

1. The figure below shows the Magic Formula

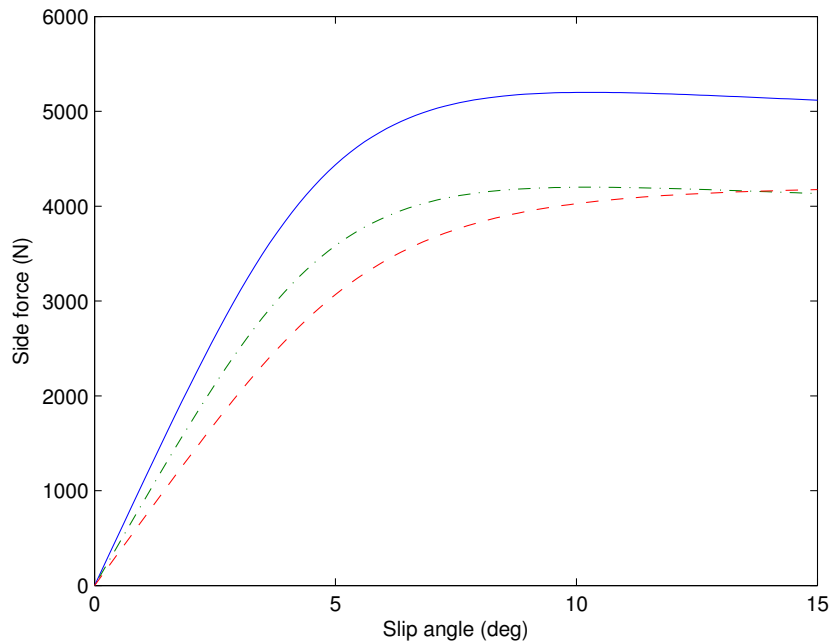
$$F_y(\alpha) = D \sin(C \arctan[B\alpha - E\{B\alpha - \arctan(B\alpha)\}])$$

plotted for three different sets of parameters

- a) B=0.16, C= 1.3, D=5200, E=-1.6
- b) B=0.15, C=1.1, D=4200, E=-1.6
- c) B=0.16, C= 1.3, D=4200, E=-1.6

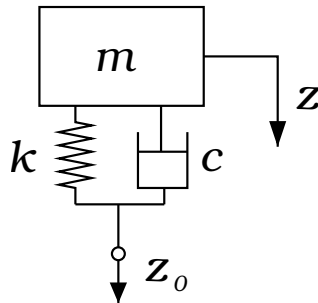
Which one is which?

Hint: You don't have to do any calculations or use a ruler to measure anything in the figure.



2. Consider a model with sprung and unsprung mass, without damping, and where  $m_s = 1200$  kg,  $m_{us} = 100$  kg,  $k_s = 50$  kN/m,  $k_{tr} = 500$  kN/m.
- a) Calculate the natural frequencies with and without the approximations in the course book.
  - b) Calculate the amplitude of the masses if the car is traveling at 50 km/h over a sinusoidal road with a wavelength of 10 m and amplitude of 5 cm.

3. In this exercise pitch and bounce motions are studied. Consider a car that weighs  $m_s = 1200$  kg, has the inertia  $I_y = 1300$  kg·m<sup>2</sup>, and has the wheelbase 230 cm. The center of gravity is 110 cm behind the front axle and the spring stiffnesses are  $k_f = 25$  kN/m front and  $k_r = 26$  kN/m rear.
- Calculate the natural frequencies.
  - Determine where the corresponding oscillation centers are located.
  - Assume that the car travels over a concrete highway with expansion joints 15 m apart. For what velocities are the bounce motion and pitch motion of the vehicle most likely to arise?
  - Assume that it is possible to move the center of gravity. Determine the location of the center of gravity such that the oscillation centers lie at the front and rear axle respectively.
  - Also determine the location of the center of gravity such that the motion consists of a vertical bouncing motion and a pitching motion with center at the center of gravity.
4. Consider a quarter-car model with a sprung mass  $m_s = 400$  kg, a spring with stiffness  $k = 30$  kN/m, and a damper with coefficient  $c = 2$  kNs/m.



The car is traveling at 70 km/h on a sinusoidal road with wavelength 15 m and amplitude of 10 mm. Determine the maximum force between tire and ground.

5. Consider a car with a rigid suspension and assume that the center of gravity of a car is located 500 mm above the ground and that the distance between the right wheels and the left wheels is 1500 mm. The car is turning fast in a sharp corner. Is there a risk that the car will roll over?

**Answers**

1. a) Solid line  
b) Dashed line  
c) Dash dotted line
2. a) 6.15 rad/s and 74.2 rad/s in both cases rounded to three significant digits.  
b) 50 mm and 41 mm
3. a) 1.03 Hz and 1.16 Hz  
b)  $-3.7$  m and 0.30 m  
c) 55 km/h and 63 km/h  
d)  $l_1 = 1.64$  m or  $l_1 = 0.66$  m  
e)  $l_1 = 1.17$  m
4. 4.47 kN.
5. No, not for reasonable values of the friction coefficient. (E.g., we can assume  $\mu \approx 1$  for dry asphalt.)