Exam

TSFS06 Diagnosis and Supervision
June 5, 2019, kl. 14.00-18.00

Approved aids: calculator

Responsible teacher: Erik Frisk

Solution language: Swedish or English

Total 40 points.

Preliminary grade limits

Grade 3: 18 points
Grade 4: 25 points
Grade 5: 30 points

Task 1. Consider the linear model

$$\begin{aligned}
\dot{x}_1 &= -x_1 + u + f_3 \\
\dot{x}_2 &= x_1 + \lambda x_2 + f_4 \\
y_1 &= x_1 + f_1 \\
y_2 &= x_2 + f_2
\end{aligned}$$

where f_i are the faults, u and y_i are known control and measurement signals respectively. The constant $\lambda > 0$ is known.

a) Prove that all four faults are detectable and that fault f_1 is isolable from fault f_4 (3 points)

b) Design a residual generator that isolates fault f_2 from fault f_1. Write the residual generator in state-space form. (3 points)

c) The system is unstable, discuss how this influences the design of residual generators. (1 points)

Help: The observable canonical form of

$$G(p) = \frac{b_1 p^{n-1} + \cdots + b_{n-1}p + b_n}{p^n + a_1 p^{n-1} + \cdots + a_{n-1}p + a_n}$$

is given by

$$\begin{aligned}
\dot{x}(t) &= \begin{pmatrix} -a_1 & 1 & 0 & \dots & 0 \\ -a_2 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -a_{n-1} & 0 & 0 & \dots & 1 \\ -a_n & 0 & 0 & \dots & 0 \end{pmatrix} x(t) + \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-1} \\ b_n \end{pmatrix} u(t) \\
y(t) &= \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \end{pmatrix} x(t)
\end{aligned}$$

Task 2. Assume 6 residuals has been designed to supervise 4 faults according to the decision structure

	f_1	f_2	f_3	f_4
r_1	X	X		
r_2	X			X
r_3		X		X
r_4			X	X
r_5	X	X		X
r_6		X	X	X

where fault f_i indicates fault in component C_i, $i = 1, \dots, 4$.

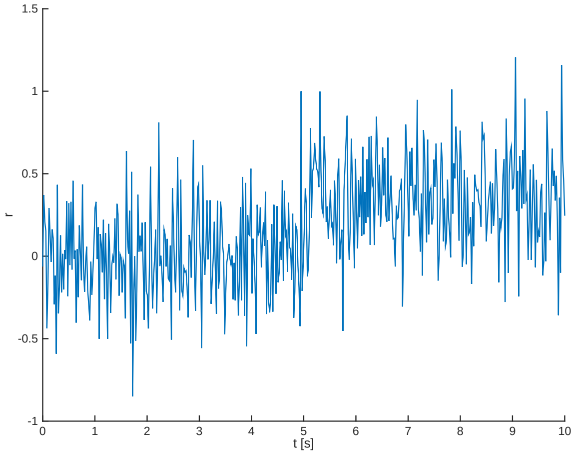
a) Assume all 6 residuals has given an alarm, i.e., exceeded its corresponding threshold. Write down the generated conflicts and indicate which that are minimal conflicts. Write the conflicts with logic notation and let $OK(C_i)$ and $\neg OK(C_i)$ denote that component i is fault-free and faulty respectively. (2 points)

b) With the alarms from the a-task, compute all minimal diagnoses. Express the diagnoses using $OK(C_i)$ and $\neg OK(C_i)$. Describe under which assumptions all minimal diagnoses characterize all diagnoses. (3 points)

c) Compute the fault isolability matrix for the 6 residuals described above. (2 points)

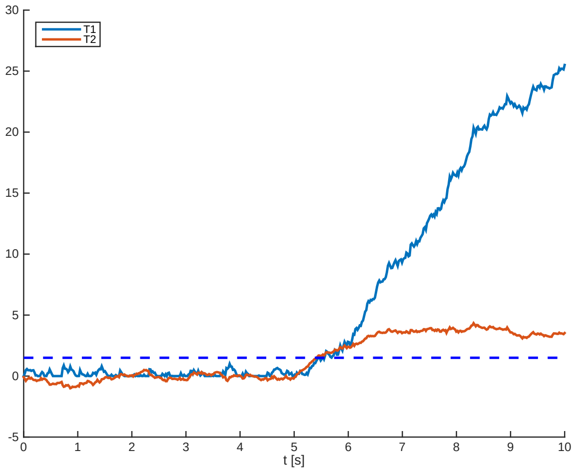
d) Describe why there are complexity problems with computing all minimal diagnoses and sketch how to modify the hitting-set algorithm to only consider k simultaneous faults. (2 points)

Task 3. Consider the residual below that monitors a fault, occurring at time $t = 5$s.



a) It is clear that it is not possible to use a fixed threshold on the residual to reliably detect the fault and at the same time avoid false alarms. Describe how low-pass filtering and the CUSUM algorithm can be used respectively to make a more reliable decision. Describe how the parameters in the algorithms are determined. (2 points)

b) In the figure below are two test quantities plotted, one designed using low-pass filtering of the residual in the a-task and one based on CUSUM. Determine which is which and motivate your answer. (1 points)



c) Assume a test with probability of detection 0.95, probability of false alarm of 0.01, and a-priori probability of a fault 0.01.

Determine the probability of a fault given an alarm. Further, assume two independent alarms are recorded, now determine the probability of a fault. (4 points)

Task 4.

a) Assume we are supervising a fault modeled by a parameter θ where $\theta = 1$ corresponds to the fault free case.

Assume a parameter estimator $\hat{\theta} = f(z)$ has been designed where z are the observed data. The estimate $\hat{\theta}$ is found to be normal distributed with expected value θ and variance σ^2. Design a test quantity and describe how the threshold for detection is determined.

Express the threshold in the cumulative distribution functions described below. (2 points)

b) Assume we are supervising the variance of a measurement signal $y \sim \mathcal{N}(0, \sigma^2)$ where the variance increases in case of a fault. Also assume that we do not batch data, but take a decision sample by sample. Formally motivate why the test quantity

$$T = y^2$$

is a good candidate test quantity and state which additional assumptions that are made. (2 points)

c) Use the expression for the cumulative distribution functions below to derive the expressions for the ROC-curve when $\sigma = \sigma_0$ in the fault free case and $\sigma = \sigma_1$ when there is a fault.

Further, sketch a principle ROC curve, it doesn't have to correspond exactly to the ROC-curve above. Illustrate how the curve will change when σ_1 increases. (3 points)

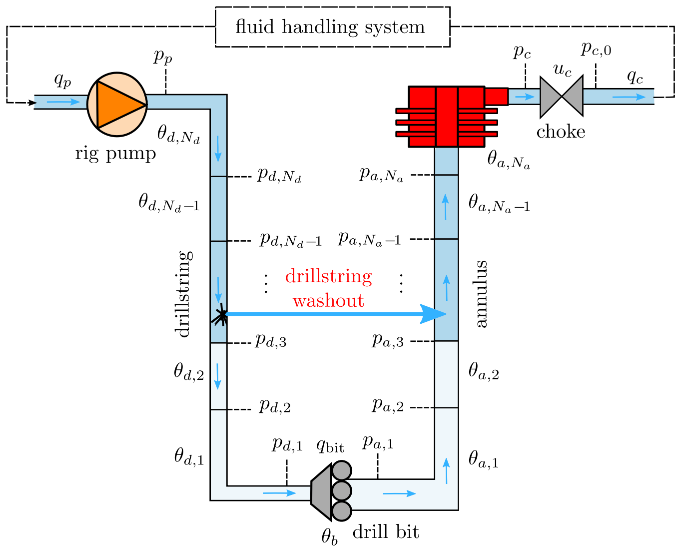
Help: Let X be a $\mathcal{N}(0, 1)$ distributed random variable, then the cumulative distribution function is denoted as

$$\Phi(x) = P(X \leq x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{s^2}{2}}\, ds$$

Let Y be a $\chi^2(n)$ distributed random variable where n is the number of degrees of freedom. Then, denote the cumulative distribution function with $F_n(y)$, i.e.,

$$F_n(y) = P(Y \leq y)$$

Task 5. The figure below[1] is an illustration of a drill in a gas/oil rig.



A simplified model of the system is given by the equations

$$\begin{aligned}
\dot{p}_p &= c_1(q_p(p_p, u_p) - q_{bit}) \\
\dot{p}_c &= c_2(q_{bit} - q_c(p_c, u_c)) \\
\dot{q}_{bit} &= c_3(p_p - p_c - \theta q_{bit}^2)
\end{aligned}$$

where p_p is the pump pressure, p_c pressure before the choke valve, q_{bit} flow past the drill head, and q_c flow after the choke valve. The parameters c_1, c_2 and c_3 can be assumed to be known.

[1]Figure taken from PhD thesis "*Model-based diagnosis of drilling incidents*", Anders Willersrud, Trondheim, June, 2015.

The parameter θ describes friction loss over the drill head, and the nominal value of θ is assumed known also. Models for the flows q_p and q_c are given by the equations

$$q_p(p_p, u_p) = g_p(u_p)\sqrt{|p_p - p_{p,0}|}$$
$$q_c(p_c, u_c) = \text{sign}(p_c - p_{c,0})g_c(u_c)\sqrt{|p_c - p_{c,0}|}$$

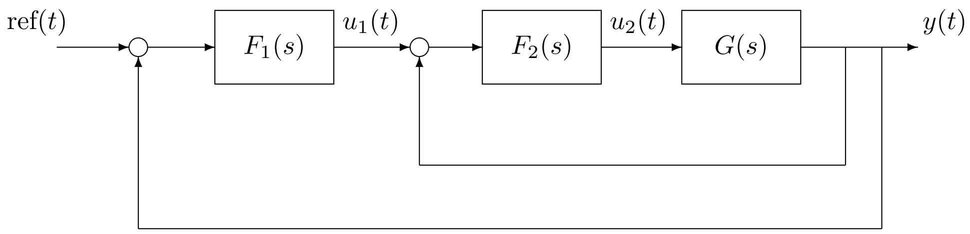
where the critical flows $p_{c,0}$, $p_{p,0}$ and the functions $g_p(u_p)$ and $g_c(u_c)$ are known. The control signal to the pump, u_p, and to the choke valve u_c are also known.

Assume that the pressures p_p (y_1) and p_c (y_2) are measured.

a) Write the model, including measurement equations, and include models for a fault in the pump (f_1) and increased friction past the drill head (f_2). (2 points)

b) Design a residual generator that isolates fault f_2 from fault f_1, describe methodology to determine any design parameters. If you use observer design techniques, you do not have to prove stability. (4 points)

Task 6. Assume a single-input single-output system $G(s)$ that is controlled in cascade by two controllers $F_1(s)$ and $F_2(s)$ according to the figure below.



The reference signal ref(t) is known. The objective is to detect a constant additive fault in the sensor $y(t)$.

a) Assume both controllers are implemented in external hardware and the corresponding control outputs u_1 and u_2 are *not* available to the diagnosis system. Describe, with motivations, how the possibilities to detect the fault is influenced by the three cases where $F_1(s)$, $F_2(s)$, and $G(s)$ respectively has a pure integrator, i.e., pole at $s = 0$. (2 points)

b) Assume that $F_2(s)$ is a PI-controller and that controller $F_1(s)$ is implemented in software such that the control signal u_1 is available to the diagnosis system. Is a constant additive sensor fault detectable?

Next, assume that the PI-controller $F_2(s)$, instead of $F_1(s)$, is implemented in software and that instead control signal u_2 is available to the diagnosis system. Is a constant additive sensor fault detectable now? (2 points)