Exam

TSFS06 Diagnosis and Supervision
June 4, 2016, kl. 14.00-18.00

Approved aids: calculator

Responsible teacher: Erik Frisk

Total 40 points.

Preliminary grade limits

Grade 3: 18 points
Grade 4: 25 points
Grade 5: 30 points

Task 1. Consider a linear model

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_1 - x_2 + u + f_3 \\
y_1 &= x_1 + f_1 \\
y_2 &= x_2 + f_2
\end{aligned}$$

where u, is a known control input, y_i known measurement signals, f_i faults, and x_i unknown states.

a) Write the model in the form

$$H(s)x + L(s)z + F(s)f = 0$$

and determine the dimension of the space of linear residual generators. (2 points)

b) Determine for all faults if they are detectable, and if so if they are strongly or weakly detectable. (1 points)

c) Design a residual generator, in state-space form, that *i*) strongly detects f_1 and *ii*) the time constant for a step fault in f_1 is 0.5 sec. (3 points)

Task 2. Assume 4 residuals has been designed to supervise 5 faults according to the decision structure

	f_1	f_2	f_3	f_4	f_5
r_1	X	X			
r_2		X	X		X
r_3	X	X		X	
r_4		X		X	

where fault f_i indicates fault in component C_i, $i = 1, \ldots, 5$.

a) Assume all 4 residuals has given an alarm, i.e., exceeded its corresponding threshold. Write down the generated conflicts and indicate which that are minimal conflicts. Write the conflicts with logic notation and let $OK(C_i)$ and $\neg OK(C_i)$ denote that component i is fault-free and faulty respectively. (2 points)

b) With the alarms from the a-task, compute the minimal diagnoses, including multiple faults. Express the diagnoses using $OK(C_i)$ and $\neg OK(C_i)$. (2 points)

c) Compute the fault isolability matrix for the 4 residuals described above. (2 points)

d) Assume that residuals with any fault sensitivity pattern with at most 3 decoupled faults, i.e., zeros in the decision structure. Specify decision structure for additional residuals that achieves full fault isolability. (2 points)

Task 3. Consider a residual designed to detect a fault f_1 but as a side effect also detects fault f_2. The internal form of the residual is

$$r = e + f_1 + 0.6\, f_2$$

where e is white noise with a zero mean Gaussian distribution with variance σ^2, i.e., $r \sim \mathcal{N}(0, \sigma^2)$. For both faults $f_i = 0$ corresponds to fault free case.

For a Gaussian random variable $X \sim \mathcal{N}(0, 1)$, the probability density function and the cumulative density function are given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad \Phi(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\, dt$$

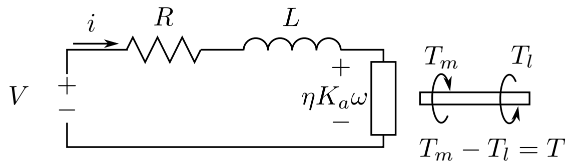
a) An alarm is generated when $|r|$ exceeds a threshold J. Derive the expression for the threshold such that the probability for false-alarm is α. Express the threshold using the cumulative distribution function Φ. (1 points)

b) Write down the power function for the residual with respect to single fault f_1 ($\beta(f_1)$) and single fault f_2 ($\beta(f_2)$) respectively. Again, utilize the cumulative density function Φ. Sketch both curves in a single plot and comment. (3 points)

c) Consider the case where fault $f_1 = 2$. Derive the expressions to plot the ROC-curve, i.e., to plot probability of detection, $P(D)$, against probability of false alarm, $P(FA)$ for different thresholds. (1 points)

d) To improve detection reliability, formulate a CUSUM test based on r for fault size $f_1 = 2$, utilizing the statistical information about the residual. (2 points)

Task 4. Consider an electric motor



A, somewhat ideal, model is given by the equations

$$e_1 : V = iR + L\frac{di}{dt} + K_a i\omega \qquad e_4 : T = T_m - T_l \qquad e_6 : y_1 = T_l$$

$$e_2 : T_m = \eta K_a i^2 \qquad e_5 : \frac{d\theta}{dt} = \omega \qquad e_7 : y_2 = i$$

$$e_3 : J\frac{d\omega}{dt} = T - b\omega \qquad e_8 : y_3 = \omega$$

where θ and ω are angle and angular velocity respectively, T net torque, T_m generated torque, T_l load torque, and i current. The known signals are the voltage V, the measurements y_1, y_2, and y_3 that measures T_l, i and ω. The known constants; b friction coefficient, K_a magnetization constant, η efficiency coefficient for torque generation, R resistance, and L inductance.

a) Model the following 5 faults: increased resistance in the motor (f_R), reduced torque generation efficiency (f_η), faults in sensors (f_1, f_2, and f_3) (1 points)

b) Design a residual generator that isolates fault f_2 from f_1. The residual generator shall be written in state-space form, with no derivatives of known signals included. (3 points)

c) Show that it is not possible to isolate fault f_η from fault f_1 with the provided sensors? Suggest additional sensor that makes isolation of fault f_η from fault f_1 possible. (3 points)

Task 5. Assume a diagnosis system with 3 residuals, r_1, r_2, and r_3 that are used to supervise 3 faults f_1, f_2, and f_3. The residuals has been designed according to the decision structure

	f_1	f_2	f_3
r_1	X	X	
r_2	X		X
r_3		X	X

Assume that thresholds has been selected such that the false alarm probability, p_{fa}, for each residual is $p_{fa} = 0.01$.

a) Model the diagnosis problem using a Bayesian network. Only fault/no fault and alarm/no alarm need to be described by the model. Show the dependency graph together with conditional probability tables for each node. Also express how the full probability model can be obtained using the individual conditional probability functions for each node.

Probability for fault f_1, f_2 and f_3 is 0.01, 0.03, and 0.005 respectively. Detection performance in the residuals is given by:

- Residuals r_1 and r_2 detects single faults with a probability 0.8 and double faults with probability 0.9
- Residual r_3 detects fault f_2 with probability 0.9, fault f_3 with probability 0.5, and double fault with probability 0.95.

Hint: Use the notation that $P(f_1)$ and $P(\neg f_1)$ denotes the probabilities $P(F_1 = \text{true})$ and $P(F_1 = \text{false})$. (4 points)

b) Determine an expression for fault f_1 given that residuals r_1 and r_2 has exceeded their corresponding thresholds, and that residual r_3 has *not* exceeded its threshold. You *do not* have to insert all numbers, it is sufficient to show how to compute. Simplify the expressions as far as possible, and should only depend on the probabilities given in the a) part. (2 points)

Task 6. Consider a system described by the first order difference equation

$$x_{t+1} = ax_t + u_t$$

where u_t is a known input signal. The measurement y_t measures the state x_t subject to zero mean white noise with variance σ^2. The control input u_t is a *known* white sequence with mean 0 and variance σ_u^2. The objective is to monitor changes in the model parameter a using a test quantity

$$T = (\hat{a} - a_0)^2$$

where a_0 is the nominal value of the parameter. Assume that $|a| < 1$ to avoid stability issues.

Formulate a least-squares estimate of a based on a batch of N data points $z_i = (y_i, u_i)$, $i = 1, \ldots, N$.

Comment on properties of your solution, e.g., with respect to normalization, as a function of the noise-to-input ratio σ^2/σ_u^2. (6 points)

Hint: Compute the expected value of the estimate. It is allowed to make the first order approximation that for random variables X and Y

$$E(\frac{X}{Y}) \approx \frac{E(X)}{E(Y)}$$