TSFS17 Elkraftsystem
Fö 5 – Begränsningar och Elnätstabilitet

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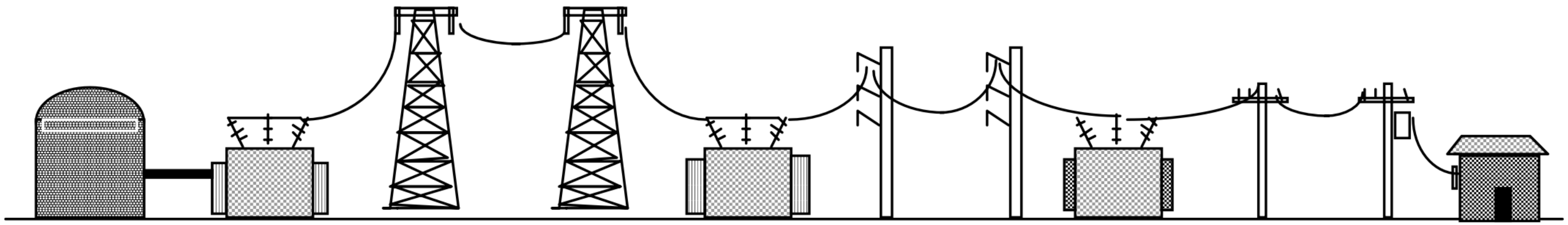
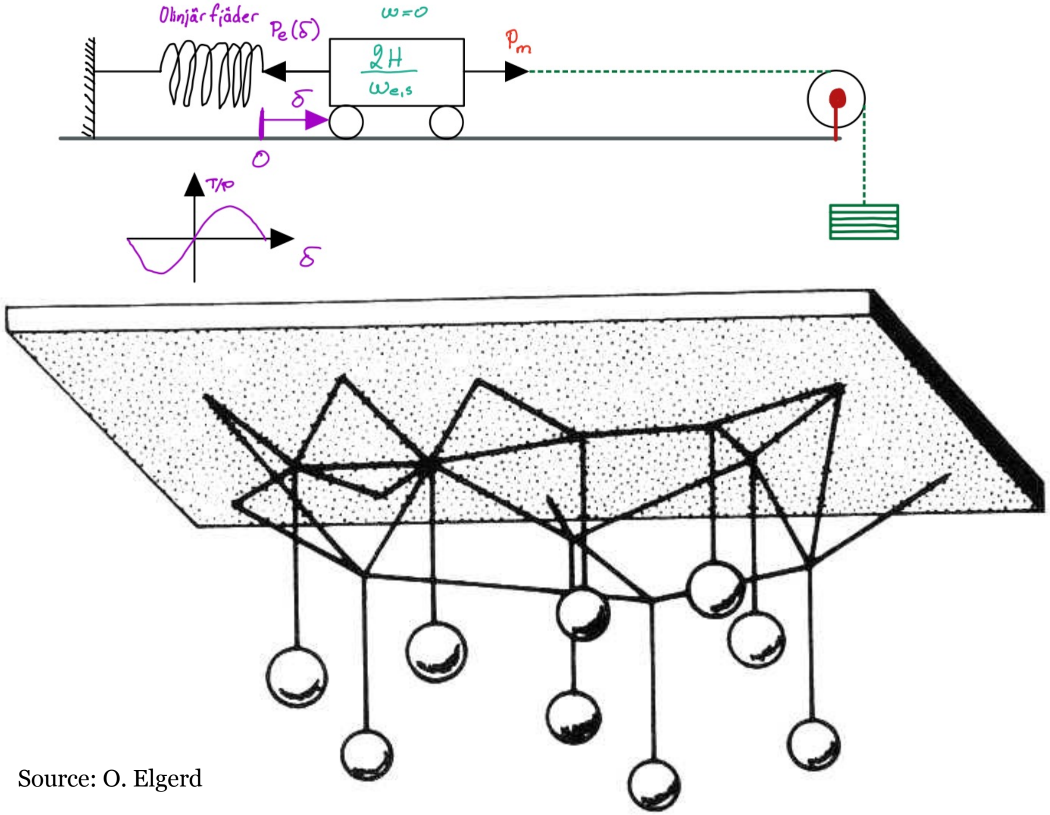
En-dimensionell bild av Elkraftsystemet



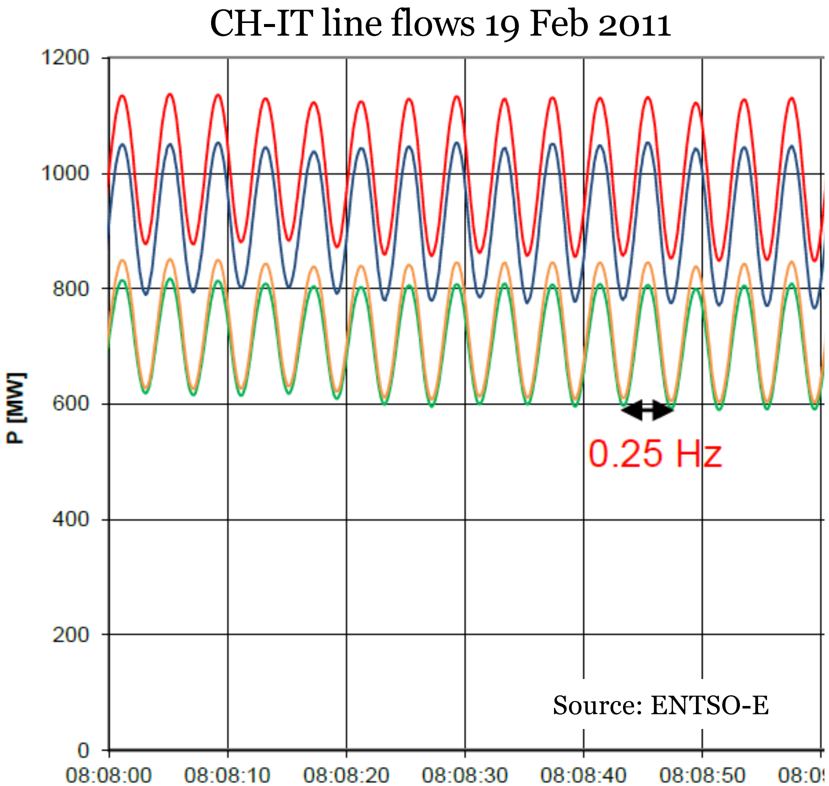
- Idag: Transmissionsnätet, begränsningar & stabilitet.

Bild från:
Renewable and Efficient Electric Power Systems
Gilbert M. Masters (2004)

Dynamik – Bild: fjäder och massa i rörelse



Source: O. Elgerd

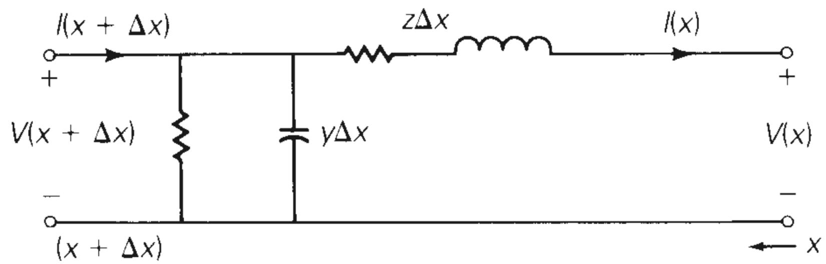


1. Elnätsstabilitet Ledningar

Stationära tillstånd och gränser

Distribuerad ledningsmodell

$z = \mathrm{R} + j\omega\mathrm{L} \quad \Omega/\mathrm{m}$
$y = \mathrm{G} + j\omega\mathrm{C} \quad \mathrm{S/m}$



- Distribuerad modell, många element

$z = \mathrm{R} + j\omega\mathrm{L} \quad \Omega/\mathrm{m}$, series impedance per unit length

$y = \mathrm{G} + j\omega\mathrm{C} \quad \mathrm{S/m}$, shunt admittance per unit length

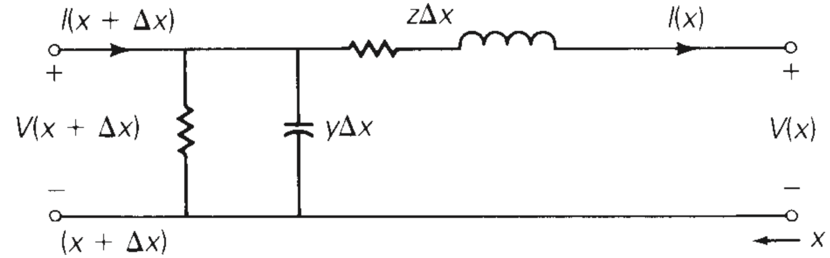
$Z = zl \quad \Omega$, total series impedance

$Y = yl \quad \mathrm{S}$, total shunt admittance

$l =$ line length m

Distribuerad ledningsmodell

$$z = R + j\omega L \quad \Omega/\text{m}$$
$$y = G + j\omega C \quad \text{S/m}$$



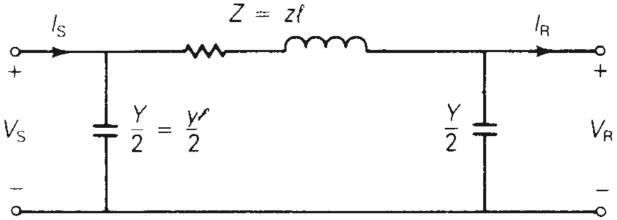
- Distribuerad modell, många element
- Differentialekvationer för V(x) & I(x)

$$\frac{dV(x)}{dx} = zI(x) \qquad \frac{dI(x)}{dx} = yV(x) \qquad \frac{d^2V(x)}{dx^2} - zyV(x) = 0$$

- Lösning $V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x} \quad \gamma = \sqrt{zy} \quad \text{m}^{-1}$

$$I(x) = \frac{A_1 e^{\gamma x} - A_2 e^{-\gamma x}}{z/\gamma} \qquad z/\gamma = z/\sqrt{zy} = \sqrt{z/y}$$

- Karaktäristisk impedans: $Z_c = \sqrt{\frac{z}{y}} \quad \Omega$
- Bestäm integrationskonstanterna A_1 & A_2 med randvillkor V_R & I_R



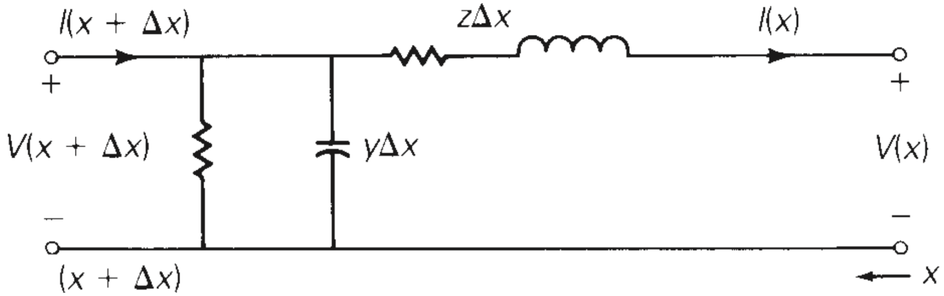
$$V(x) = \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right)V_R + Z_c\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right)I_R$$

$$I(x) = \frac{1}{Z_c}\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right)V_R + \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right)I_R$$

$$V(x) = \cosh(\gamma x)V_R + Z_c \sinh(\gamma x)I_R$$

$$I(x) = \frac{1}{Z_c}\sinh(\gamma x)\ V_R + \cosh(\gamma x)I_R$$

Surge Impedance - Överspänningsimpedans



- Förlustfri ledning $R = G = 0$

$$z = j\omega \mathrm{L} \quad \Omega/\mathrm{m}$$
$$y = j\omega \mathrm{C} \quad \mathrm{S/m}$$

- Karaktäristiska impedansen, blir Reel, kallas "Surge Impedance"

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{j\omega \mathrm{L}}{j\omega \mathrm{C}}} = \sqrt{\frac{\mathrm{L}}{\mathrm{C}}} \quad \Omega$$

- Utbredningskonstanten blir rent imaginär

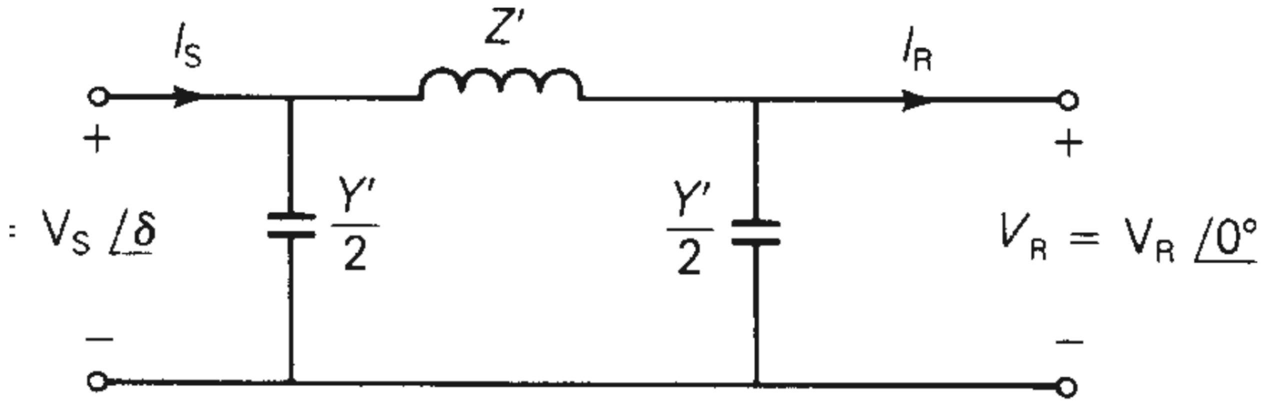
$$\gamma = \sqrt{zy} = \sqrt{(j\omega \mathrm{L})(j\omega \mathrm{C})} = j\omega\sqrt{\mathrm{LC}} = j\beta \quad \mathrm{m}^{-1}$$

- Våglängd: $f\lambda = \frac{1}{\sqrt{LC}} \approx 3 \cdot 10^8$, 50 Hz ger $\lambda \approx 6000$ km

Ekvivalenta Pi-Krets parametrar

Z' betecknar ekvivalenta Pi-kretsens Z
En lång ledning ger mer impedans och fasvrider mer.

$$Z' = jZ_c \sin(\beta l) = jX' \quad \Omega$$

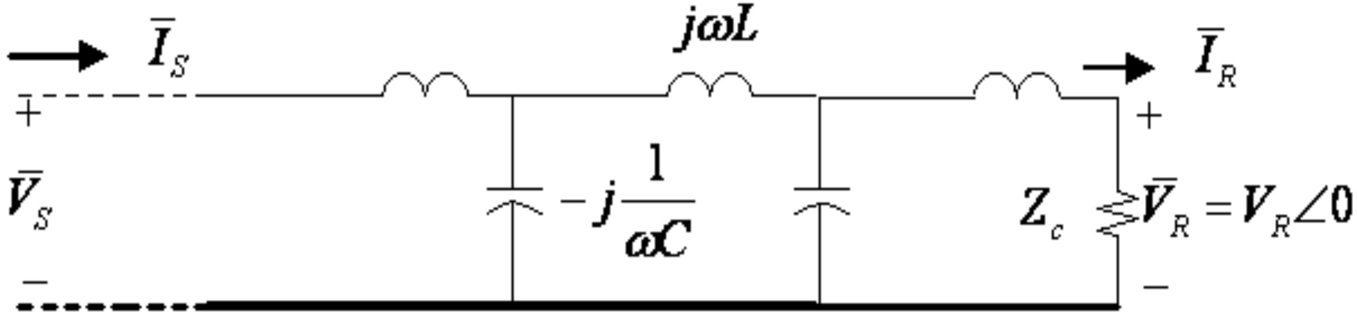


$$\frac{Y'}{2} = \frac{Y}{2}\frac{\tanh(j\beta l/2)}{j\beta l/2} = \frac{Y}{2}\frac{\sinh(j\beta l/2)}{(j\beta l/2)\cosh(j\beta l/2)}$$

$$= \left(\frac{j\omega Cl}{2}\right)\frac{j\sin(\beta l/2)}{(j\beta l/2)\cos(\beta l/2)} = \left(\frac{j\omega Cl}{2}\right)\frac{\tan(\beta l/2)}{\beta l/2}$$

$$= \left(\frac{j\omega C' l}{2}\right) \quad \text{S}$$

Surge Impedance Loading (SIL)



- Belasta ledningen med Z_C
- SIL definieras som effekt vid märkspänning V_{rated}

$$SIL = \frac{V_{rated}^2}{Z_C}$$

- Impedansmatchning – Ingen vågreflektion

Spänning, Impedans och SIL

Table 4-2

Surge Impedance and Three-Phase Surge Impedance Loading [2, 6]

Nominal Voltage	$Z_c(\Omega)$	$SIL(MW)$
230 kV	375	140 MW
345 kV	280	425 MW
500 kV	250	1000 MW
765 kV	255	2300 MW

Samma information olika böcker.
Att skicka aktiv effekt ökar med V^2

V_{rated} (kV)	$Z_C = \sqrt{L/C}$ (Ω)	$SIL = V_{rated}^2/Z_C$ (MW)
69	366–400	12–13
138	366–405	47–52
230	365–395	134–145
345	280–366	325–425
500	233–294	850–1075
765	254–266	2200–2300

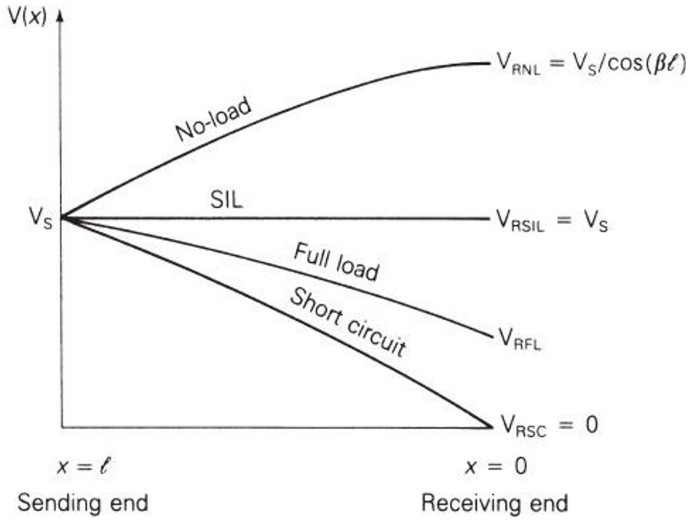
Spänningsprofiler (upp till $\frac{1}{4}$ våglängd)

1. At no load, I_{RNL}= 0, so the voltage increases from V_S = (cos βl)V_{RNL} at sending end to V_{RNL} at receiving end

2. Voltage profile at SIL is flat

3. For a short circuit at the load, V_{RSC} = 0, so the voltage decreases from V_S = (sin βl)($Z_c I_{RSC}$) at sending end to V_{RSC} = 0 at receiving end

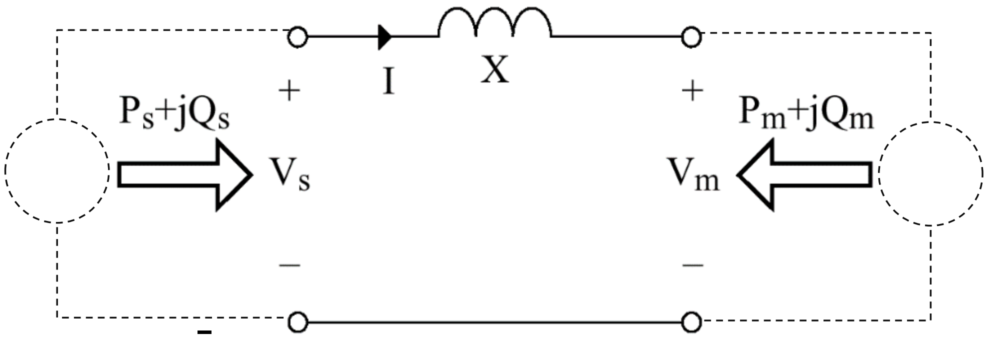
4. The full-load voltage profile, which depends on the specification of full-load current, lies above the short-circuit voltage profile

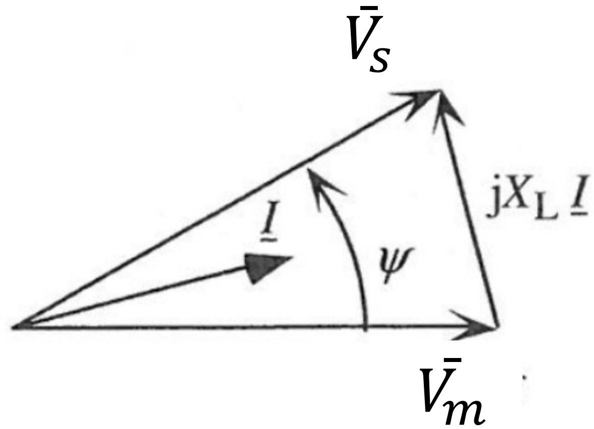


2. Ledningskapacitet och Stabilitet

Princip för långdistans effektöverföring

X har inga aktiva förluster → $P_m = -P_s$





$$\bar{S}_s = P_s + jQ_s = 3\frac{\bar{V}_s}{\sqrt{3}}\bar{I}_s^*$$

$$\bar{S}_s = 3\frac{\bar{V}_s}{\sqrt{3}}\left(\frac{\bar{V}_s - \bar{V}_m}{\sqrt{3}jX}\right)^* = j\frac{\bar{V}_s\bar{V}_s^*}{X} - j\frac{\bar{V}_s\bar{V}_m^*}{X} = j\frac{V_s^2}{X} - j\frac{\bar{V}_s\bar{V}_m^*}{X} = j\frac{V_s^2}{X} + \frac{V_sV_m}{X}(-j\cos\Psi + \sin\Psi)$$

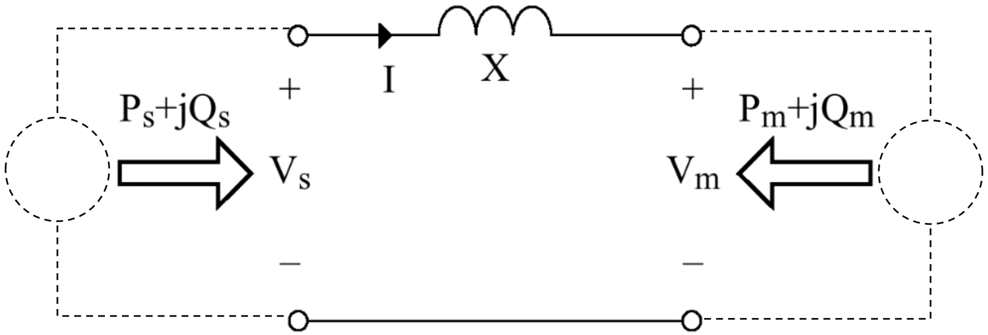
$$|P_s| = |P_m| < P_{max} = \frac{V_sV_m}{X}$$

Termen rent imaginär ingår i Q_s

Om $V_s = V_m$, $\Psi > 0$ aktiv effekt överförs.
Om $|V_s| > |V_m|$ överförs reaktiv effekt från s till m, och vise versa.
Frekvens styr aktiv effekt, spänning styr reaktiv effekt

Gräns för långdistans effektöverföring

X har inga aktiva förluster → $P_m = -P_s$



$$P_s = \frac{V_s V_m}{X} (\sin \Psi)$$

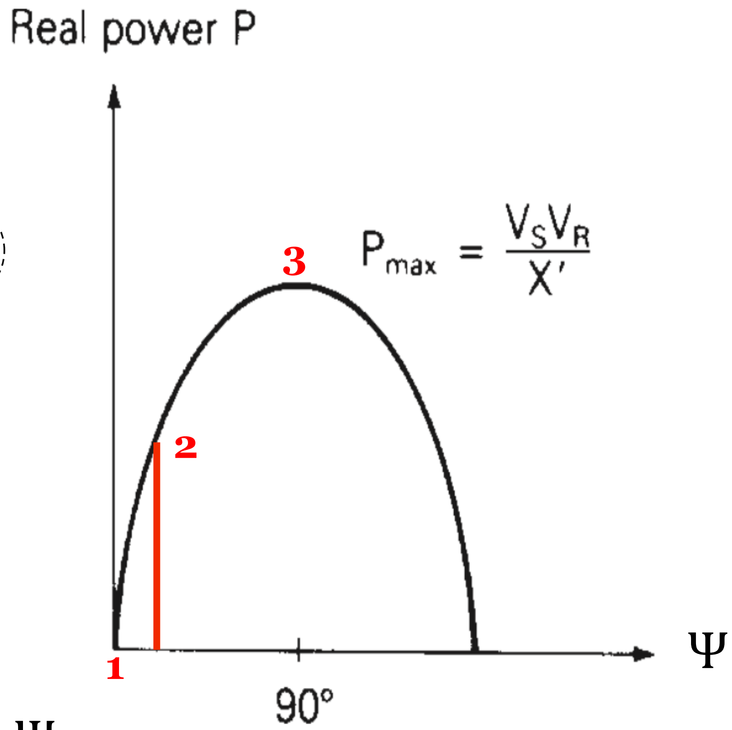


FIGURE 5.11

Real power delivered by a lossless line versus voltage angle across the line

1. Obeastad linje
2. Ökad belastning ger ökad vinkel Ψ
3. Över maxgränsen. Generator och förbrukare tappar synkronisering.

Uttryck kapacitet mha SIL

- Ekvationer från bok $\delta = \Psi$

$$P = \frac{V_S V_R \sin\delta}{Z_c \sin\beta l} = \left(\frac{V_S V_R}{Z_c}\right) \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

- Byt till per enhet p.u.

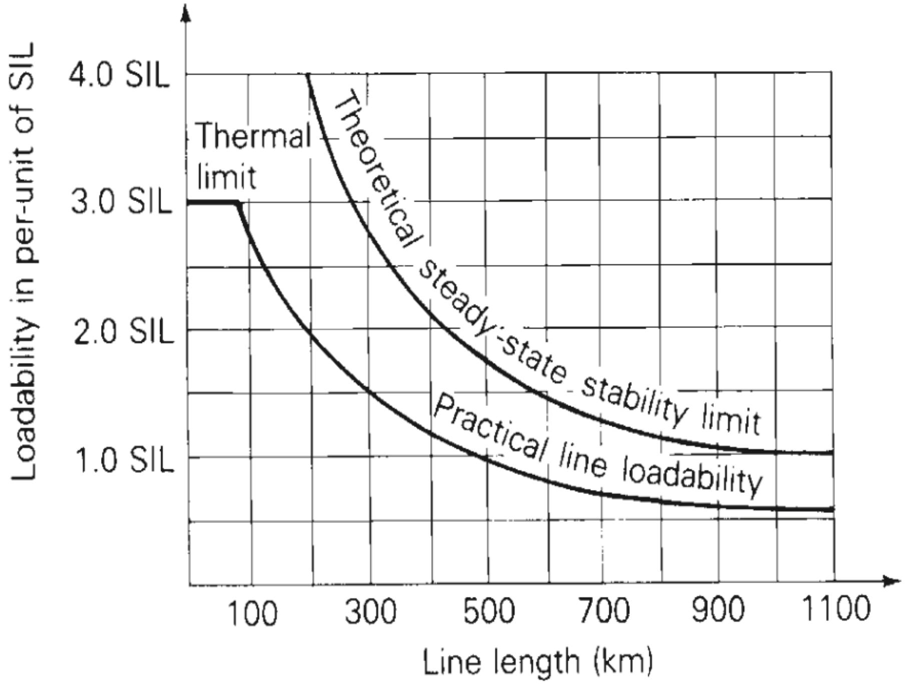
$$P = \left(\frac{V_S}{V_{rated}}\right)\left(\frac{V_R}{V_{rated}}\right)\left(\frac{V_{rated}^2}{Z_c}\right) \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

- Resultat
  - Ökar med kvadraten på spänningen
  - Minskar med ledningslängd

$$= V_{Sp.u.} V_{Rp.u.} (SIL) \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)} \quad W$$

Ledningens belastningsgränser

- Power lines are not operated to deliver their theoretical maximum power
  - Theoretical max power: rated terminal voltages and an angular displacement $\Psi = 90°$
- Practical loadability:
  - Voltage-drop limit $V_R/V_S \leq 0.95$
  - Maximum angular displacement of 30 to 35° across the line
- For short lines less than 25 km long, loadability is limited by the thermal rating of the conductors or by terminal equipment ratings, not by voltage drop or stability considerations



Vad begränsar transmissionsledningens kapacitet

Table 4-3

Loadability of Transmission Lines [6]

Line Length (km)	Limiting Factor	Multiple of SIL
0 - 80	Thermal	> 3
80 - 240	5% Voltage Drop	1.5 - 3
240 - 480	Stability	1.0 – 1.5

- En ledning är inuduktor, långa ledare får stor induktans.
- Induktans äter upp reaktiv effekt. Kan inte skicka reaktiv effekt långt.
- Serie kompensering.

3. Elnätsstabilitet

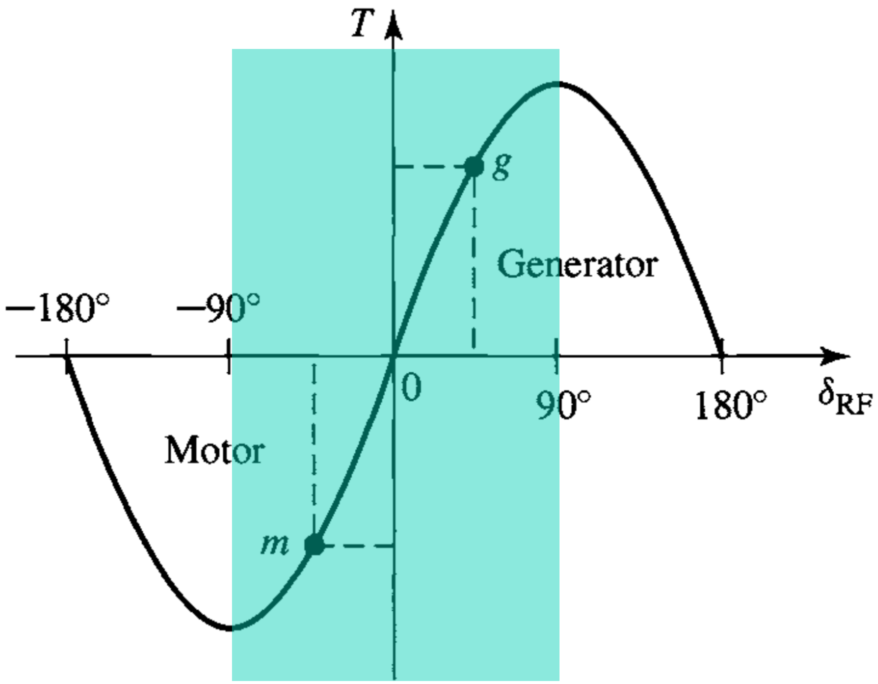
Dynamiska tillstånd och förlopp

Synchronous Generator until now

- Steady state
  - All generators run synchronously (think tandem bike)
  - $\omega_m = \omega_{m,s(ynchronous)} \Leftrightarrow \omega_e \Leftrightarrow$ 50 Hz, $P_m = T_m \omega_m$
  - $P_e = P_m \Leftrightarrow 0 = P_m - P_e$ and $P_e(E,V,X_d,\delta)$ and $Q_e(E,V,X_d,\delta)$
- Electromagnetic dynamics at short-circuit
  - Subtransient period during the first ms $\Leftrightarrow X''_d$
  - Transient period during the following s $\Leftrightarrow X'_d$
  - Steady state $\Leftrightarrow X_d$
- Today electromechanical dynamics in the 1 Hz range

Demo

Lastvinkel och Rotorns moment, T



Maskinens stabila arbetsområde

$$T = \frac{\pi}{2}\left(\frac{\text{poles}}{2}\right)^2 \Phi_R F_f \sin\delta_{RF}$$

Φ_R	resultant air-gap flux per pole
F_f	mmf of the dc field winding
δ_{RF}	electrical phase angle between magnetic axes of Φ_R and F_f

Maskinens effekt

$$P = \omega\, T$$

Mekanisk turbineffekt

Balansekvationer

Tillstånd motsvarar energi. Förändring motsvarar effekt.
Tillstånd kan inte ändras snabbare än vad högerled och tröghet medger

Newton 2 Linjär

$$m\frac{dv}{dt} = F_{acc} - F_{br}$$

$$W = \frac{1}{2}mv^2$$

Kondensator

$$C\frac{dV}{dt} = i_{in} - i_{ut}$$

$$W = \frac{1}{2}CV^2$$

Newton 2 Rotation

$$J\frac{d\omega}{dt} = T_{acc} - T_{br}$$

$$W = \frac{1}{2}J\omega^2$$

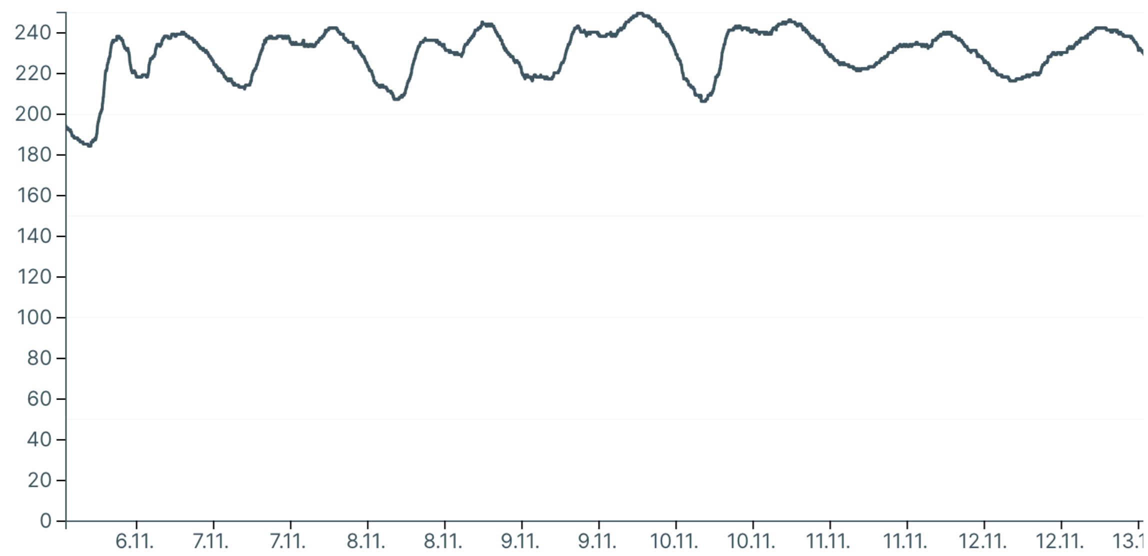
Induktans

$$L\frac{di}{dt} = u_{öka} - u_{minska}$$

$$W = \frac{1}{2}Li^2$$

Masströghet J i nordiska nätet.

- Finngrid presenterar data



	Name	Minimum	Maximum	Average
●	Inertia	184	250	229 GWs

The Swing Equation – in per unit

- General torque balance for rotor (Newton's second law Ma=F)

$$J\frac{d\omega_m}{dt} = T_m - T_e$$

- Multiply torque balance by ω_m ➔ Tω=P on right-hand side

- Divide by S_{base} to get p.u.:

$$\frac{\omega_m}{S_{base}} J \frac{d\omega_m}{dt} = P_m(p.u.) - P_e(p.u.)$$

- Use $\omega_m \approx \omega_{m,s}$ on left-hand side:

$$\frac{\omega_m}{S_{base}} J \frac{d\omega_m}{dt} \approx \frac{\omega_{m,s}}{S_{base}} J \frac{d\omega_m}{dt}$$

- p magnetic rotor poles

- Complicated! Use ω_e as state (next slide)

$$\omega_m(\text{mech. rad/s}) = \frac{2}{p}\omega_e(\text{elec. rad/s})$$

The inertia constant H

$$\frac{\omega_{m,s}}{S_{base}} J \frac{d\omega_m}{dt} = \frac{2}{\omega_{m,s}} \frac{\frac{1}{2} J \omega_{m,s}^2}{S_{base}} \frac{d\omega_m}{dt} = \frac{2}{\omega_{e,s}} \frac{\frac{1}{2} J \omega_{m,s}^2}{S_{base}} \frac{d\omega_e}{dt} = \frac{2H}{\omega_{e,s}} \frac{d\omega_e}{dt}$$

$$\frac{\frac{1}{2} J \omega_m^2}{S_{base}} = \frac{\text{Kinetic energy of rotating masses}}{\text{Generator MVA rating}} = H$$

Unit: Ws/VA=s

The per unit swing equation:

$$\frac{2H}{\omega_{e,s}} \frac{d\omega_e}{dt} = P_m(p.u.) - P_e(p.u.)$$

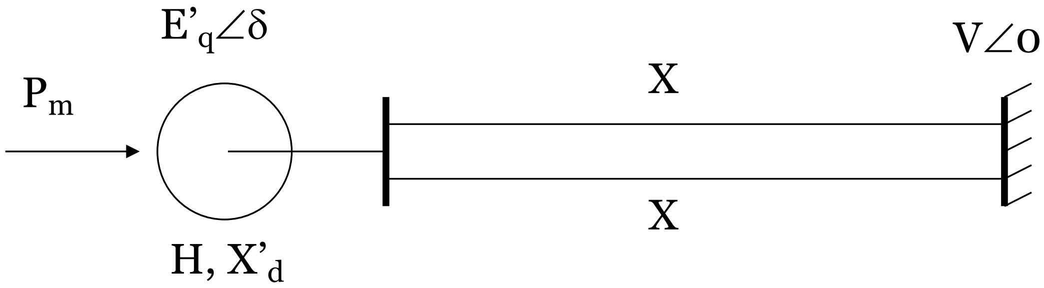
H on different MVA bases

- Machine base
  - Steam turbines 4-9 s
  - Gas turbines 3-4 s
  - Hydro turbines 2-4 s
  - Synchronous compensator 1-1.5 s
- Common base
  - H ~ generator size (kW-GW)
  - Infinite bus has infinite H → fixed frequency (and phase)

Narrow range!

"Single Machine Infinite Bus"

Represents one generator connected to a large system



<u>"Classical model":</u>

- **Swing equation for dynamics**
- Fixed E'_q behind X'_d (Thévenin!)
- Constant P_m
- No damping, no saliency

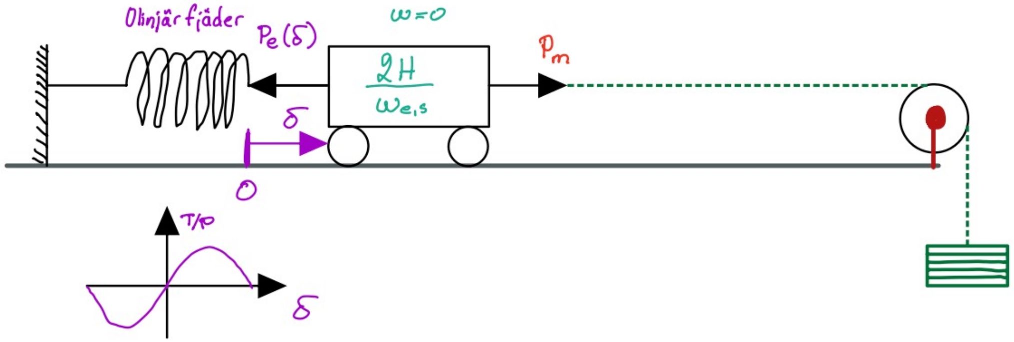
<u>"Infinite bus" generator:</u>

- Infinite H
- Fixed voltage $V\angle 0$
- Zero Thévenin impedance

"Classical" dynamic generator model

Synchronous generator connected to infinite bus:

$$\begin{cases} \frac{2H}{\omega_{e,s}} \frac{d\omega_e}{dt} = P_m - P_e(\delta) \\ \frac{d\delta}{dt} = \omega_e - \omega_{e,s} \end{cases}$$



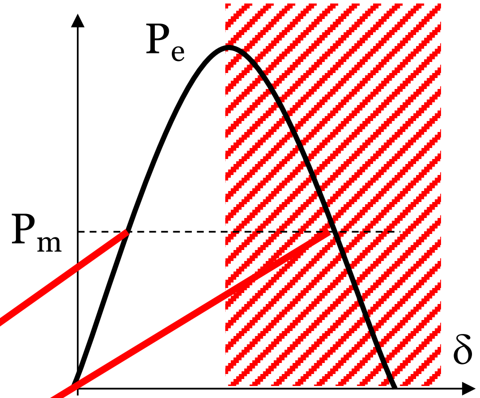
- δ in rad, ω_e in rad/s, $\omega_{e,s}$ typically 100π rad/s
- E'_q and X'_d for slow transients in $P_e(\delta)$ with V and X//X //=in parallell
- Second order system with poor damping
- Electro-mechanical or "swing" dynamics

Two equilibrium points

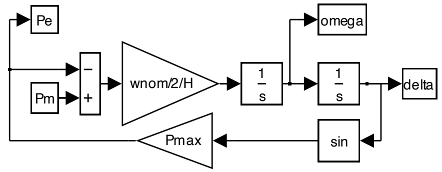
$$P_m = P_e(\delta) = \frac{E'_q V}{X_{eq}} \sin\delta = P_{max} \sin\delta$$

Two solutions for δ:

$$\delta = \begin{cases} \delta_0 = \arcsin\left(\dfrac{P_m}{P_{max}}\right) \\ 180° - \delta_0 \end{cases}$$



$$\begin{cases} \dfrac{2H}{\omega_{e,s}} \dfrac{d\omega_e}{dt} = P_m - P_e(\delta) \\ \dfrac{d\delta}{dt} = \omega_e - \omega_{e,s} \end{cases}$$



- Synchronizing torque $dP_e/d\delta$

  Try disturbance like small increase in P_m and walk around Simulink model:

  → increase in ω_e → increase in δ → increase in P_e?

  For δ<90°, $dP_e/d\delta>0$ → increase in P_e → decrease in ω_e → stable equilibrium

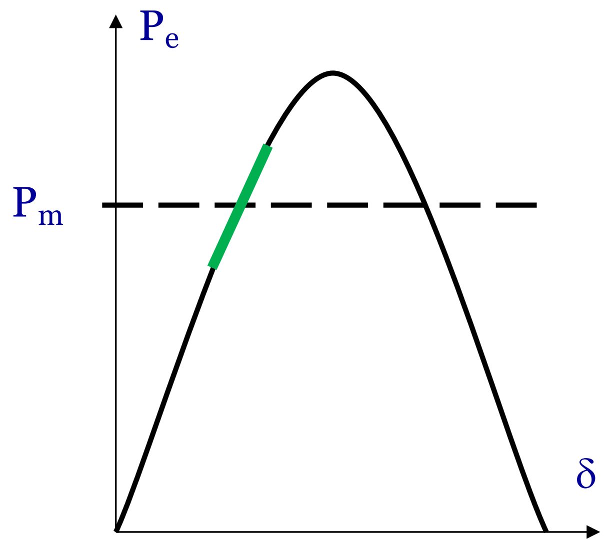
  For δ>90°, $dP_e/d\delta<0$ → decrease in P_e → increase in ω_e → unstable equilibrium point (UEP)

Dynamic response

Temporary short-circuit near generator, P_e zero during fault

Response?

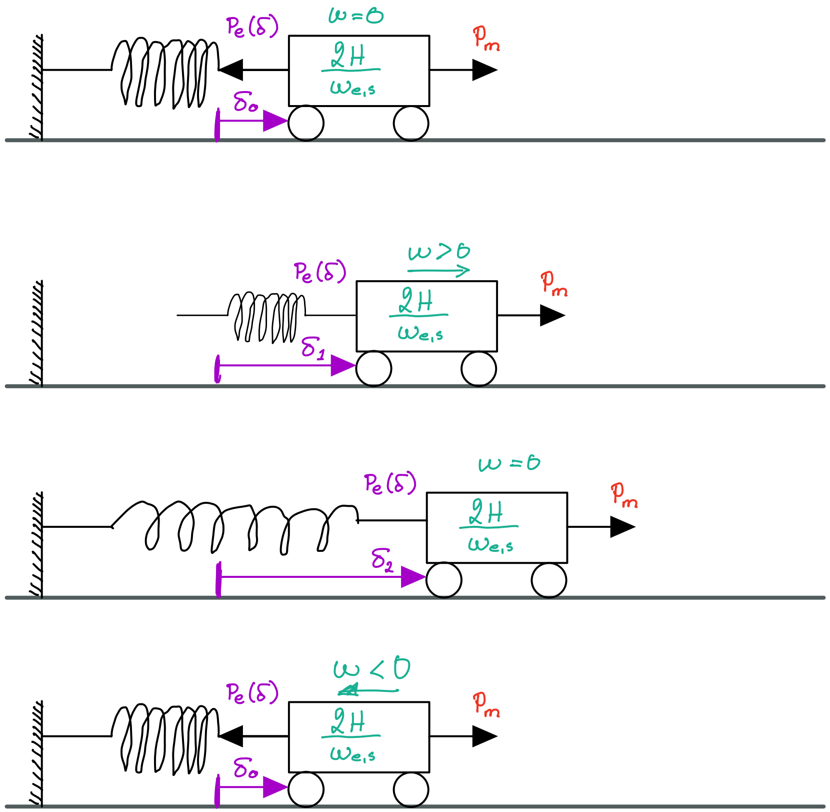
1. Second order system
2. No damping
3. Oscillator! δ and ω oscillate
4. δ(t) will lag ω(t)



Small disturbance → sinusoids (se slide1) → linear model OK

Second order response

- P_e=0 at short-circuit near gen (source feeds just X → Q)
- Step in P_m-P_e
- Mechanical states slow
- Start at δ_0 and $P_e(\delta_0)$
- Acceleration during fault
- Fault removed at $\delta=\delta_1$=*clearing angle*
- Overshoot to δ_2 and $P_e(\delta_2)$
- Oscillate around equilibrium δ_0 so $P_e(\delta_0)=P_m$



Second order response

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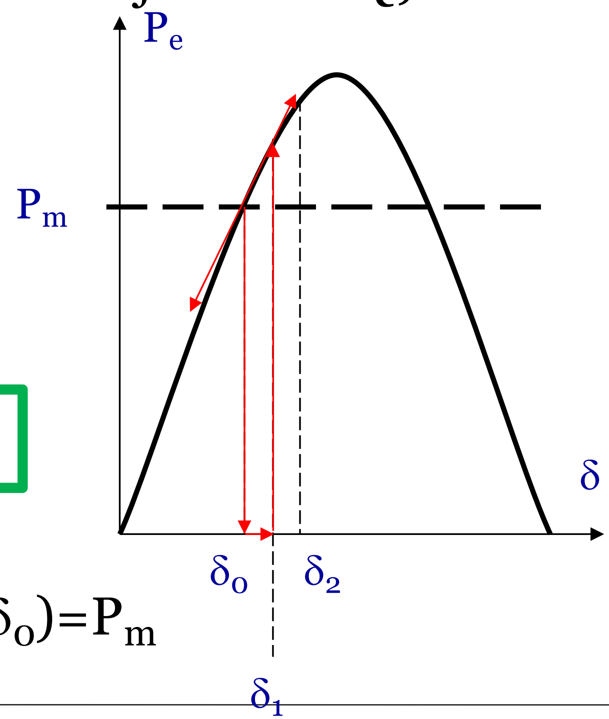


Transient or large disturbance angle stability

- δ_0 must be less than steady state limit 90^o
- δ_2 also has limit – transient angle stability limit

- Questions:
- How large can δ_2 be?
- What happens when it becomes too large?
- What is the largest disturbance that is OK?

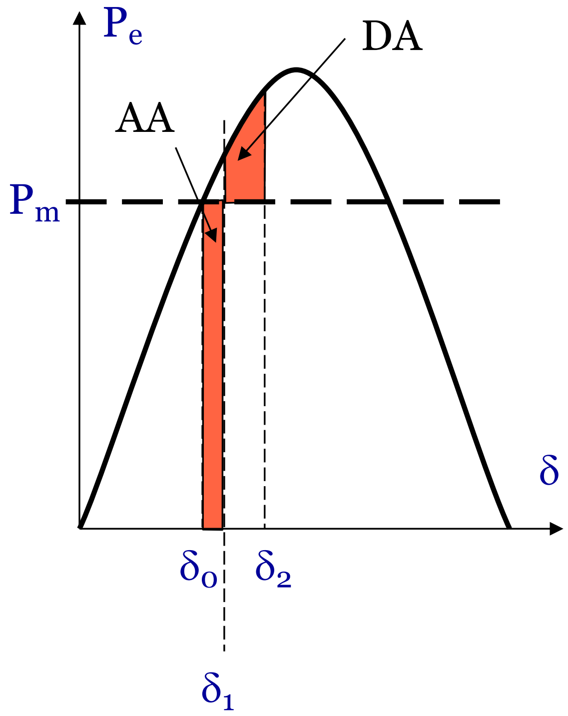
PW Example 12.5 7th ed Demo
tcl=0.05-0.1895

Beyond stability limit

- $d\omega/dt$ never becomes zero
- Rotor accelerates even more
- Machine transiently unstable = loses synchronism
- Must disconnect and resynchronise

"The Equal Area Criterion"

- Short-circuit: Pe=zero
  Mark areas between $P_e(\delta)$ and P_m in interval δ_o to δ_2
- Accelerating Area: Below P_m
- Decelerating Area : Above P_m
- For stable system AA=DA



EAC derivation

- Textbook 12.3

$$\frac{2H}{\omega_{s,e}}\frac{d^2\delta}{dt^2} = P_m - P_e$$

Trick1: multiply with dδ/dt

$$\frac{2H}{\omega_{s,e}}\frac{d^2\delta}{dt^2}\frac{d\delta}{dt} = \left(P_m - P_e\right)\frac{d\delta}{dt}$$

Rewrite LHS!

$$\frac{H}{\omega_{s,e}}\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \left(P_m - P_e\right)\frac{d\delta}{dt}$$

Trick2: multiply with dt

Integrate both sides over relevant δ range

$$\frac{H}{\omega_{s,e}}\int_{\delta_0}^{\delta_2} d\left(\frac{d\delta}{dt}\right)^2 = \int_{\delta_0}^{\delta_2}\left(P_m - P_e\right)d\delta$$

Solve LHS

$$\frac{H}{\omega_{s,e}}\left[\left(\frac{d\delta}{dt}\right)^2\right]_{\delta_0}^{\delta_2} = 0 - 0 = \int_{\delta_0}^{\delta_2}\left(P_m - P_e\right)d\delta$$

Split δ range

$$\int_{\delta_0}^{\delta_1}\left(P_m - P_e\right)d\delta + \int_{\delta_1}^{\delta_2}\left(P_m - P_e\right)d\delta = 0$$

Make integrals equal

$$AA = \int_{\delta_0}^{\delta_1}\left(P_m - P_e\right)d\delta = \int_{\delta_1}^{\delta_2}\left(P_e - P_m\right)d\delta = DA$$

2. Charts

Examples of LiU colours in charts.

Use as inspiration. Clear and simple charts always work best.

TSFS 17 Elkraftsystem

Föreläsning

https://isy.gitlab-pages.liu.se/fs/courses/TSFS17/

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