$
- 4 possible switch states:

T+	T-	
Off	Off	v_{A0} def by i_o. $i_o>0$: v_{A0} = $-V_d/2$ $i_o<0$: v_{A0} = $+V_d/2$
On	Off	v_{A0} = $+V_d/2$
Off	**On**	v_{A0} = $-V_d/2$
On	**On**	Short circuit. Forbidden state

$$\hat{V}_{o1} = m_a \frac{V_d}{2}$$





PWM switching scheme, half-bridge

- Constant f_s
- Amplitude modulation ratio

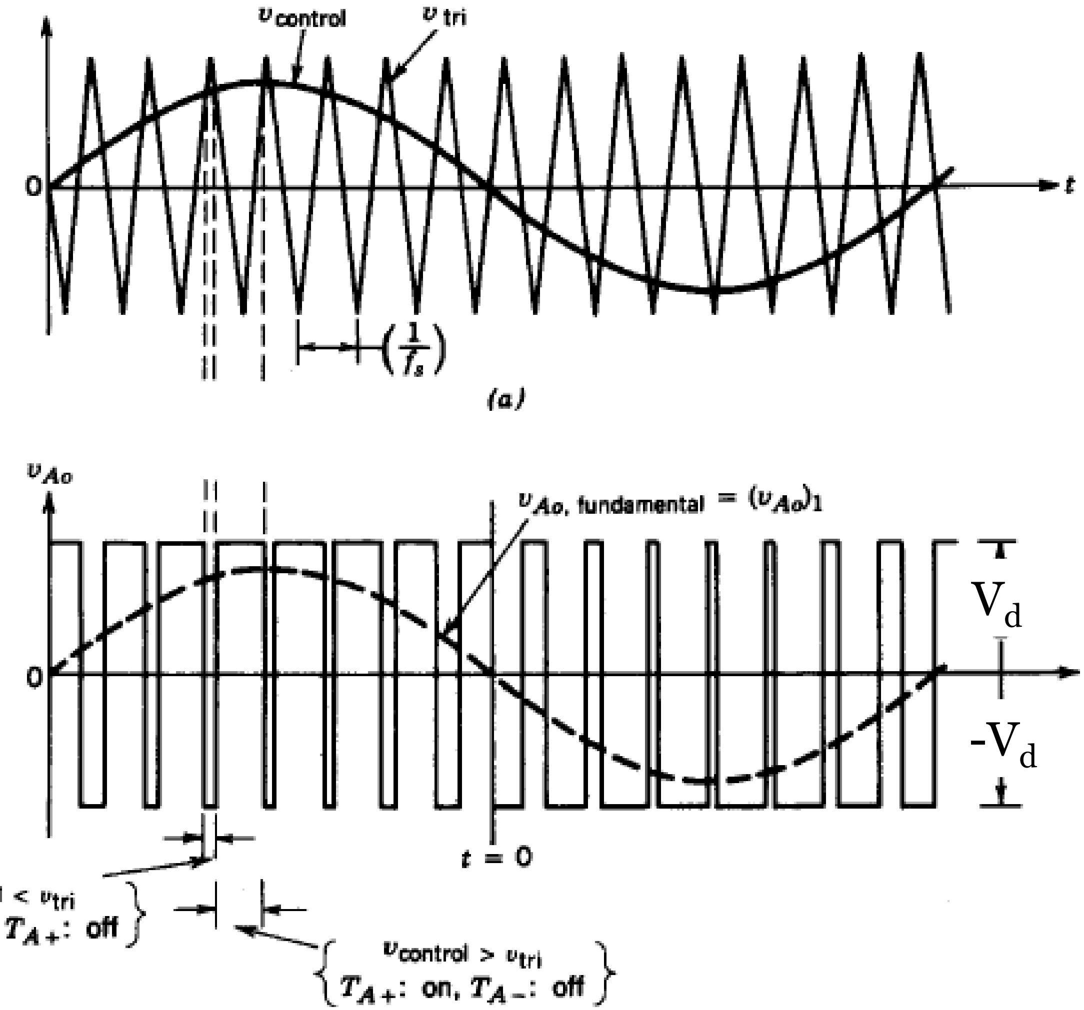
$$m_a = \frac{\hat{V}_{control}}{\hat{V}_{tri}}$$

- Fundamental output

$$\hat{V}_{o1} = m_a \frac{V_d}{2}$$

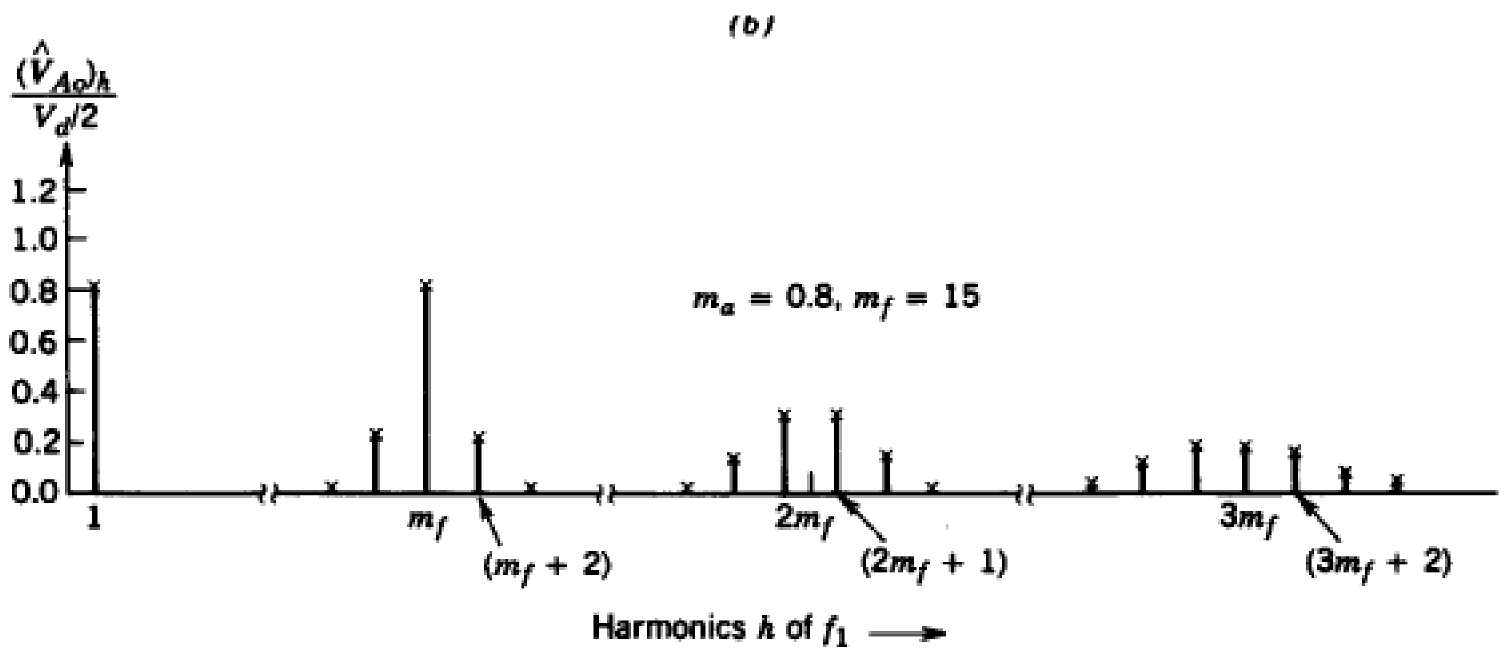
- Frequency modulation ratio (pulse number)

$$m_f = \frac{f_s}{f_1}$$



PWM modulation harmonics

- Harmonics as sidebands around multiples of switching frequency



Harmonics in half-bridge vs. m_a and m_f>9

- For m_f < 9 harmonics is almost independent of m_f
- Choose m_f odd integer
  - Odd symmetry
  - Half-wave symmetry
  - Only odd harmonics
  - Even harmonics = 0
  - With $v_A = \hat{V}_A \sin \omega t$ all harmonics $\sin h\omega t$
- **Table data for half-bridge** $\frac{(\hat{V}_o)_h}{V_d/2}$

Table 8-1 Generalized Harmonics of v_{Ao} for a Large m_f.

h \ m_a	0.2	0.4	0.6	0.8	1.0
1 Fundamental	0.2	0.4	0.6	0.8	1.0
m_f	1.242	1.15	1.006	0.818	0.601
$m_f \pm 2$	0.016	0.061	0.131	0.220	0.318
$m_f \pm 4$					0.018
$2m_f \pm 1$	0.190	0.326	0.370	0.314	0.181
$2m_f \pm 3$		0.024	0.071	0.139	0.212
$2m_f \pm 5$				0.013	0.033
$3m_f$	0.335	0.123	0.083	0.171	0.113
$3m_f \pm 2$	0.044	0.139	0.203	0.176	0.062
$3m_f \pm 4$		0.012	0.047	0.104	0.157
$3m_f \pm 6$				0.016	0.044
$4m_f \pm 1$	0.163	0.157	0.008	0.105	0.068
$4m_f \pm 3$	0.012	0.070	0.132	0.115	0.009
$4m_f \pm 5$			0.034	0.084	0.119
$4m_f \pm 7$				0.017	0.050

Note: $(\hat{V}_{Ao})_h/\frac{1}{2}V_d$ [$= (\hat{V}_{AN})_h/\frac{1}{2}V_d$] is tabulated as a function of m_a.

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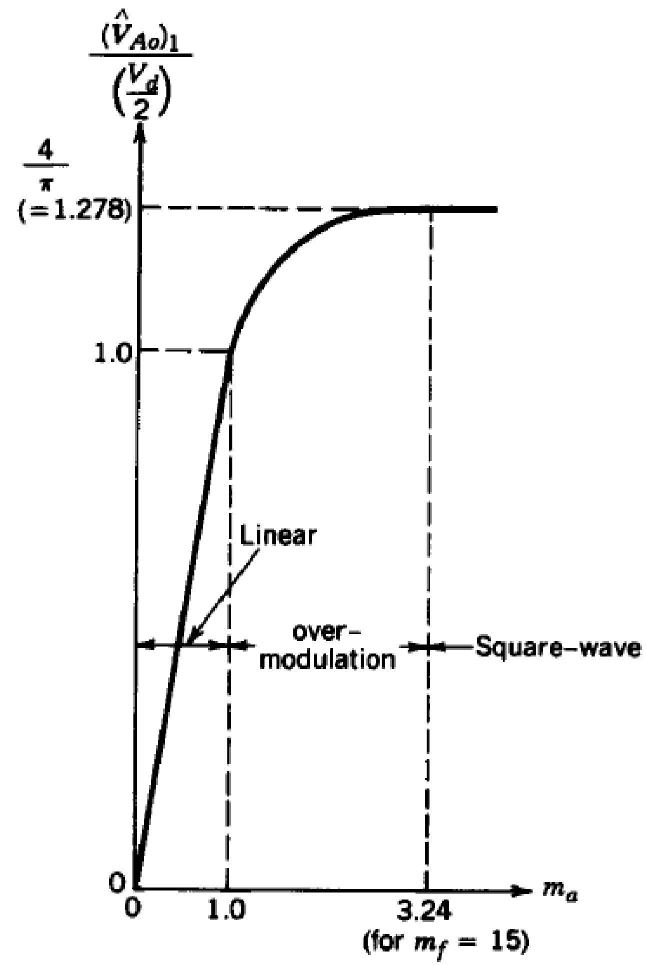
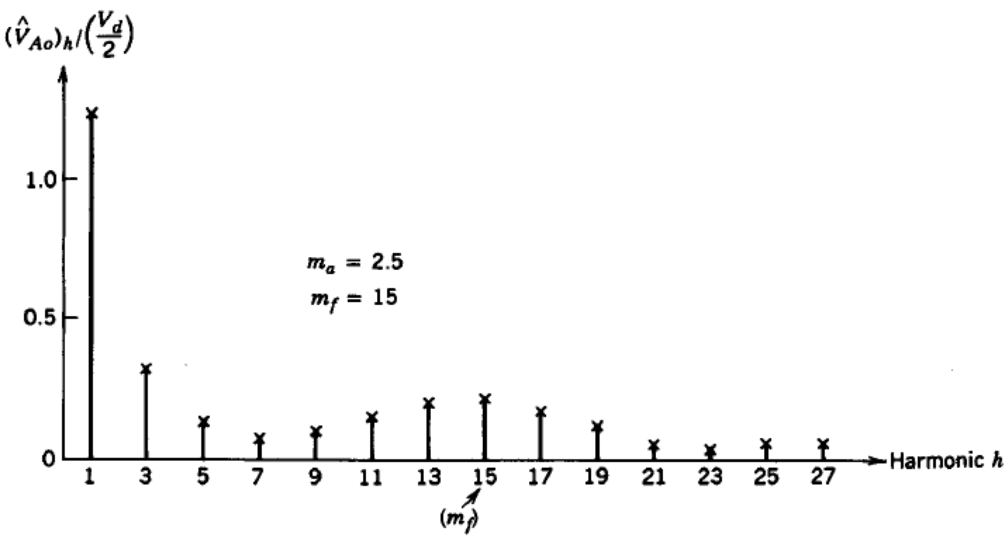
Over-modulation

- $m_a > 1$
- Increased harmonics with over-modulation



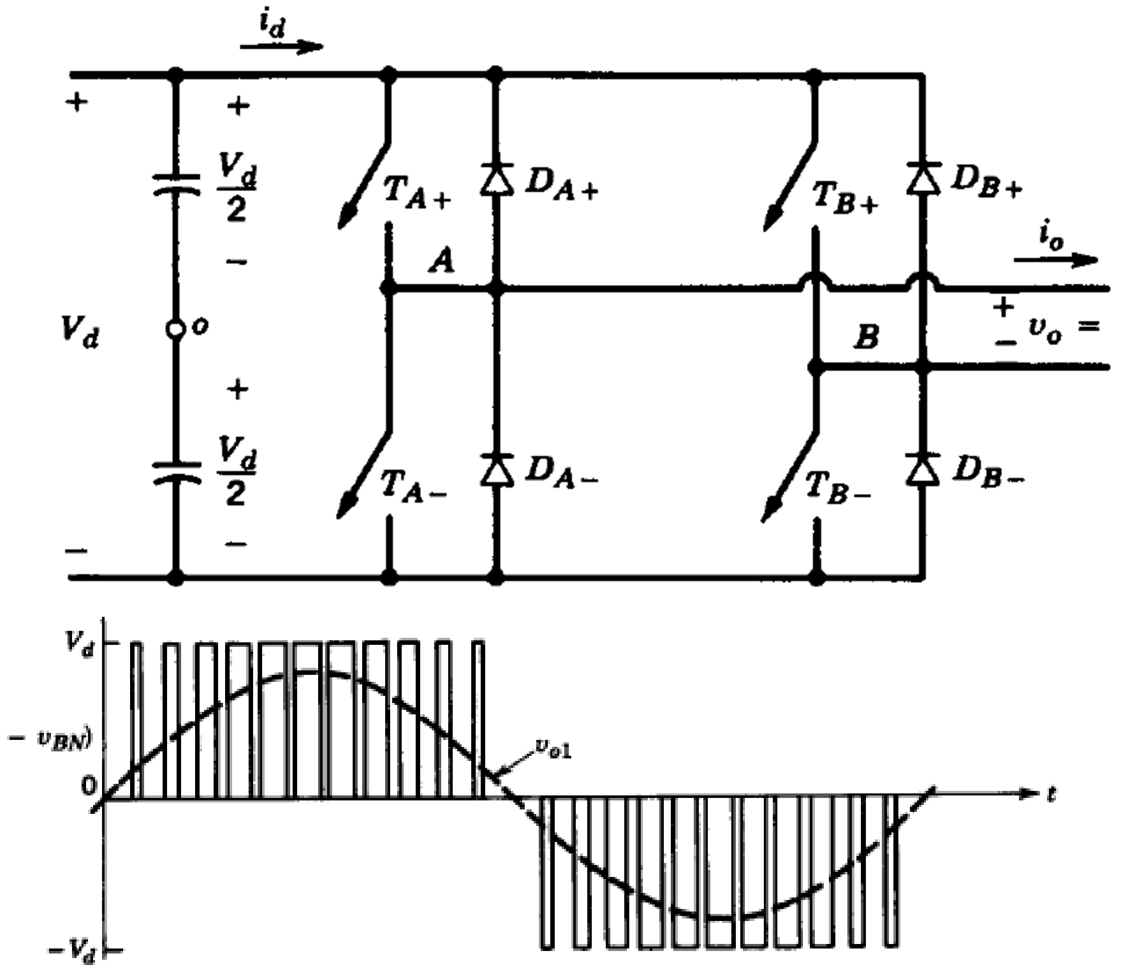


Full-bridge (3-level) converter

- Maximum output voltage doubled compared to half-bridge inverter
- No need for midpoint voltage

$$\hat{V}_{o1} = m_a V_d$$

T_{A+}	T_{A-}	T_{B+}	T_{B-}	
Off	Off	Off	Off	Output isolated. Unless $v_o > V_d$ by external source
On	Off	**On**	Off	$v_o = 0$
On	Off	Off	**On**	$v_o = +V_d$
Off	**On**	**On**	Off	$v_o = -V_d$
Off	**On**	Off	**On**	$v_o = 0$
On	**On**	x	x	Short circuit, Forbidden states
x	x	**On**	**On**	Short circuit, Forbidden states



Harmonics in full-bridge vs. m_a and m_f>9

- For m_f < 9 harmonics is almost independent of m_f
- Choose m_f odd integer
  - Odd symmetry
  - Half-wave symmetry
  - Only odd harmonics
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  - With $v_A = \hat{V}_A \sin \omega t$ all harmonics $\sin h\omega t$
- **Table data for full-bridge** $\frac{(\hat{V}_o)_h}{V_d}$

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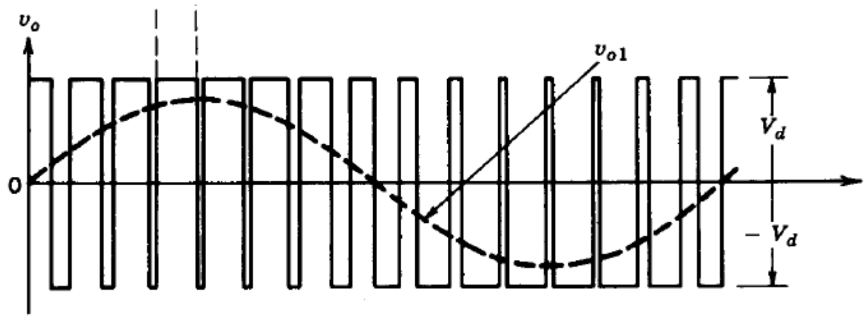
Note: $(\hat{V}_{Ao})_h/\frac{1}{2}V_d$ [= $(\hat{V}_{AN})_h/\frac{1}{2}V_d$] is tabulated as a function of m_a.

PWM switching strategies, full-bridge

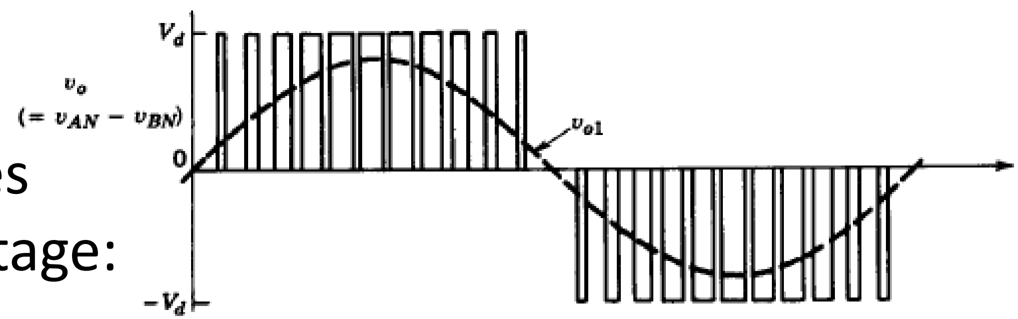
- Fundamental output

$$\hat{V}_{o1} = m_a V_d$$

- Bipolar voltage switching
  - Only two switching states used giving output voltage: $+V_d$ or $-V_d$

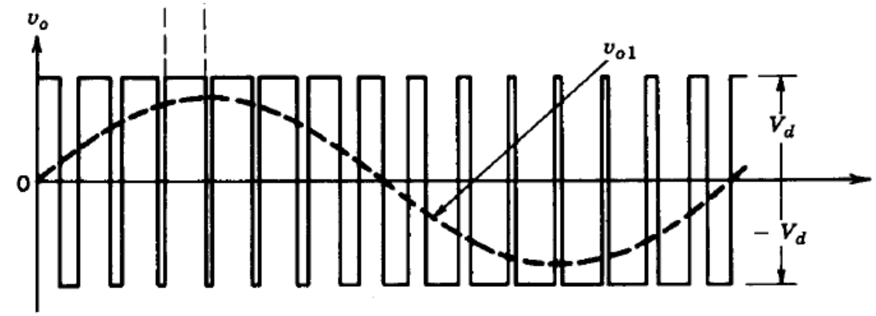


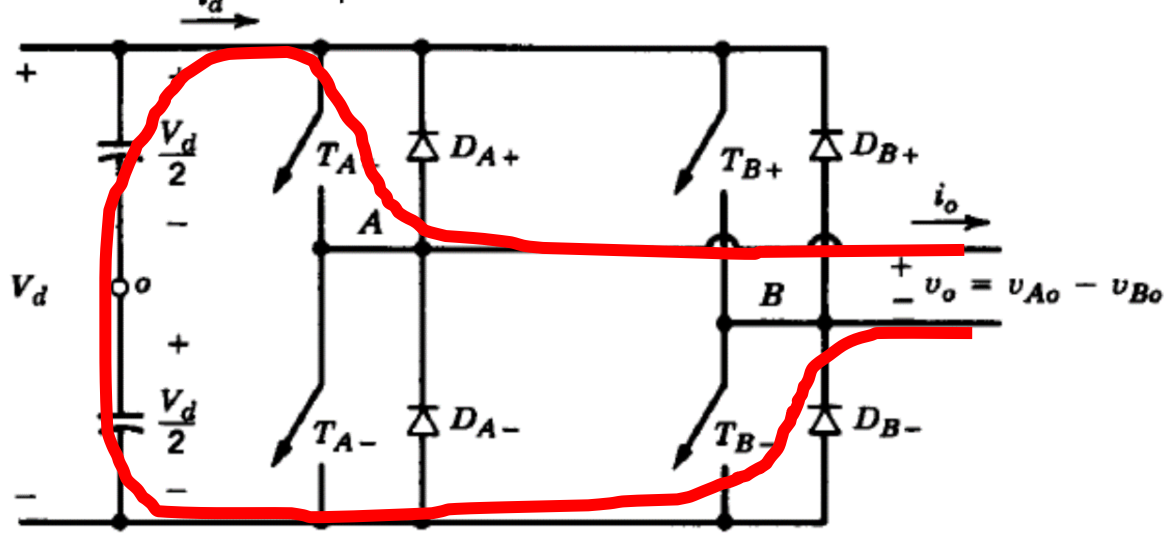
- Unipolar switching
  - All four switching states used giving output voltage: $+V_d$, 0 or $-V_d$



PWM bipolar switching

- Bipolar voltage switching
  - Both pairs (TA+, TB-) and (TA-, TB+) controlled simultaneous
- 2 possible switch configurations
  1. **T_{A+} on, T_{B-} on: $v_o = v_A - v_B = +V_d$**
  2. T_{A-} on, T_{B+} on: $v_o = v_A - v_B = -V_d$





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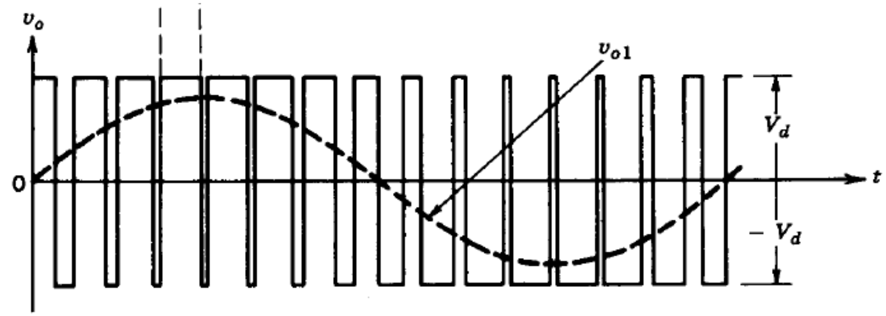
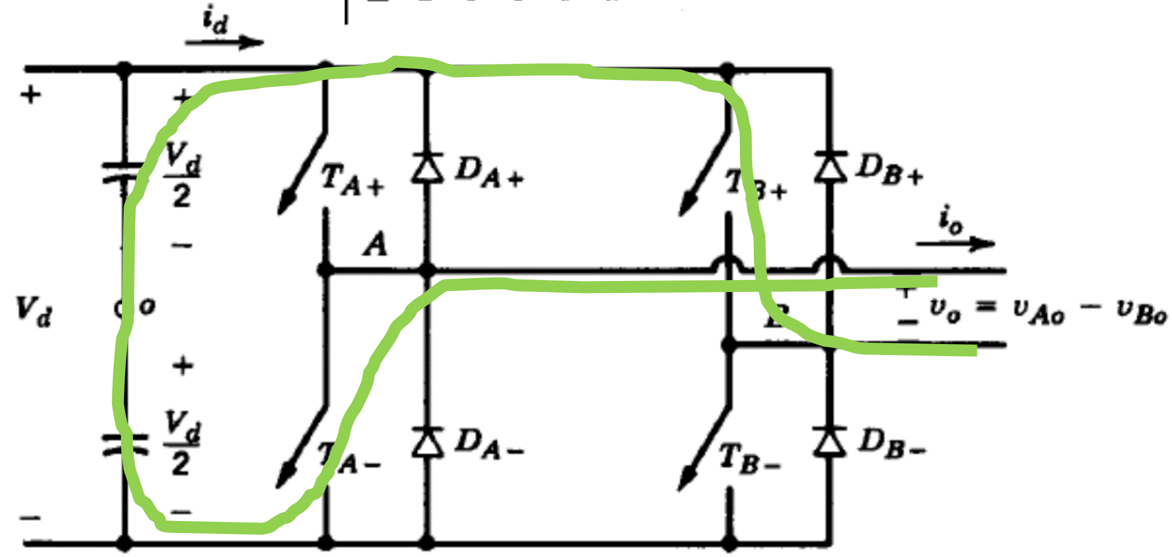




PWM bipolar switching

- Both legs switch at the same time

$m_a < 1.0$

$$\hat{V}_{o1} = m_a V_d$$

$m_a > 1.0$

$$V_d < \hat{V}_{o1} < \frac{4}{\pi} V_d$$

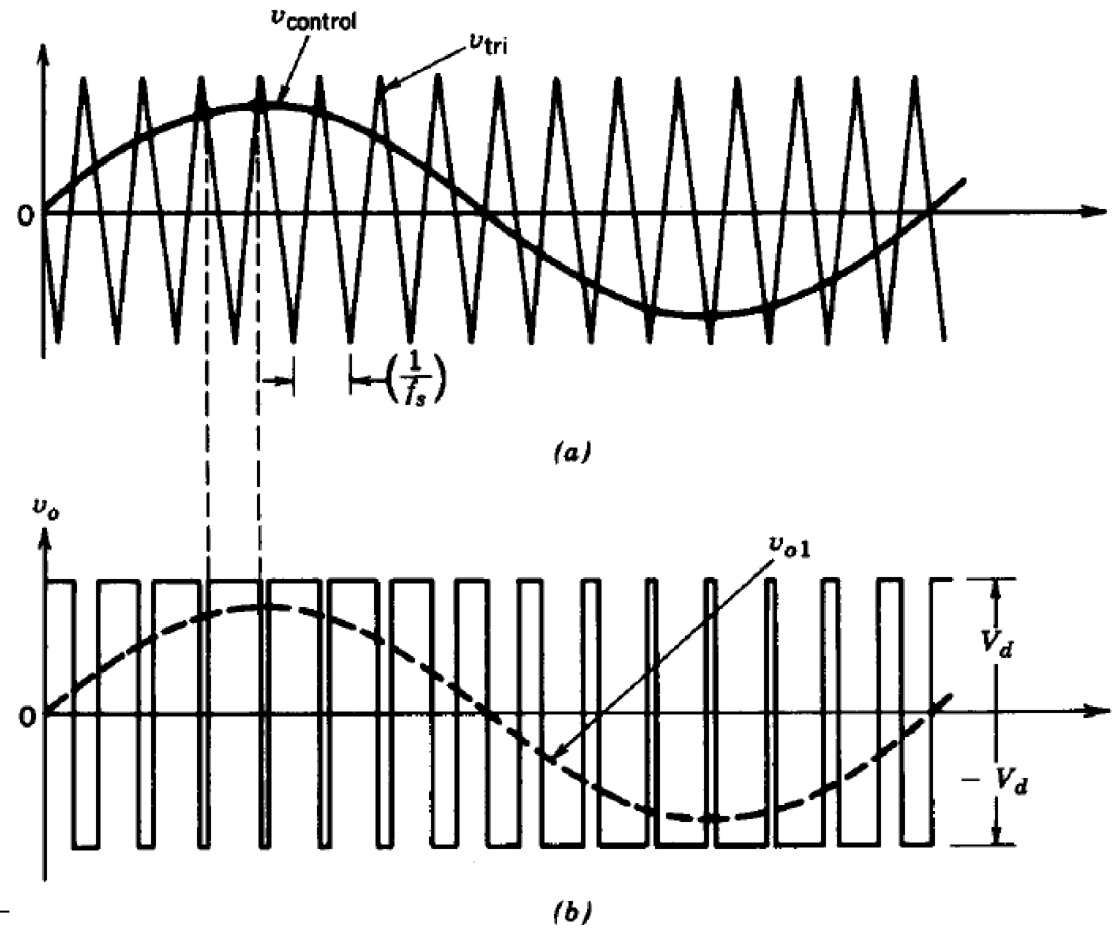
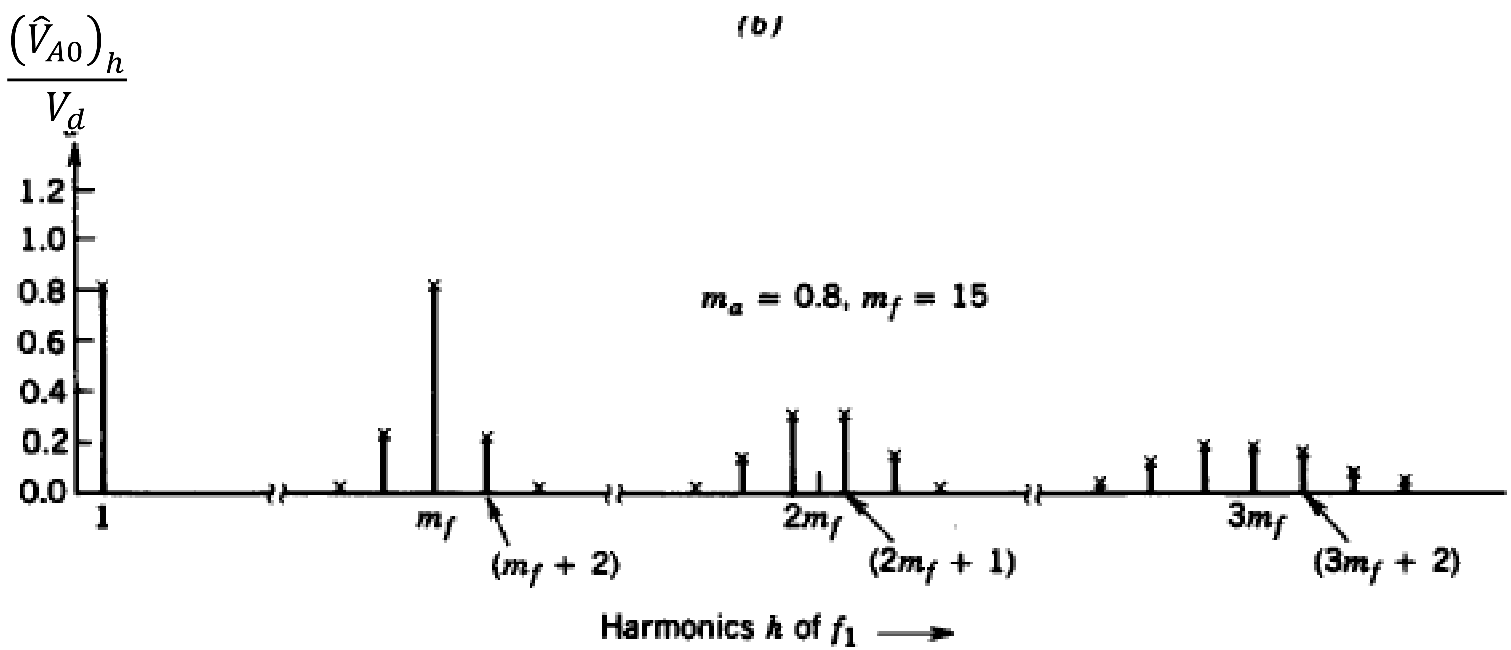


Figure 8-12 PWM with bipolar voltage switching.

PWM bipolar switching harmonics

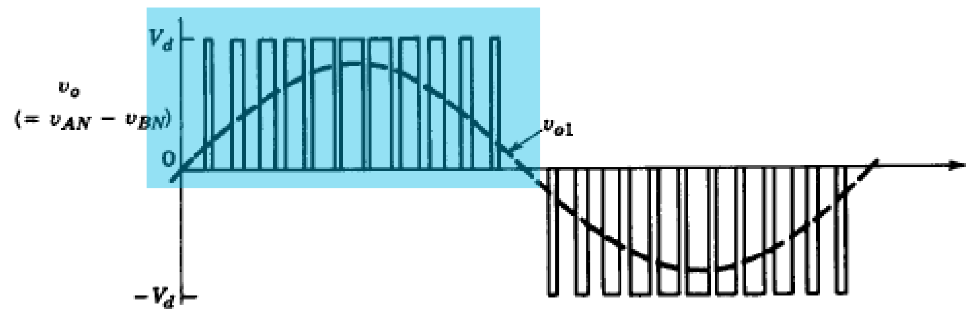
- Harmonics as sidebands around multiples of switching frequency

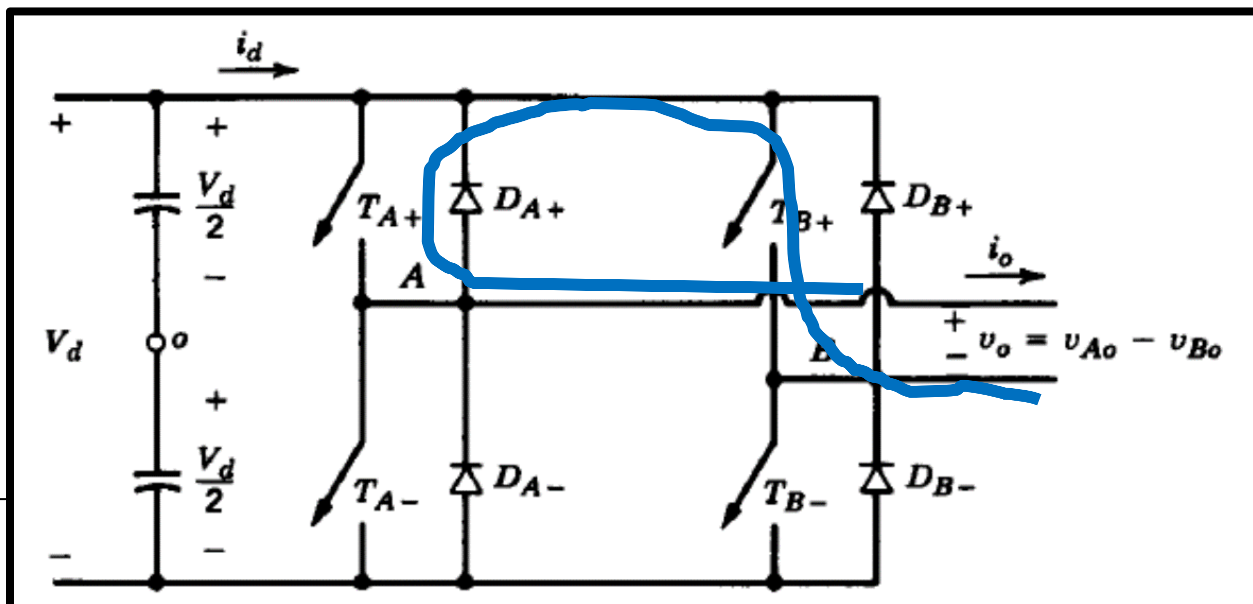


Unipolar (3-level) voltage switching, positive half-cycle

- Switches in each inverter leg (A and B) are controlled independently of the other leg

1. **T_{A+} on, T_{B+} on: $v_o = v_A - v_B = 0$**
2. T_{A+} on, T_{B-} on: $v_o = v_A - v_B = +V_d$
3. T_{A-} on, T_{B-} on: $v_o = v_A - v_B = 0$
4. T_{A-} on, T_{B+} on: $v_o = v_A - v_B = -V_d$





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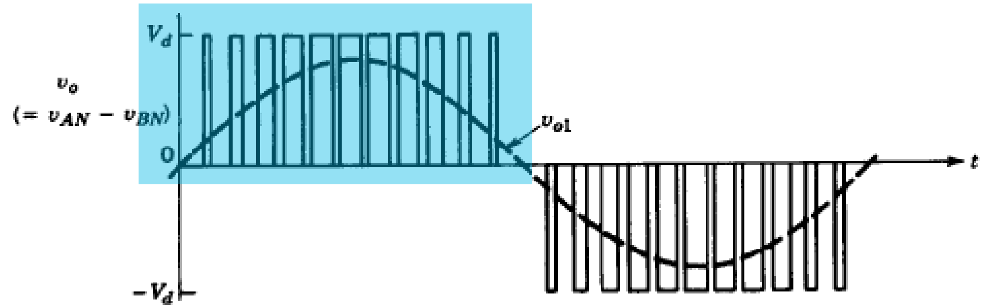
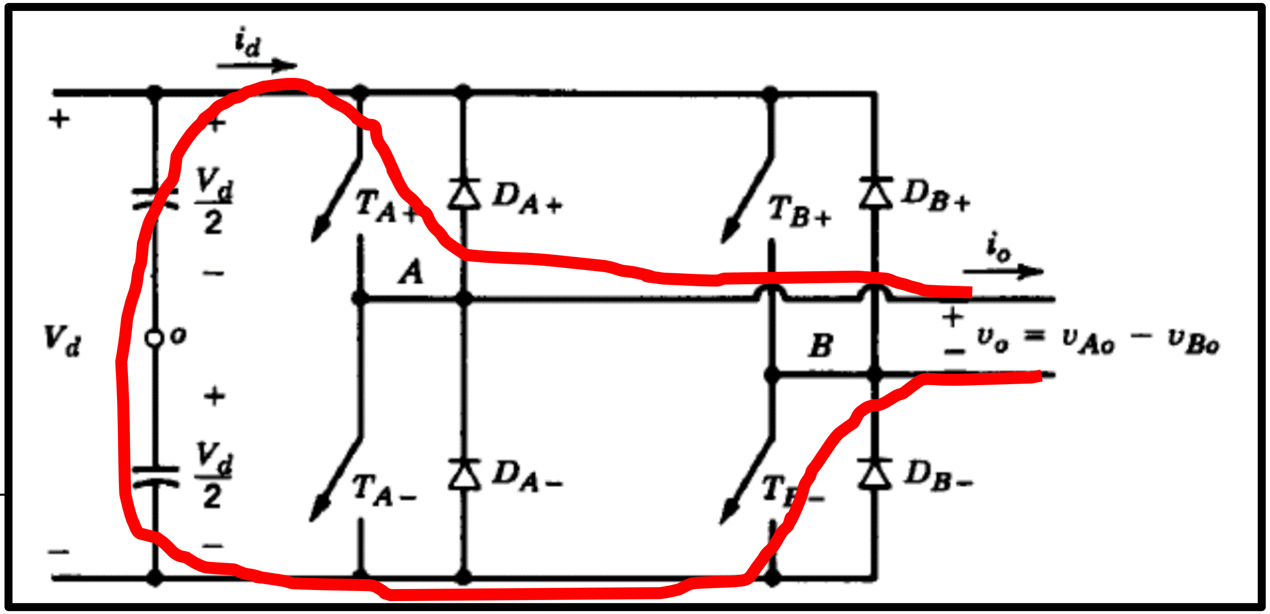
3. T_{A-} on, T_{B-} on: $v_o = v_A - v_B = 0$
4. T_{A-} on, T_{B+} on: $v_o = v_A - v_B = -V_d$

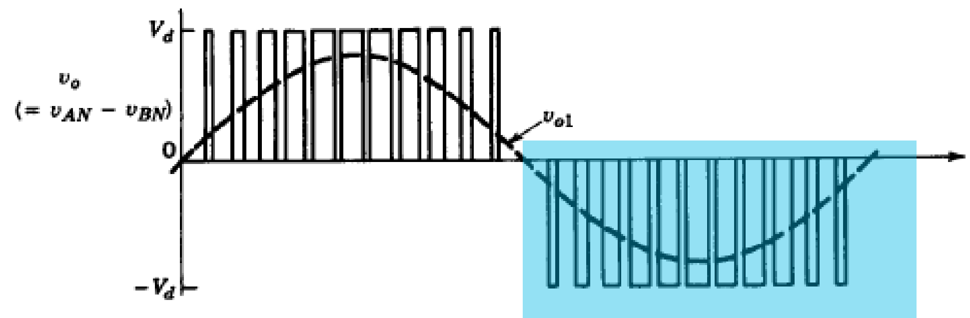


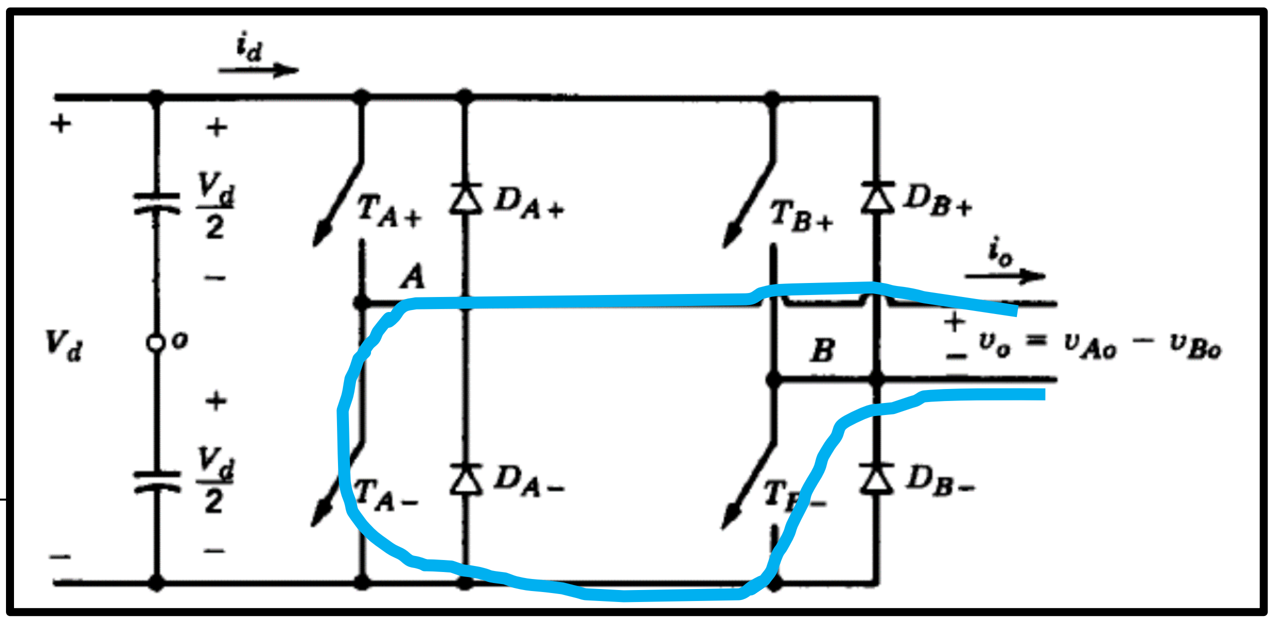


Unipolar (3-level) voltage switching, negative half-cycle

- Switches in each inverter leg (A and B) are controlled independently of the other leg

1. T_{A+} on, T_{B+} on: $v_o = v_A - v_B = 0$
2. T_{A+} on, T_{B-} on: $v_o = v_A - v_B = +V_d$
3. **T_{A-} on, T_{B-} on: $v_o = v_A - v_B = 0$**
4. T_{A-} on, T_{B+} on: $v_o = v_A - v_B = -V_d$

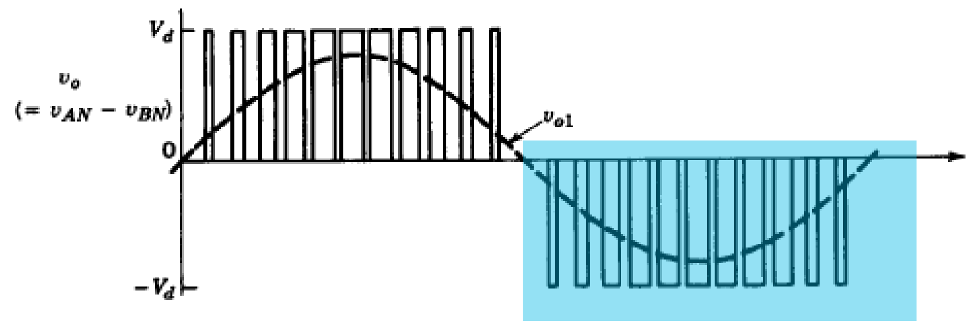


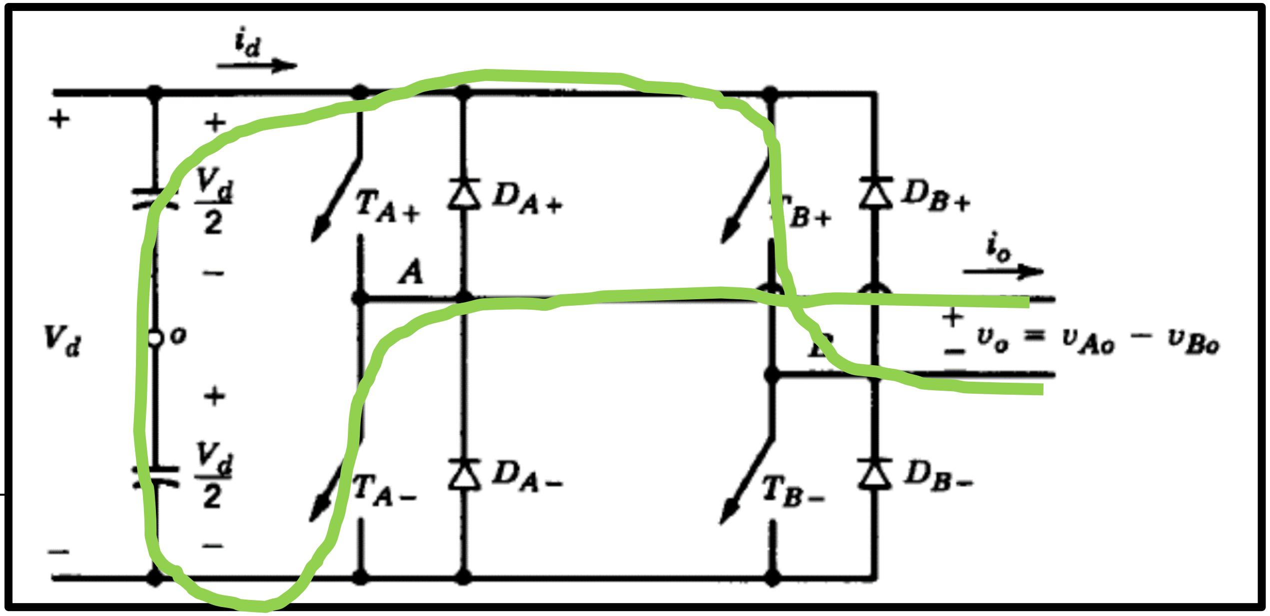


Unipolar (3-level) voltage switching, negative half-cycle

- Switches in each inverter leg (A and B) are controlled independently of the other leg

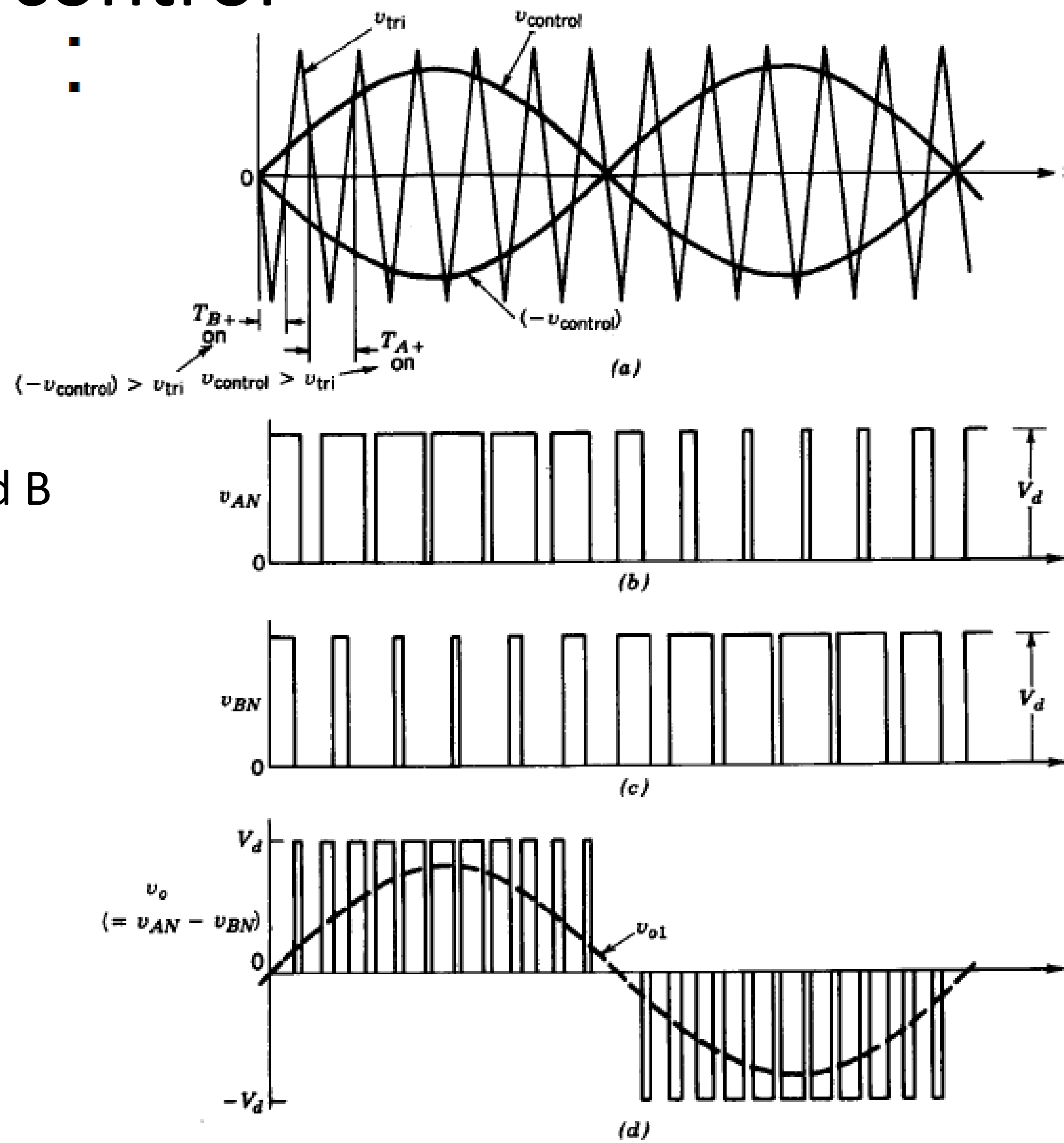
1. T_{A+} on, T_{B+} on: $v_o = v_A - v_B = 0$
2. T_{A+} on, T_{B-} on: $v_o = v_A - v_B = +V_d$
3. T_{A-} on, T_{B-} on: $v_o = v_A - v_B = 0$
4. **T_{A-} on, T_{B+} on: $v_o = v_A - v_B = -V_d$**





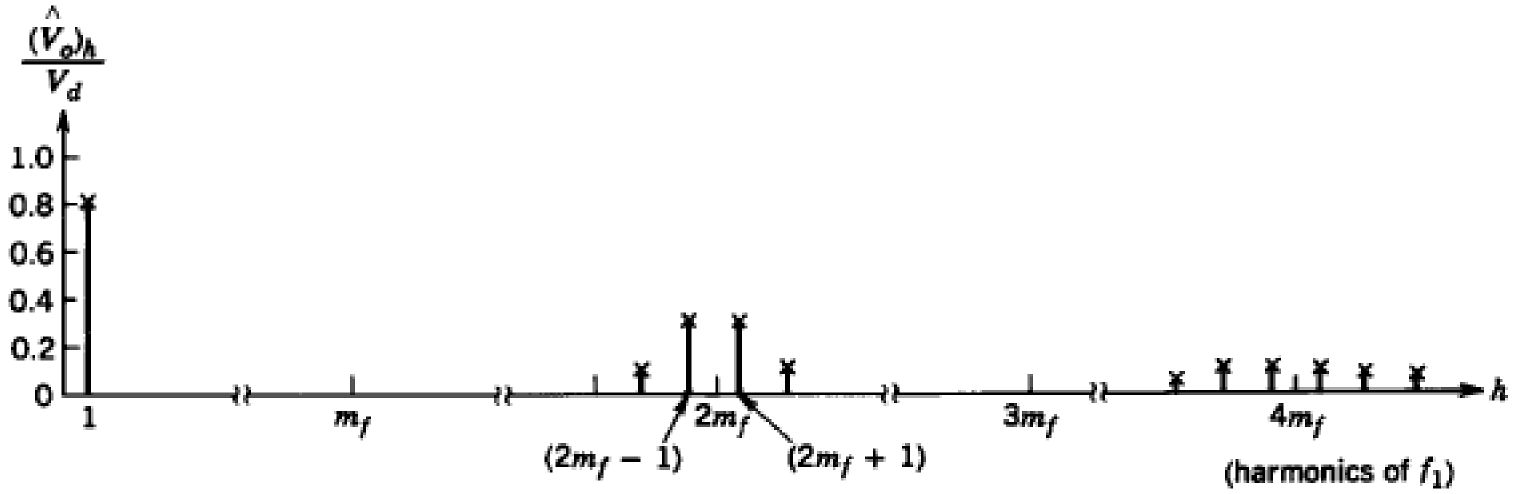
Unipolar PWM-control

- One leg controlled by $v_{control}$: $\hat{v}_{AN} = m_a \frac{V_d}{2}$
- Other leg controlled by $-v_{control}$: $\hat{v}_{BN} = m_a \frac{V_d}{2}$
- Output voltage, the difference between A and B
  $v_o = v_{AN} - v_{BN}$
- The fundamental component adds
  $\hat{V}_{o1} = m_a V_d$
- Some harmonic voltages cancel out



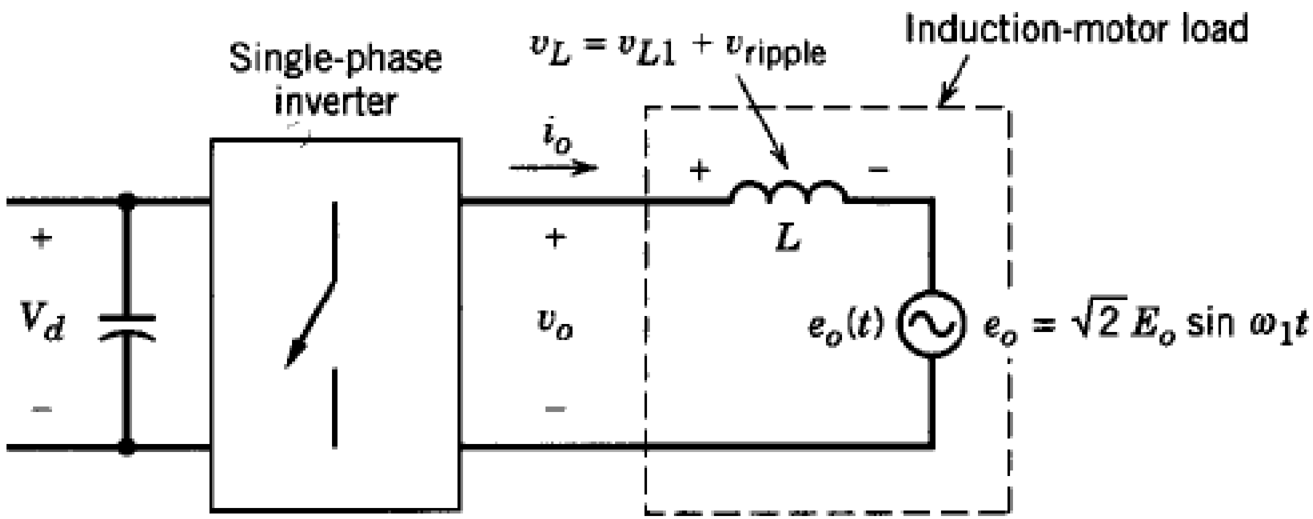
PWM unipolar switching harmonics

- Harmonics at twice the switching frequency
- m_f even makes switching frequency harmonic cancel out



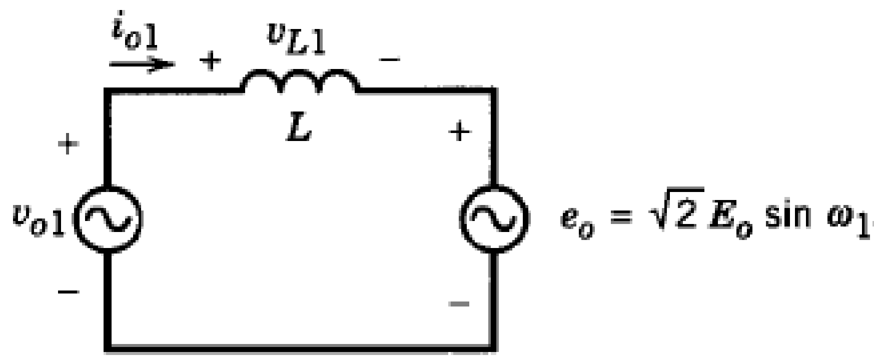
Ripple in single-phase inverter output

- Assume induction-motor load
- Counter electromotive force (emf) e_0

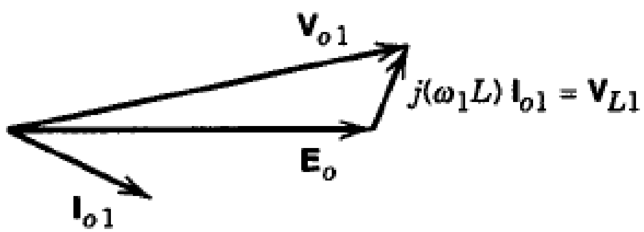


Ripple in single-phase inverter output, cont.

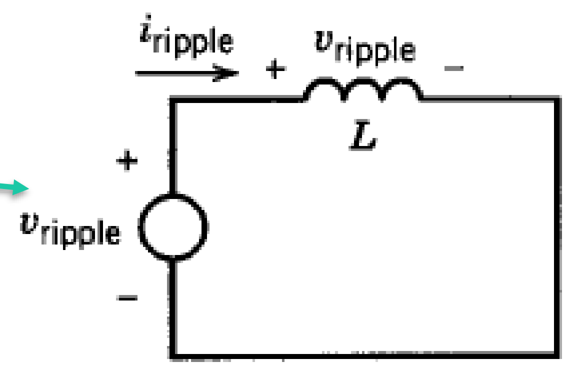
- Superposition gives two circuits



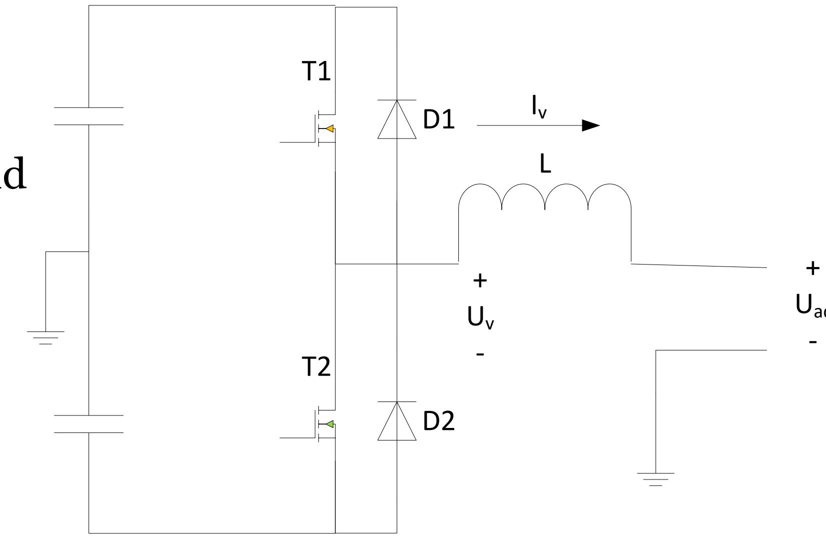
1. Fundamental frequency components



2. All switching voltage harmonics (v_{ripple}) across L. No switching voltage harmonics in the output voltage



Exercise 8-100



- In a half-bridge converter with U_d=2 V and L = 2 mH switching is done with m_a=0.8 and m_f=5
- $u_{ac}(t) = 0.8\sin(2\pi 50t)$

a) Construct graphically the output voltage and current, u_v and i_v. Assume $i_v(0)=0$.

$$u_L = L\frac{di_L}{dt}$$

$$\Delta i_v = \frac{u_v - u_{ac}}{L}\Delta t$$

b) Determine the largest harmonic current component. (Use table of harmonics in U_v)

c) Estimate the current ripple magnitude from the largest voltage harmonic

$$\left(\hat{I}_v\right)_h = \frac{\left(\hat{V}_{AN}\right)_h - (U_{ac})_h}{h\omega L}$$

Harmonics in PWM vs. m_a and m_f>9

- For m_f < 9 harmonics is almost independent of m_f
- Choose m_f odd integer
  - Odd symmetry
  - Half-wave symmetry
  - Only odd harmonics
  - Even harmonics = 0
  - With $v_A = \hat{V}_A \sin \omega t$ all harmonics $\sin h\omega t$
- Table data for half-bridge $$\frac{(\hat{V}_o)_h}{V_d/2}$$

Table 8-1 Generalized Harmonics of v_{Ao} for a Large m_f.

h \ m_a	0.2	0.4	0.6	0.8	1.0
1 Fundamental	0.2	0.4	0.6	0.8	1.0
m_f	1.242	1.15	1.006	0.818	0.601
$m_f \pm 2$	0.016	0.061	0.131	0.220	0.318
$m_f \pm 4$					0.018
$2m_f \pm 1$	0.190	0.326	0.370	0.314	0.181
$2m_f \pm 3$		0.024	0.071	0.139	0.212
$2m_f \pm 5$				0.013	0.033
$3m_f$	0.335	0.123	0.083	0.171	0.113
$3m_f \pm 2$	0.044	0.139	0.203	0.176	0.062
$3m_f \pm 4$		0.012	0.047	0.104	0.157
$3m_f \pm 6$				0.016	0.044
$4m_f \pm 1$	0.163	0.157	0.008	0.105	0.068
$4m_f \pm 3$	0.012	0.070	0.132	0.115	0.009
$4m_f \pm 5$			0.034	0.084	0.119
$4m_f \pm 7$				0.017	0.050

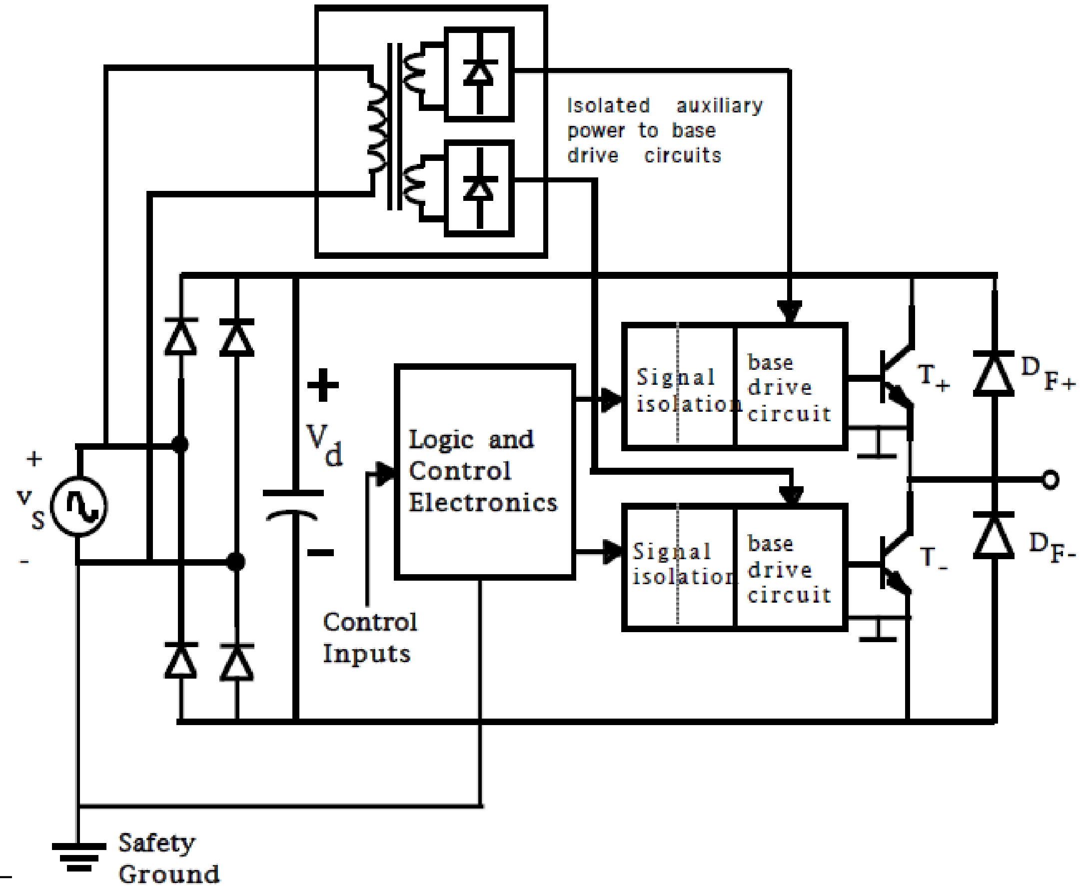
Note: $(\hat{V}_{Ao})_h/\frac{1}{2}V_d$ [$= (\hat{V}_{AN})_h/\frac{1}{2}V_d$] is tabulated as a function of m_a.

Lecture 5

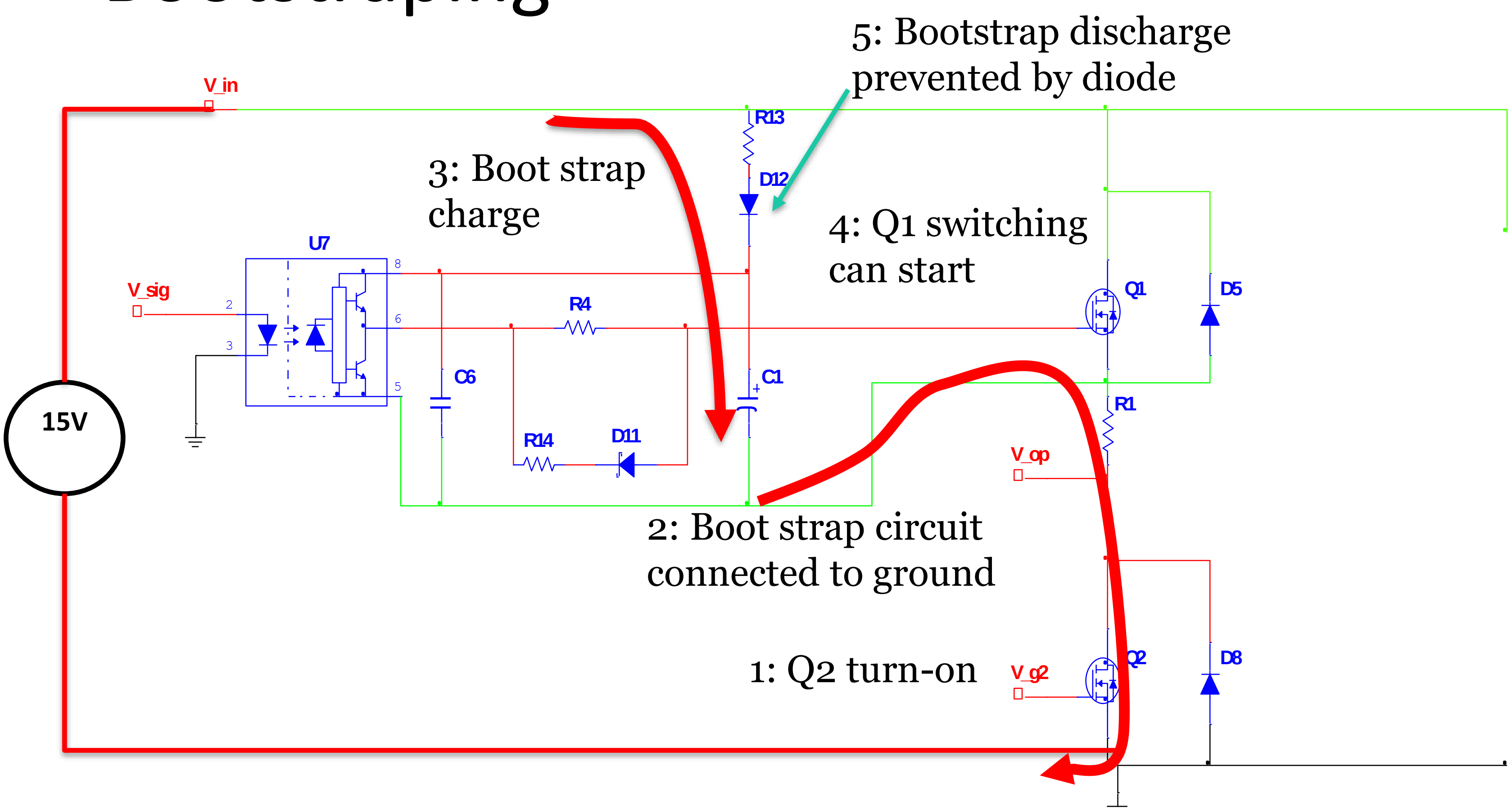
Gate drive supply – Boot strapping

Electrical isolation of drive circuit

- V_d – potential varies with input $v_S(t)$ relative to safety ground
- Signal isolation to base drive circuit necessary



Bootstraping



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