

# Vehicle Dynamics and Control

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Lecture 3

## Section 1

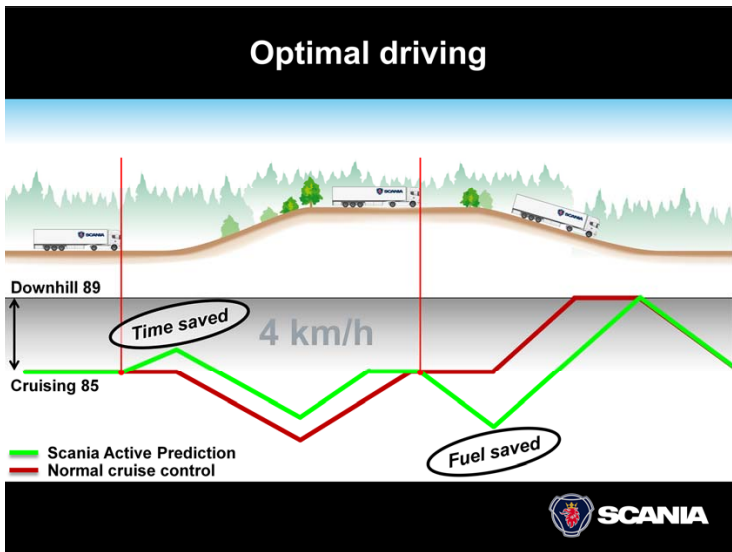
# Adaptive Cruise Control

## *“RunSmart Predictive Cruise: How it Works*

*Unlike standard cruise control, where the truck tries to maintain a set speed regardless of the terrain ahead, RunSmart Predictive Cruise looks up to one mile ahead of the truck’s location and anticipates road grades by using GPS and 3D digital map technology. The system adjusts the actual speed of the truck for maximum fuel efficiency based on the terrain while staying within 6 percent of the set speed.”*

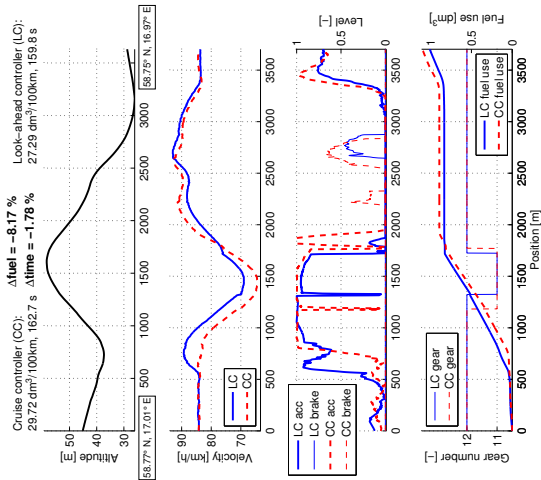
Press release from Freightliner Trucks, March 19 2009

# Longitudinal control: Cruise control



# Longitudinal control: Cruise control

Comparison between Look-ahead Controller (LC) and Conventional controller (CC)



## Some examples

- CC Cruise Control
- ACC Adaptive Cruise Control
- CA Collision avoidance
- ABS Anti-Blockier-System

Use radar or other sensors to measure the distance to other vehicles.

Control brakes and acceleration

Three different modes

- Cruise control
- Keep distance to a vehicle in front of you
- Brake to avoid collision

without preceding vehicle

maintain constant speed

with preceding vehicle

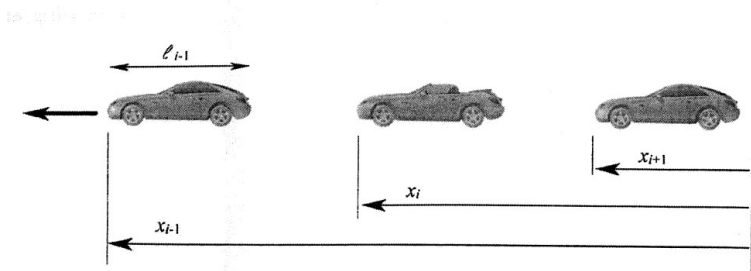
maintain safe distance





# ACC: String stability

In a long caravan with ACC in all vehicles, string stability is important



# ACC String stability

Consider a caravan where  $x_i$ ,  $i = 1, 2, \dots$  the positions of the vehicles

Define

$$\delta_i = x_i - x_{i-1} + L_{des}$$

where  $L_{des}$  is the desired distance.

A simple longitudinal model of the vehicle

$$\ddot{x}_i = u_i$$

where the acceleration  $u_i$  is the control signal

Assume that the following control strategy is used

$$u_i = -k_p \delta_i - k_v \dot{\delta}_i$$

It is possible to show that the transfer function relating the spacing errors of two consecutive vehicles is

$$G(s) = \frac{\delta_i(s)}{\delta_{i-1}(s)} = \frac{k_v s + k_p}{s^2 + k_v s + k_p}$$

The gain is

$$|G(i\omega)| = \sqrt{\frac{k_p^2 + k_v^2 \omega^2}{(k_p - \omega^2)^2 + k_v^2 \omega^2}}$$

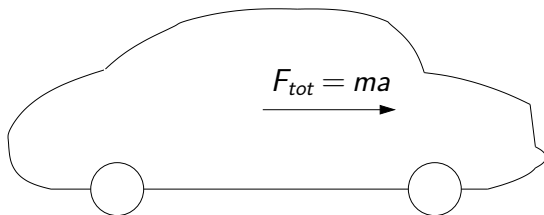
and it is straightforward to show that  $|G(i\omega)| > 1$  if  $\omega < \sqrt{2k_p}$ . This means that the amplitude of a low frequency oscillation increases when it is transferred backwards in the caravan.

Source: *Vehicle Dynamics and Control*, Rajesh Rajamani

## Section 2

# Stopping Distance

From the previous lecture:



Equation of longitudinal motion:

$$m \frac{dV}{dt} = F - R_r - R_g - R_a$$

## Position as independent variable

In the cruise control application, the grade resistance  $R_g$  is a function of position. In this and many other cases it is natural to use position as independent variable.

The right-hand side of differential then becomes

$$m \frac{dv}{dt} = m \frac{dv}{dx} \frac{dx}{dt} = mv \frac{dv}{dx} = \frac{m}{2} \frac{d(v^2)}{dx} = \frac{d(mv^2/2)}{dx}$$

and we obtain

$$\frac{m}{2} \frac{d(v^2)}{dx} = F - R_r - R_g - R_a$$

Note that the previously introduced models for  $R_r$  and  $R_a$  are linear functions of  $v^2$ .

It follows that

$$d(mv^2/2) = (F - R_r - R_a) dx - mg dh$$

where

- $d(mv^2/2)$ : change of kinetic energy.
- $(F - R_r - R_a) dx$ : work
- $mg dh = mg \sin \theta_s dx$ : change of potential energy

## Stopping distance: First case

Given an initial speed  $V$ , the objective is to determine the stopping distance  $S$ . From the previous slides:

$$\frac{m}{2} \frac{d(v^2)}{dx} = -F_b - R_r - R_g - R_a$$

Before the general case is analyzed, some special cases will be considered  
First case: Neglect all forces except  $F_b$ . In this case we get:

$$\frac{m}{2} d(v^2) = -F_b dx$$

Calculate the integrals from start to stop

$$\int_{V^2}^0 \frac{m}{2} d(v^2) = - \int_0^S F_b dx$$

Note how the intervals of the integrals were chosen!



# Stopping distance: First case

The results is

$$\frac{mV^2}{2} = F_b S$$

i.e.

Initial kinetic energy = Stopping distance  $\times$  Brake force

and

$$S = \frac{mV^2}{2F_b}$$

## Stopping distance: Second case

The second case includes the grade resistance  $mg \sin \theta_s$ :

$$\int_{V^2}^0 \frac{m}{2} d(v^2) = - \int_0^S (F_b + mg \sin \theta_s) dx$$

and the result is

$$\frac{mV^2}{2} = F_b S + mg \sin \theta_s S$$

i.e.

Initial kinetic energy = Stopping distance  $\times$  Brake force  
+ Change in potential energy

and

$$S = \frac{mV^2/2}{F_b + mg \sin \theta_s}$$

## Stopping distance: General case

$$\frac{m}{2} \frac{d(v^2)}{dx} = -F_b - mg \sin \theta_s - f_r mg \cos \theta_s - C_{ae} v^2$$

It is a separable differential equation

$$\frac{m}{2} \int_{V^2}^0 \frac{d(v^2)}{F_b + mg \sin \theta_s + f_r mg \cos \theta_s + C_{ae} v^2} = - \int_0^S dx$$

and

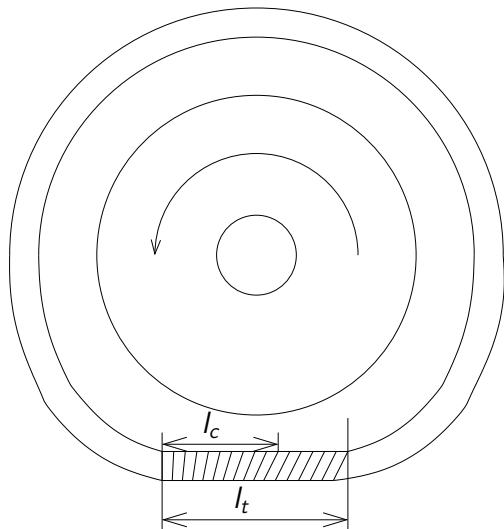
$$\begin{aligned} S &= \frac{m}{2C_{ae}} \log \left( \frac{F_b + mg \sin \theta_s + f_r mg \cos \theta_s + C_{ae} V^2}{F_b + mg \sin \theta_s + f_r mg \cos \theta_s} \right) \\ &= \frac{m}{2C_{ae}} \log \left( 1 + \frac{C_{ae} V^2}{F_b + mg \sin \theta_s + f_r mg \cos \theta_s} \right) \end{aligned}$$

## Section 3

# Tyre modelling: The Brush Model, cont'd

# Brush model

From Lecture 1



## Brush model: Normal pressure

It was assumed that the normal pressure was constant in the contact region.

According to Figure 1.15 a parabola shaped distribution seem more reasonable, i.e.,

$$\frac{dF_z}{dx} = Cx(l_t - x)$$

In the adhesion region:

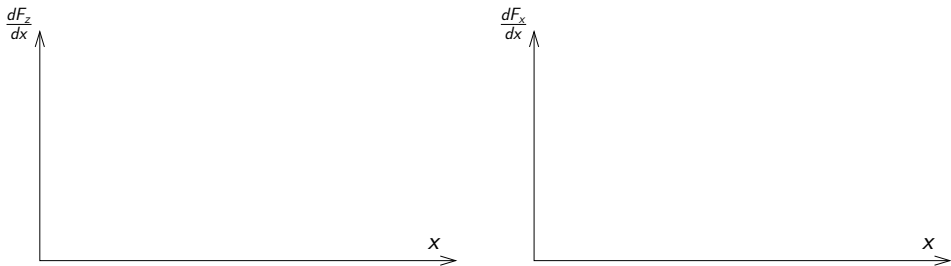
$$\frac{dF_x}{dx} = k_t \cdot i \cdot x$$

and

$$\frac{dF_x}{dx} < \mu_p \frac{dF_z}{dx}$$

# Brush model: Normal pressure

Sketch the normal and longitudinal force distributions:



The longitudinal force  $F_x$  is the area under the curve to the right.

Figure 1.16 shows the longitudinal force as a function of slip.

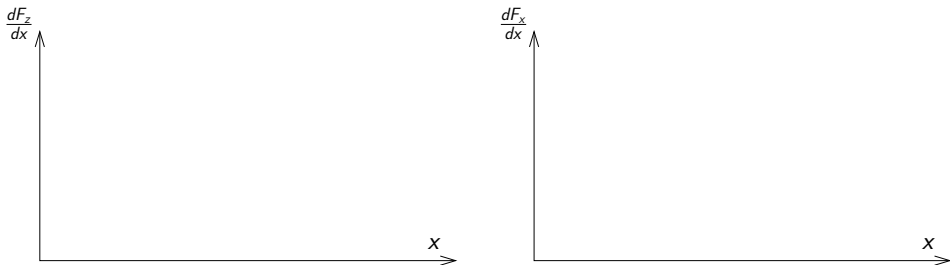
It can be seen that the force reaches a peak value and then decreases.

Assume that the sliding friction  $\mu_s$  is lower than the friction  $\mu_p$  in the adhesive region, and that the normal force distribution is constant in the contact region.



# Brush model: Sliding friction

Sketch the normal and longitudinal force distributions:



The longitudinal force  $F_x$  is the area under the curve to the right.

## Section 4

Application: Estimations of Coefficient of Friction  $\mu$

# Estimation of coefficient of friction $\mu$

The coefficient of friction has been an important part when analyzing the braking and acceleration performance. Now, one example will be presented on how to estimate the coefficient.

The approach is based on the approximation

$$\frac{F_x}{W} = K(\mu) \cdot i$$

where it is assumed that the gradient  $K$  is a function of  $\mu$ . If we first find an estimate of  $K$ , then we can calculate  $\mu$ .

To be able to calculate  $K$ , we will first estimate

- Longitudinal force  $F_x$
- Normal force  $W$
- Longitudinal slip  $i$

## Friction: Longitudinal force $F_x$

Assume that longitudinal acceleration is measured.

Longitudinal model

$$ma = F_x - R_a - R_r - R_g$$

The longitudinal force is now given by

$$F_x = m(a + g \sin \theta_s) + R_a + R_r$$

where

- $m$  is estimated mass
- $a + g \sin \theta_s$  is measured by the accelerometer
- $R_a$  and  $R_r$  is calculated using empirical models

Normal force

$$W_f = \frac{l_2}{L}mg - \frac{h}{L}(R_a + m(a + g \sin \theta_s))$$

$$W_r = \frac{l_1}{L}mg + \frac{h}{L}(R_a + m(a + g \sin \theta_s))$$

## Friction: Longitudinal slip $i$

Assume that the car is front-wheel driven.

Then there is no slip at the rear-wheel:

$$i_r = 1 - \frac{V_x}{\omega_r r_r} = 0$$

Modern cars have sensors measuring angular speed with high precision, since this information is needed by the ABS-system.

The sensors at the rear wheels can therefore be used to calculate  $V_x$  and then the slip at the front wheel can be calculated

$$i_f = 1 - \frac{V_x}{\omega_f r_f}$$

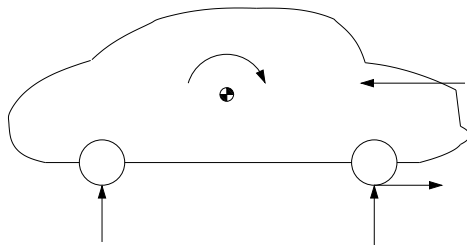
Now, the  $F_{xf}$ ,  $W_f$ , and  $i_f$  at the front wheels are known and it possible to estimate  $K$  and  $\mu$

## Section 5

# Brake Force Distribution, cont'd

# Brake force distribution

From lecture 2



$$\frac{K_{bf}}{K_{br}} = \frac{l_2 + h\mu}{l_1 - h\mu}$$

$$K_{bf} + K_{br} = 1$$



# Electronic Brake-force Distribution

In this case we get

$$W_f = \frac{1}{L}(Wl_2 + h(F_b + F_r))$$

och

$$W_r = \frac{1}{L}(Wl_1 - h(F_b + F_r))$$

If we neglect aerodynamic resistance we get

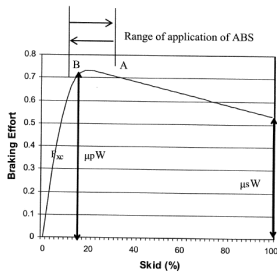
$$F_b + F_r = m(a + g \sin \theta)$$

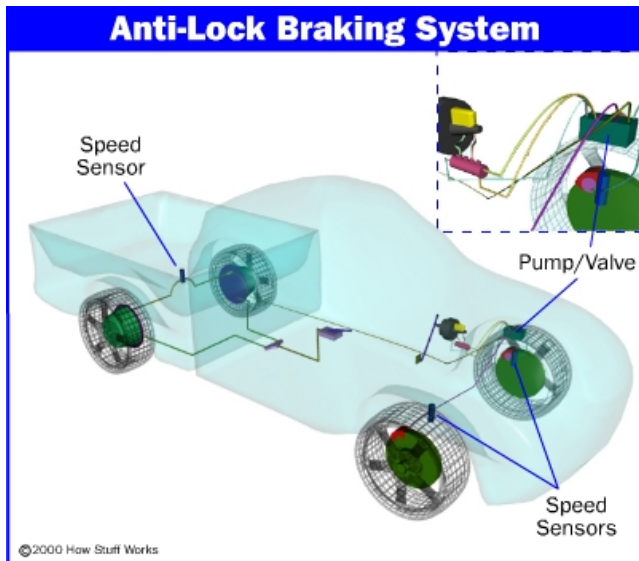
Assume that we measure the longitudinal acceleration. How can we use this information to distribute the brake forces so that all wheels start sliding at the same time, without knowing  $\mu$ ?

## Section 6

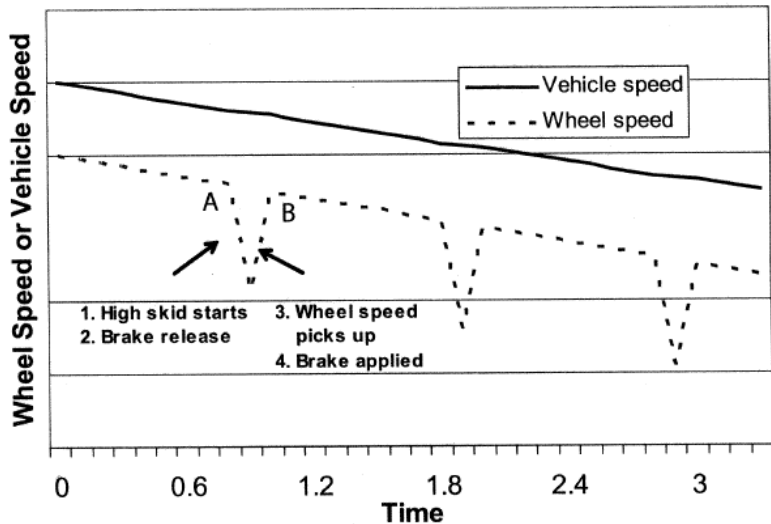
# Antilock Braking Systems (ABS)

# ABS: Introduction





# ABS: A simple control strategy



When the wheels locks you loose

- Brake force
- Stability
- Ability to control the vehicle

The objective of the ABS system is to prevent wheel lock-up

# ABS: Detecting wheel lock-up

If the wheels do not slide, then

$$\dot{\omega}r \approx a \leq \mu g$$

This can be used to detect when the wheels are locking, either by measuring the acceleration  $a$  or using an estimate of  $\mu$ .

Another option is to use the skid

$$i_s = 1 - \frac{r\omega}{V}$$

where  $V$  is estimated.

Result of experiment; slip=20%; bang-bang control

