Linköping University, ISY, Vehicular systems

Tutorial Compendium
Power Elecrtonics
TSTE25

LINKÖPINGS UNIVERSITET

Linköping 2023

Part I

Exercises

Chapter 1

Power Basics and Circuit Theory

Exercise 1.1 (3-4 in textbook)

In the current waveforms in Fig. 1.1, if A = 10 and $u = 20°$, determine the total RMS values by:

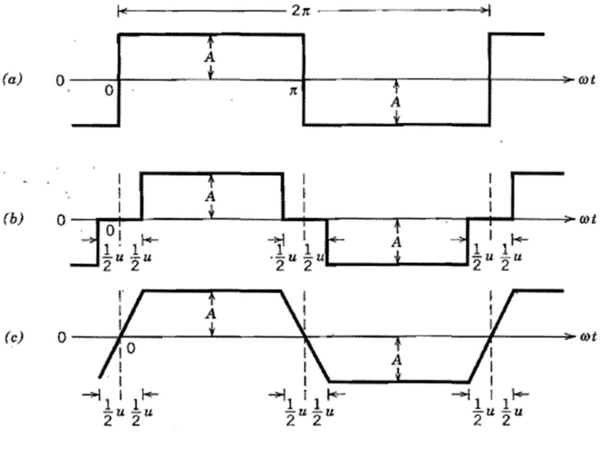

Figure 1.1: Problem 1.1

(a) inspection, observe the waveforms visually and order the waveforms based on their RMS values.

(b) using the definition of the RMS value as given by

$$I_e = \sqrt{\frac{1}{T}\int_0^T i^2(\omega t)\ d\omega t}. \tag{1.1}$$

Exercise 1.2 (1-2 in textbook)

Consider a linear regulated dc power supply (Fig. 1.2). The instantaneous input voltage corresponds to the lowest waveform in Fig. 1.2, where $V_{d,min} = 20\,\mathrm{V}$ and $V_{d,max} = 30\,\mathrm{V}$. Approximate this waveform by a triangular wave consisting of two linear segments between the above two values. Let $V_o = 15\,\mathrm{V}$ and assume that the output load is constant. Calculate the energy efficiency in this part of the power supply due to losses in the transistor.

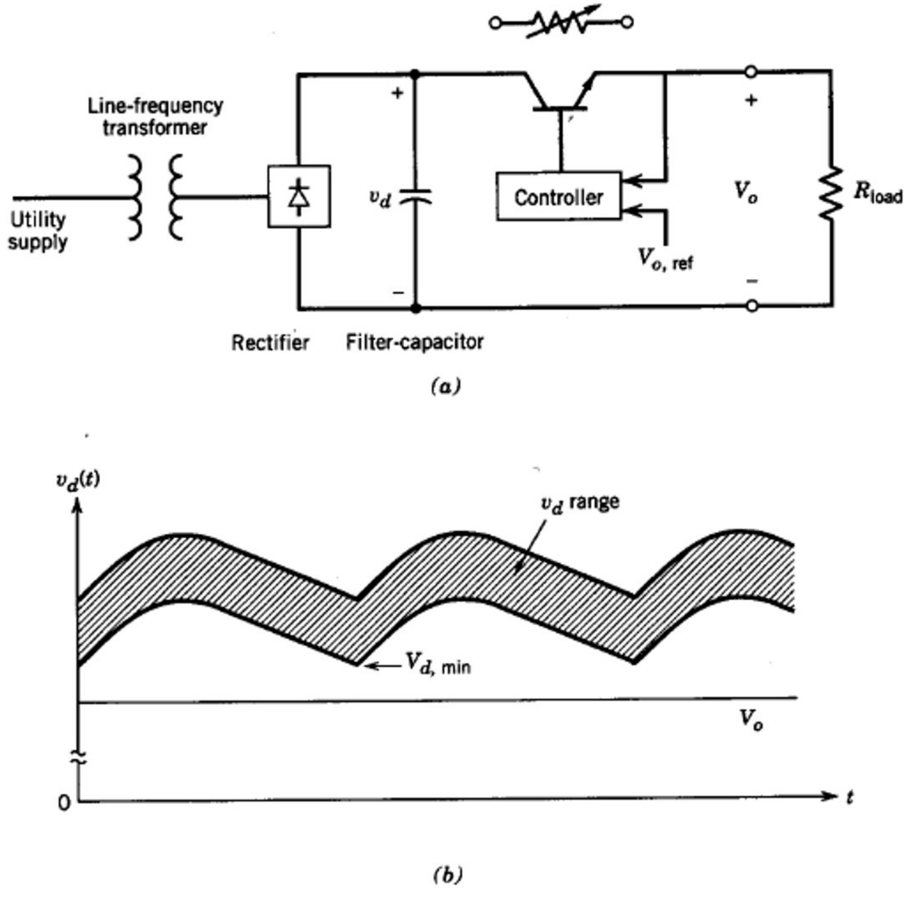

Figure 1.2: Exercise 1.2

Exercise 1.3 (3-100 in textbook)

A 50Hz ac-voltage source V_s has a voltage of $230\,\mathrm{V_{rms}}$ and is feeding a load with inductance and resistance of unknown values. The current $i(t)$ is given by (1.3). The angle is given with V_s as a reference, i.e., $\angle V_s = 0$.

(a) Express I as a complex phasor.

(b) Calculate active (P) and reactive (Q) power.

(c) What is the power factor?

(d) Determine L and R.

$$i(t) = 14.14 \cdot \cos\left(\omega t - \frac{\pi}{6}\right). \tag{1.2}$$

Exercise 1.4 (5-3 in textbook)

The voltage v across a load and the current i into the positive-polarity terminal are as follows (where ω_1 and ω_3 are not equal):

$$\begin{aligned} v(t) &= V_d + \sqrt{2}V_1\cos(\omega_1 t) + \sqrt{2}V_1\sin(\omega_1 t) + \sqrt{2}V_3\cos(\omega_3 t) && [\mathrm{V}] \\ i(t) &= I_d + \sqrt{2}I_1\cos(\omega_1 t) + \sqrt{2}I_3\cos(\omega_3 t - \phi_3) && [\mathrm{A}] \end{aligned}$$

Calculate the following

(a) The instantaneous power ($s(t)$) to the load and mark the active and reactive power components.

(b) The average power to the load.

Chapter 2

Diode converters

Exercise 2.1

Draw the circuit diagram of a half-wave rectifier with a voltage source (v_s) that has an inductance of L_s connected to a constant current load. Furthermore:

(a) Sketch the output voltage, diode current, and inductor voltage.

(b) Explain the commutation process and determine the commutation angle.

(c) Create a simple Simulink model. (This will not be asked for the exam but it can be a good exercise to get started with Matlab/Simulink.)

Exercise 2.2

Draw the circuit diagram of a full-wave (full-bridge) rectifier with a voltage source (v_s) that has an inductance of L_s connected to a constant current load. Furthermore:

(a) Sketch the output voltage, diode current, and inductor voltage.

(b) Explain the commutation process and determine the commutation angle.

(c) Create a simple Simulink model. (This will not be asked for the exam but it can be a good exercise to get started with Matlab/Simulink.)

Exercise 2.3 (5-4 in textbook)

In the single-phase diode rectifier circuit shown in Fig. 2.1 with zero L_s and a constant dc current $I_d = 10\,\text{A}$, calculate the average power supplied to the load:

(a) If v_s is sinusoidal voltage with $V_s = 120\,\text{V}$ at $60\,\text{Hz}$.<

(b) If v_x has the pulse waveform shown in Fig. P5-4.

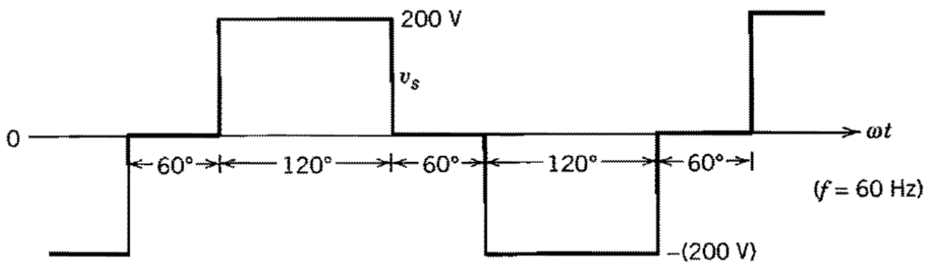

Figure P5-4

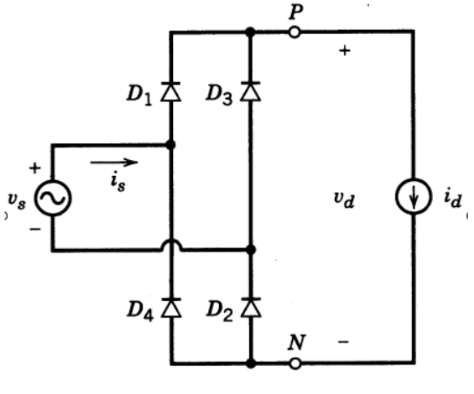

Figure 2.1: Exercise 2.1

Exercise 2.4 (5-5 in textbook)

Consider the basic commutation circuit in Fig. 2.2 with $I_d = 10\,\text{A}$.

(a) With $V_s = 120\,\text{V}$ at $60\,\text{Hz}$ and $L_s = 0\,\text{H}$, calculate V_d and the average power P_d.

(b) With $V_s = 120\,\text{V}$ at $60\,\text{Hz}$ and $L_s = 5\,\text{mH}$, calculate the commutation angle u, V_d and P_d.

(c) With the data in Exercise 2.1(b), calculate u, V_d and P_d with $I_d = 10\,\text{A}$.

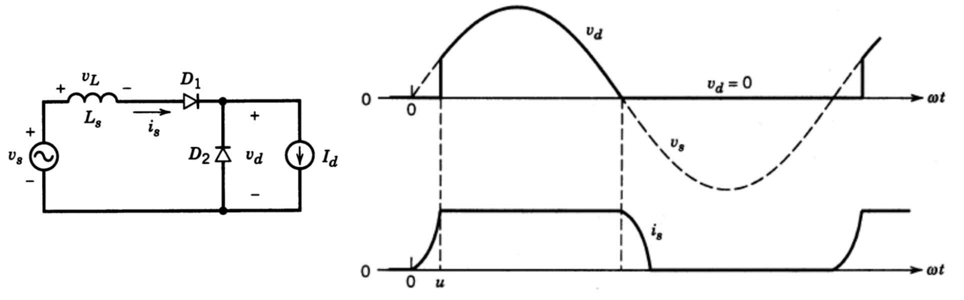

Figure 2.2: Exercise 2.2

Exercise 2.5 **(5-8 in textbook)**

In the single-phase rectifier circuit shown (same as exercise 2.3(a)), $V_s = 120\,\mathrm{V}$ at $60\,\mathrm{Hz}$, $L_s = 1\,\mathrm{mH}$, and $I_d = 10\,\mathrm{A}$.

(a) Calculate u, V_d, and P_d.

(b) What is the percentage voltage drop in V_d due to L_s?

Chapter 3

DC-DC Step-down (Buck) and Step-up (Boost) Converters

Exercise 3.1 (7-1 in textbook)

In a step-down converter, consider all components to be ideal. Let $v_o \approx V_o$ be held constant at 5 V by controlling the switch duty ratio D. Calculate the minimum inductance L required to keep the converter operation in a continuous-conduction mode under all conditions if: V_d is 10-40 V, $P_o \geq 5$ W, and $f_s = 50$ kHz.

Exercise 3.2 (7-2 in textbook)

In a step-down converter, consider all components to be ideal. Assume $V_o = 5$ V, $f_s = 20$ kHz, $L = 1$ mH, and $C = 470\,\mu$F. Calculate the peak-peak output voltage ripple (ΔV_o) if $V_d = 12.6$ V and $I_o = 200$ mA.

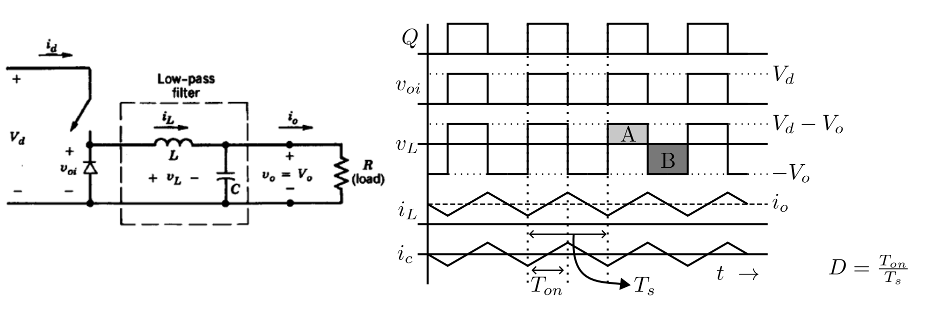

Figure 3.1: Step-down (buck) converter operation and schematic.

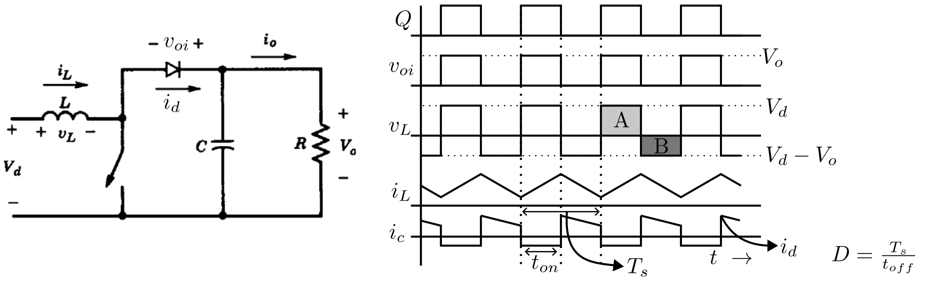

Figure 3.2: Step-uo (boost) converter operation and schematic.

Exercise 3.3 (7-7 in textbook)

In a step-up converter, consider all components to be ideal. Let V_d be between 8 to 16 V, $V_o = 24$ V (regulated), $f_s = 20$ kHz, and C = 470 μF. Calculate L_{min} that will keep the converter operating in a continuous-conduction mode if $Po \geq 5$ W.

Exercise 3.4 (7-8 in textbook)

In a step-up converter, consider all components to be ideal, $V_d = 12$ V, $V_o = 24$ V, $I_o = 0.5$ A, L = 150 μH, C = 470 μF, and $f_s = 20$ kHz. Calculate the output peak-to-peak voltage ripple (ΔV_o).

Chapter 4
DC-AC Inverters

Exercise 4.1 (7-100 in textbook)

In the half-bridge example with $U_d = 30\,\mathrm{V}$, answer the questions below. Fig. 4.1 shows switched voltage u_v and its 50 Hz component u_{v1}, inductor current i_v, and load voltage u_{ac}.

(a) What switching frequency is used?

(b) Estimate the inductance value?

(c) Estimate the peak fundamental current.

(d) Modulation index m_a.

(e) Estimate the active and reactive power on the load side. Consider the current i_v to be in phase with u_{ac}.

(f) Estimate the phase angle of the fundamental current with respect to the fundamental component of the switched converter side voltage, u_{v1}.

(g) Calculate P and Q on the converter.

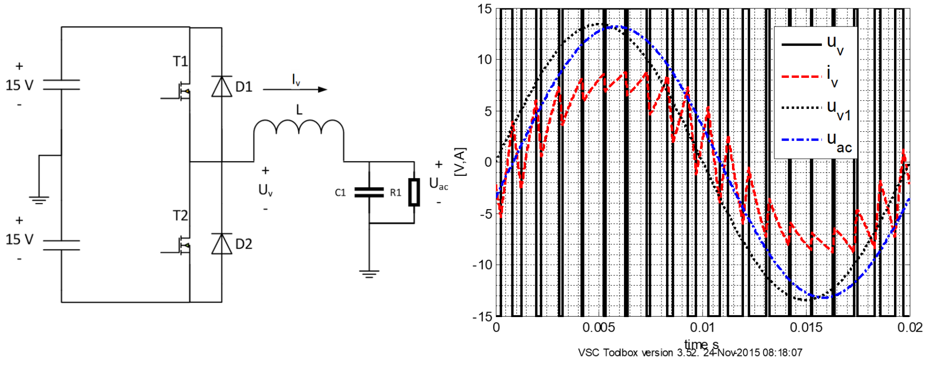

Figure 4.1: Half-bridge inverter example.

Exercise 4.2 (7-101 in textbook)

In a single-phase half-bridge PWM inverter, the input dc voltage varies in a range of 295 to 325 V. Because of the low distortion required in the output v_o, $m_a < 1$.

(a) What is the highest output voltage, $V_{o1,max}$ that can be obtained for the given input voltage range?

(b) To what values should m_a be controlled to keep the output voltage at the rated value, $V_{o1,nom}$ obtained in (a) for the given input voltage range?

(c) The nameplate volt-ampere rating is specified as 2000 VA, i.e., $V_{o1,nom} I_{o1,max} = 2000$ VA, where i_o is assumed to be sinusoidal. Calculate the peak voltage and current of the switches.

Exercise 4.3 (8-1 in textbook)

In a single-phase full-bridge PWM inverter, the input dc voltage varies in a range of 295 to 325 V. Because of the low distortion required in the output v_o, $m_a < 1$.

(a) What is the highest output voltage, $V_{o1,max}$ that can be obtained for the given input voltage range?

(b) The nameplate volt-ampere rating is specified as 2000 VA, i.e., $V_{o1,nom} I_{o1,max} = 2000$ VA, where i_o is assumed to be sinusoidal. Calculate the peak voltage and current of the switches.

(c) Compare with results for a half-bridge.

Chapter 5

MOSFET Switching, Losses and Thermal Modeling

Exercise 5.1 (5-100 in textbook)

For a step-down converter shown in Fig. 5.1, the dV_{ds}/dt of a MOSFET during turn-on is defined by V_d (assume $V_{ds(on)}$=0) V and t_{fv}. Assume $V_d = 100\,\text{V}$, $t_{fv} = 200\,\text{ns}$, the gate-drain capacitance (C_{gd}) of 120 pF, and miller plateau voltage (V_{gp}) of 4 V, calculate the gate resistance (R_g) for the gate drive with $V_{gg} = 10\,\text{V}$, which gives a dV_{ds}/dt as specified.

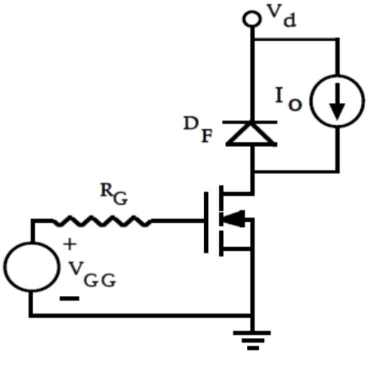

Figure 5.1: Step-down converter.

Exercise 5.2 (29-6 in textbook)

A MOSFET used in a step-down converter has an on-state loss of 50 W and a switching loss given by $10^{-3} \cdot f_s$ (in W) where f_s is the switching frequency in Hz. The junction-to-case thermal resistance ($R_{\theta jc}$) is 1 K/W and the maximum junction temperature ($T_{j,max}$) is 150°C. Assuming that the case temperature (T_c) is 50°C, estimate the maximum allowable switching frequency.

Exercise 5.3 (29-7 in textbook)

The MOSFET in Exercise 5.2 is mounted on a heat sink and the ambient temperature $T_a = 35°$C. If the switching frequency (f_s) is 25 kHz, what is the maximum allowable value of the case-to-ambient thermal resistance $R_{\theta ca}$ of the heat sink, when maximum junction temperature $T_{j,max} = 150°$. Assume all other parameters given in Exercise 5.2 remain the same except for the case temperature which can change.

Exercise 5.4 (12-101 in textbook)

A full-bridge inverter giving 50Hz output is having $I_o = 17$ A RMS. Use the datasheet of MOSFET IRF540 for thermal and electrical data.

(a) Determine the MOSFET T1 on-state losses assuming an average duty cycle of 50%.

(b) Calculate the MOSFET T1 case and junction temperature at 25°C ambient. Neglect switching losses. Consider a MOSFET without a heatsink.

(c) Calculate the required thermal resistance ($R_{\theta ca}$) of the heatsink to keep the case temperature below 80°C for losses according to (a). Neglect switching losses.

(d) What is the junction temperature in (c)?

Exercise 5.5 (12-102 in textbook)

The inverter in Exercise 5.4 is fed by $U_d = 15$ V and operated with PWM at 50 kHz switching frequency. Use the datasheet of MOSFET IRF540 for thermal and electrical data.

(a) Determine the switching losses and total losses including on-state from 12-101. Current and voltage rise/fall times are: $t_{ri} = 38$ ns, $t_{fv} = 690$ ns, $t_{rv} = 24$ ns, $t_{fi} = 32$ ns.

(b) Calculate the MOSFET case and junction temperature at 25°C ambient considering the MOSFET has no heatsink.

(c) Calculate the required thermal resistance (R_θ) of a heatsink to keep the case temperature below 80°C for losses according to (a) and (b).

(d) What is the junction temperature in (c)? Consider thermal resistance junction-case and case-sink.

Exercise 5.6 (22-13 in textbook)

A MOSFET step-down converter such as shown in Fig. 5.2 operates at a switching frequency of 30 kHz with a 50% duty cycle at an ambient temperature of 50°C. The power supply $V_d = 100\,\text{V}$ and the load current $I_o = 100\,\text{A}$. The free-wheeling diode is ideal but a stray inductance, L_p, of 100 nH is in series with the diode. The MOSFET characteristics are listed below:

- V_{ds}^{max} (Break down V_ds)$= 150\,\text{V}$,
- $T_{j,max} = 150°\text{C}$,
- $R_{\theta ja} = 1\,\text{K/W}$,
- $R_{ds(on)} = 0.01\,\Omega$,
- $t_{ri} = t_{fi} = 50\,\text{ns}$,
- $t_{rv} = t_{fv} = 200\,\text{ns}$, and
- $I_{d,max} = 125\,\text{A}$.

Is the MOSFET overstressed in this application and if so, how? Be specific and quantitative in your answer.

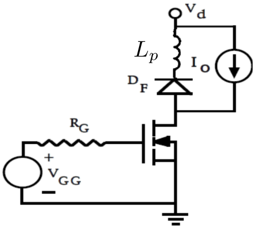

Figure 5.2: Step-down converter with stray inductance.

Chapter 6

DC-AC Inverter Harmonic Calculations

Exercise 6.1 (8-100 in textbook)

In a half-bridge converter with $U_d = 2\,\mathrm{V}$ and a filter inductor $L = 2\,\mathrm{mH}$. Switching is done with modulation index, $m_a = 0.8$ and $m_f = 5$. The instantaneous output voltage is

$$u_o(t) = 0.8\cos\left(2\pi\, 50\, t\right). \tag{6.1}$$

(a) Construct graphically (or simulate) the output voltage and current, u_v and i_v. Assume $i_v(0) = 0$.

$$u_L = L\frac{di}{dt}, \quad \Delta i_v = \frac{u_v - U_o}{L}\Delta t.$$

(b) Determine the largest harmonic current component (Use the table of harmonics in U_v. Table 6.1).

(c) Estimate the current ripple magnitude from the largest voltage harmonic.

$$\left(\hat{I}_v\right)_h = \frac{\left(\hat{U}_v\right)_h - \left(\hat{U}_o\right)_h}{h\omega L},$$

where h is the harmonic order (harmonic number, 1 - fundamental, 2 - second harmonic, m_f - switching frequency).

Exercise 6.2 (8-101 in textbook)

In a full-bridge converter, shown in Fig. 6.1, with $U_d = 2\,\mathrm{V}$ and $L = 2\,\mathrm{mH}$, PWM is done with a 50 Hz reference having m_a =0.8 and zero phase angle, $m_f = 5$, and the instantaneous output voltage $u_{ac}(t)$ is

$$u_{ac}(t) = 1 \cdot \sin(\omega t) \tag{6.2}$$

(a) Determine the RMS of the fundamental frequency components of u_v and i_v.

(b) Determine the largest harmonic current component considering both bipolar and unipolar PWM. (Use table 8-1 for harmonics in U_v)

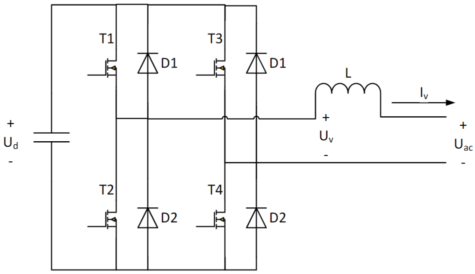

Figure 6.1: Full-bridge inverter with filter internal resistance.

Exercise 6.3 (8-102 in textbook)

In a full-bridge converter, shown in Fig. 6.2, with U_d=15 V and an inductor $L_1 =$ 2 mH with resistive losses $R_1 = 0.35\,\Omega$. Unipolar PWM is used with a 150 Hz reference having $m_a = 0.8$ and $m_f = 19$, and the load is $R = 10\,\Omega$.

(a) Determine the fundamental voltage of U_{out}.

(b) Determine the two dominating harmonic voltage of U_{out}.

(c) Estimate the THD of U_{out} based on the two dominating harmonic voltages.

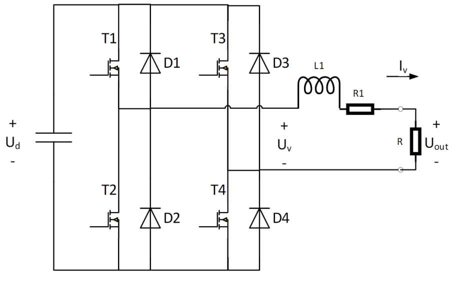

Figure 6.2: Full-bridge inverter with filter internal resistance.

Table 6.1: Generalized harmonics of a half-bridge inverter output voltage for a large m_f.

$h \downarrow \quad m_a \rightarrow$	0.2	0.4	0.6	0.8	1
1 Fundamental	0.2	0.4	0.6	0.8	1
m_f	1.242	1.15	1.006	0.818	0.6023
$m_f \pm 2$	0.061	0.061	0.131	0.22	0.318
$m_f \pm 4$					0.018
$2m_f \pm 1$	0.19	0.326	0.37	0.314	0.181
$2m_f \pm 3$		0.024	0.071	0.139	0.212
$2m_f \pm 5$				0.013	0.033
$3m_f$	0.335	0.123	0.083	0.171	0.133
$3m_f \pm 2$	0.044	0.139	0.203	0.176	0.062
$3m_f \pm 4$		0.012	0.047	0.104	0.157
$3m_f \pm 6$				0.016	0.044
$4m_f \pm 1$	0.163	0.1.57	0.088	0.105	0.068
$4m_f \pm 3$	0.012	0.070	0.132	0.115	0.009
$4m_f \pm 5$			0.034	0.084	0.119
$4m_f \pm 7$				0.017	0.05

Note: output voltage ($\hat{V}_o$) is $\hat{V}_o = m_a\, V_d/2$.

Part II

Solutions

Answers for Chapter 1

Power Basics and Circuit Theory

Exercise 1.1 (3-4 in textbook)

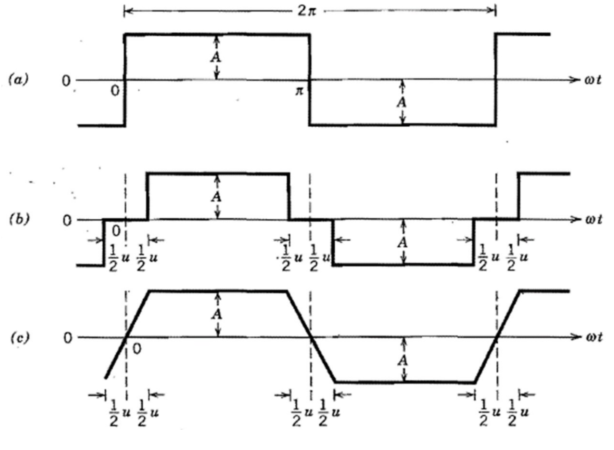

Figure 1.1: Exercise 1.1

A = 10 A, and $u = 20°$. The RMS current is defined as

$$I_e = \sqrt{\frac{1}{T}\int_0^T i^2(t)\ dt} = \sqrt{\frac{1}{\omega t}\int_0^{2\pi} i^2(\omega t)\ d\omega t.$$

(a) Due to half-wave symmetry, it is sufficient to calculate the RMS for one half-cycle.

$$\begin{aligned} I_e &= \sqrt{\frac{1}{\pi}\int_0^{\pi} \mathrm{A}^2 \; d\omega t} \\ &= \mathrm{A}\sqrt{\frac{1}{\pi}\pi} \\ &= \mathrm{A} \\ &= 10\,\mathrm{A} \end{aligned}$$

(b) Due to half-wave symmetry, it is sufficient to calculate RMS for one half-cycle.

$$\begin{aligned} I_e &= \sqrt{\frac{1}{\pi}\int_{\frac{u}{2}}^{\pi-\frac{u}{2}} \mathrm{A}^2 \; d\omega t} \\ &= \mathrm{A}\sqrt{\frac{1}{\pi}\left(\pi-u\right)} \\ &= \mathrm{A}\sqrt{\left(1-\frac{u}{\pi}\right)} \\ &= 10\sqrt{\left(1-\frac{20^\circ}{180^\circ}\right)} \\ &= 9.4\,\mathrm{A} \end{aligned}$$

(c) Due to half-wave symmetry, it is sufficient to calculate RMS for one half-cycle.

$$\begin{aligned} I_e &= \sqrt{\frac{1}{\pi}\left(2\int_0^{\frac{u}{2}} \left(\frac{\mathrm{A}\omega t}{u/2}\right)^2 \; d\omega t + \int_{\frac{u}{2}}^{\pi-\frac{u}{2}} \left(\mathrm{A}\right)^2 \; d\omega t\right)} \\ &= \mathrm{A}\sqrt{\frac{1}{\pi}\left(2\left[\frac{\left(\omega t\right)^3}{3\left(u/2\right)^2}\right]_0^{\frac{u}{2}} + \left(\pi-u\right)\right)} \\ &= \mathrm{A}\sqrt{\frac{1}{\pi}\left(\frac{u}{3}+\left(\pi-u\right)\right)} \\ &= 10\sqrt{\left(1+\frac{20^\circ}{540^\circ}-\frac{20^\circ}{180^\circ}\right)} \\ &= 9.6\,\mathrm{A} \end{aligned}$$

Exercise 1.2 (1-2 in textbook)

The output voltage and current are

$$V_o = 15\,\mathrm{V}, I_o = \text{constnat}.$$

The average output power is

$$\underline{P}_o = 15 I_o. \tag{1.1}$$

The input voltage v_{in} has the limits, $V_{d,min} = 20\,\mathrm{V}$ and $V_{d,max} = 30\,\mathrm{V}$, and $v_{in}(t)$ is shown in Fig. 1.2.

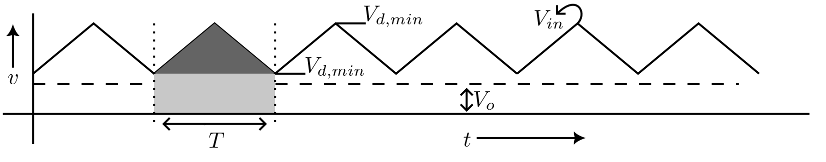

Figure 1.2: Exercise 1.2

The instantaneous input power is

$$p_{in}(t) = v_i n(t) i_i n(t) = v_i n(t) I_o$$

becuase the input current, $i_{in}(t) = I_o$.

The average input power is

$$\begin{aligned}
\underline{P}_{in} &= \frac{1}{T} I_o \int_0^T v_{in}(t)\ dt \\
&= \frac{1}{T} I_o \int_0^T \left(V_{d,min}\, t + \frac{1}{2} \left(V_{d,max} - V_{d,min} \right) t \right)\ dt,
\end{aligned}$$

where $V_{d,min} T$ is the area under the rectangle and $1/2\,(V_{d,max} - V_{d,min})\,T$ is the area of the triangle with height of $V_{d,max} - V_{d,min}$ and base (or length) T.

$$\begin{aligned}
\underline{P}_{in} &= \frac{1}{T} I_o \left[V_{d,min} T + \frac{1}{2} \left(V_{d,max} - V_{d,min} \right) T \right], \\
&= \frac{1}{T} I_o \left(V_{d,min} + V_{d,max} \right), \\
\therefore \underline{P}_{in} &= 25 I_o. \qquad (1.2)
\end{aligned}$$

From (1.1) and (1.2), the efficiency of the converter is

$$\eta = \frac{\underline{P}_o}{\underline{P}_{in}} = \frac{15 I_o}{25 I_o} = 60\%.$$

This means that about 40% of the power is dissipated as heat to the surroundings.

Exercise 1.3 **(3-100 in textbook)**

The instantaneous current through the circuit is

$$i(t) = 14.14 \cdot \cos\left(\omega t - \frac{\pi}{6} \right).$$

(a) Express I as a complex phasor.

$$I = \frac{14.14}{\sqrt{2}}\, e^{-\frac{\pi}{6}j} \quad [\mathrm{A}]$$

or as a complex number,

$$I = 8.66 - 5j \quad [\mathrm{A}]$$

(b) Calculate active (P) and reactive (Q) power.

We know that, $V_s = 230\,\mathrm{V}$ RMS or,

$$V_s = 230\, e^{0j} \quad [\mathrm{V}]$$

The total power (S) is

$$\begin{aligned} S &= V_s \,\mathrm{conj}\,(I)\,, \\ &= 230\,\mathrm{conj}\,(8.66 - 5j)\,, \\ &= 230\ (8.66 + 5j)\,, \\ &= 1991.6 + 1149.8j \quad [\mathrm{VA}] \end{aligned}$$

The active (or real) power (P) is

$$P = \mathrm{Re}\,(S) = 1991.6\,\mathrm{W}$$

The reactive (or apparent) power (Q) is

$$Q = \mathrm{Im}\,(S) = 1149.8\,\mathrm{var}$$

(c) What is the power factor?

The power factor ($\cos\phi$) is defined as the ratio of P to S.

$$\cos\phi = \frac{P}{S} = 0.87.$$

(d) Determine L and R.

The total impedance is

$$Z = \frac{V_s}{I} = \frac{230}{8.66 - 5j} = 19.92 + 11.5j \quad [\Omega].$$

The resistance (R) is

$$R = \mathrm{Re}\,(Z) = 19.92\,\Omega.$$

The inductive reactance (X) is

$$X = \mathrm{Im}\,(Z) = 11.5\,\Omega.$$

We also know that

$$X = 2\pi f L,$$

where f is the frequency and L is the inductance. Therefore,

$$\begin{aligned} 2\pi f L &= \mathrm{Im}\,(Z) \\ \implies L &= \frac{\mathrm{Im}\,(Z)}{2\pi f}. \end{aligned}$$

Assuming that $f = 50\,\mathrm{Hz}$, then

$$L = \frac{11.5}{2\pi 50} = 36.61\,\mathrm{mH}.$$

Exercise 1.4 (5-3 in textbook)

$$\begin{aligned} v(t) &= V_d + \sqrt{2}V_1\cos(\omega_1 t) + \sqrt{2}V_1\sin(\omega_1 t) + \sqrt{2}V_3\cos(\omega_3 t) && [\mathrm{V}] \\ i(t) &= I_d + \sqrt{2}I_1\cos(\omega_1 t) + \sqrt{2}I_3\cos(\omega_3 t - \phi_3) && [\mathrm{A}] \end{aligned}$$

the instantaneous power is

$$\begin{aligned} s(t) &= v(t)\cdot i(t), \\ &= \Big(V_d + \sqrt{2}V_1\cos(\omega_1 t) + \sqrt{2}V_1\sin(\omega_1 t) + \sqrt{2}V_3\cos(\omega_3 t)\Big) \\ &\quad \cdot \Big(I_d + \sqrt{2}I_1\cos(\omega_1 t) + \sqrt{2}I_3\cos(\omega_3 t - \phi_3)\Big) \\ &= V_d I_d + V_d\sqrt{2}I_1\cos(\omega_1 t) + V_d\sqrt{2}I_3\cos(\omega_3 t - \phi_3) \\ &\quad + \sqrt{2}V_1 I_d\cos(\omega_1 t) + 2V_1 I_1\cos^2(\omega_1 t) + 2V_1 I_3\cos(\omega_3 t - \phi_3)\cos(\omega_1 t) \\ &\quad + \sqrt{2}V_3 I_d + 2V_3 I_1\cos(\omega_1 t)\cos(\omega_3 t) + 2V_3 I_3\cos(\omega_3 t - \phi_3)\cos(\omega_3 t) \end{aligned}$$

Using basic trigonometric identities such as,

$$\begin{aligned} \cos A\cos B &= \frac{1}{2}\left(\cos\left(A - B\right) + \cos\left(A - B\right)\right) \\ \cos^2 A &= \frac{1}{2}\left(\cos\left(2A\right) + 1\right), \end{aligned}$$

we get,

$$\begin{aligned} s(t) = {} & V_d I_d + V_d\sqrt{2}I_1\cos(\omega_1 t) + V_d\sqrt{2}I_3\cos(\omega_3 t - \phi_3) \\ & + \sqrt{2}V_1 I_d\cos(\omega_1 t) + V_1 I_1\left(\cos(2\omega_1 t) + 1\right) \\ & \quad + V_1 I_3\left(\cos(\omega_3 t - \omega_1 t - \phi_3) + \cos(\omega_3 t + \omega_1 t - \phi_3)\right) \\ & + \sqrt{2}V_3 I_d\cos(\omega_3 t) + V_3 I_1\left(\cos(\omega_1 t - \omega_3 t) + \cos(\omega_1 t + \omega_3 t)\right) \\ & \quad + V_3 I_3\left(\cos\left(-\phi_3\right) + \cos\left(2\omega_3 t - \phi_3\right)\right) \end{aligned}$$

Rearranging

$$\begin{aligned}
s(t) = {} & V_d I_d + V_1 I_1 + V_3 I_3 \cos(\phi_3) && \text{DC component} \\
& + V_d \sqrt{2} I_1 \cos(\omega_1 t) + \sqrt{2} V_1 I_d \cos(\omega_1 t) && \omega_1 \text{ component} \\
& + V_1 I_1 \cos(2\omega_1 t) && 2\omega_1 \text{ component} \\
& + V_d \sqrt{2} I_3 \cos(\omega_3 t - \phi_3) + \sqrt{2} V_3 I_d \cos(\omega_3 t) && \omega_3 \text{ component} \\
& + V_1 I_3 \cos(\omega_3 t - \omega_1 t - \phi_3) + V_3 I_1 \cos(\omega_1 t - \omega_3 t) && \omega_3 - \omega_1 \text{ component} \\
& + V_1 I_3 \cos(\omega_3 t - \omega_1 t - \phi_3) + V_3 I_1 \cos(\omega_1 t + \omega_3 t) && \omega_3 + \omega_1 \text{ component} \\
& + \cos(2\omega_3 t - \phi_3) && 2\omega_3 \text{ component}
\end{aligned}$$

(a) The real or average power P is

$$P = V_d I_d + V_1 I_1 + V_3 I_3 \cos(\phi_3)$$

(b) The active (or real) power component is the DC component and the reactive power is everything else.

Answers for Chapter 2

Diode converters

Exercise 2.1

The half-wave rectifier with an AC voltage source, V_s, and source inductance, L_s, connected to a constant current load with current (I_d) with a is shown in Figure 2.1(a).

(a) To sketch the waveforms, the equivalent circuits when the diode D_1 is 'on' and 'off' are shown in Figures 2.1(b), (c) and (d). Figures 2.1(b) and (c) are the equivalent circuits D_1 is in conduction (tuned-on), and Figure 2.1(d) is the equivalent circuit when D_1 is not in conduction. Note that the diode D_2 provides a path for the current to flow when D_1 is not in conduction, this is known as free-wheeling.

From Figure 2.1(b), it is clear that the output voltage $v_d = v_s$ and the current though the source, $i_s = I_d$, and from Figure 2.1(d), $v_d = 0\,\mathrm{V}$ and the current though the source, $i_s = 0\,\mathrm{A}$. However, due to L_s, $i_s(t)$ cannot change instantaneously (i.e., it will not have a square wave shape). The equivalent circuit when D_1 starts conduction is shown in Figure 2.1(c), i_d slowly starts increasing towards I_d, i_{d2} starts decreasing towards $0\,\mathrm{A}$ and $v_d = 0\,\mathrm{V}$ since D_2 is in conduction. At $\omega t = u$, $i_s = I_d$, and $v_d = v_s$ as in Figure 2.1(b). The waveforms are given in Figure 2.2.

(b) Commutation is the periodic reversal of the current direction.

To determine the commutation angle, we first apply Kirchhoff's voltage law to the i_s loop in Figure 2.1(c),

$$v_s(t) = v_L(t) + v_d(t), \qquad \Longrightarrow v_s(t) = v_L(t),$$

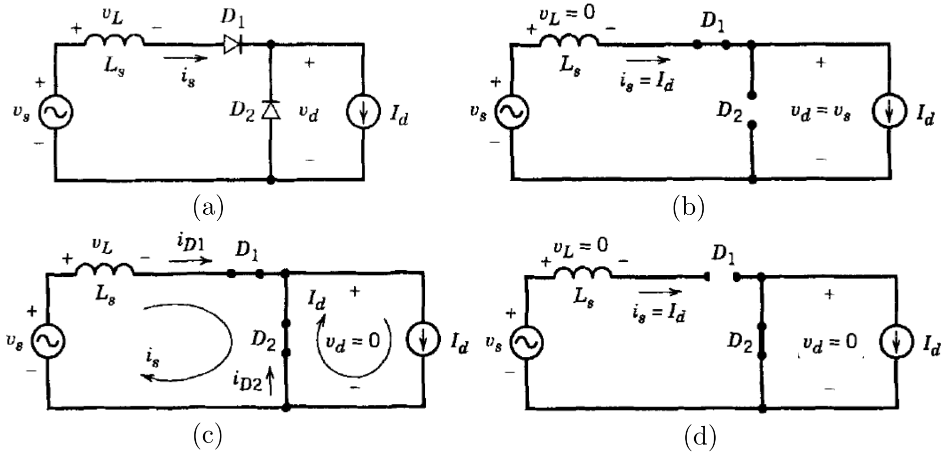

Figure 2.1: Half-wave rectifier equivalent circuit diagram.

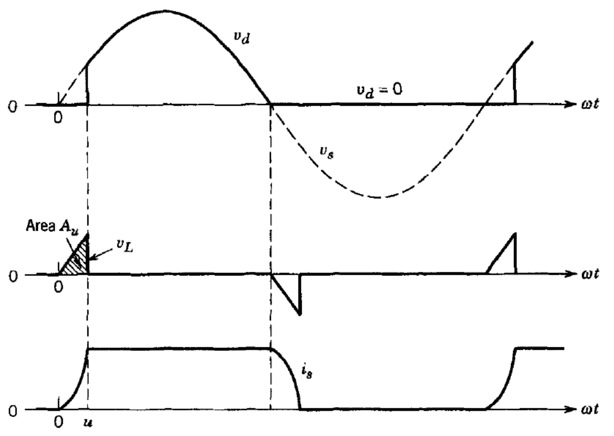

Figure 2.2: Half-wave rectifier waveforms.

since $v_d = 0\,\mathrm{V}$. The voltage drop across the inductor is

$$v_L(t) = L_s \frac{di_s(t)}{dt}. \tag{2.1}$$

Assuming a pure sinusoidal input voltage,

$$v_L(t) = \hat{v}_s \sin(\omega t). \tag{2.2}$$

From (2.1) and (2.2), we get

$$\hat{v}_s \sin(\omega t) = L_s \frac{di_s(t)}{dt}, \qquad \Longrightarrow \hat{v}_s \sin(\omega t)\, dt = L_s di_s(t),$$

$$\Longrightarrow \hat{v}_s \sin(\omega t)\, d(\omega t) = \omega L_s di_s(t), \qquad \Longrightarrow \int_0^u \hat{v}_s \sin(\omega t)\, d(\omega t) = \omega L_s \int_0^{I_d} di_s,$$

$$\Longrightarrow -\hat{v}_s \left[\cos(\omega t)\right]_0^u = \omega L_s I_d, \qquad \Longrightarrow \hat{v}_s (1 - \cos(u)) = \omega L_s I_d$$

$$\text{i.e., } \cos(u) = 1 - \frac{\omega L_s I_d}{\hat{v}_s} \qquad \text{(or) } \cos(u) = 1 - \frac{\omega L_s I_d}{\sqrt{2} V_s},$$

where $\hat{v}_s$ and V_s are the peak and RMS values of the input voltage (v_s).

Exercise 2.2

The full-wave (full-bridge) rectifier with an AC voltage source, V_s, and source inductance, L_s, connected to a constant current load with current (I_d) with a is shown in Figure 2.3(a).

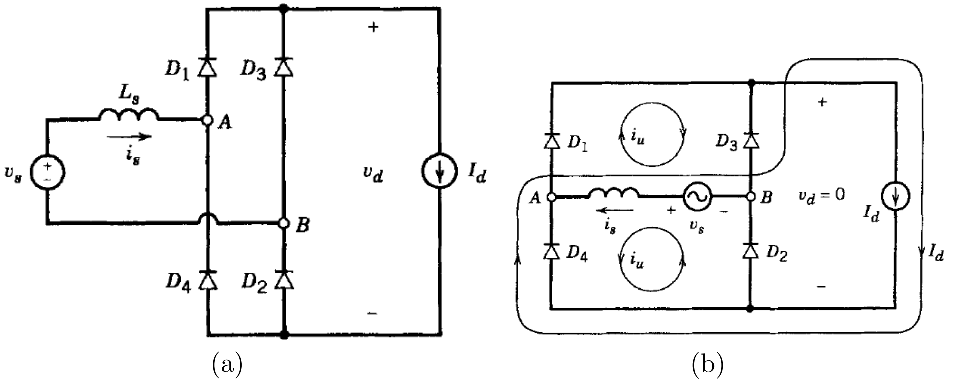

Figure 2.3: Full-wave rectifier.

(a) In Figure 2.3(a), during the positive half-cycle of the input voltage, v_s, D_1 and D_2 are in conduction. Therefore, the output voltage, $v_d = v_s$.

During the negative half-cycle of the input voltage, v_s, D_3 and D_4 are in conduction. Therefore, the output voltage, $v_d = -v_s$.

Atr $t = 0$, due to L_s, the current, i_s, does not change instantaneously. i_s slowly starts increasing towards I_d and here $v_d = 0\,\text{V}$ since D_3 and D_4 are still in conduction (see Figure 2.3(b)). At $\omega t = u$, $i_s = I_d$, and $v_d = v_s$. The vice-versa happens when D_3 and D_4 start to conduct at $\omega t = \pi$.

The waveforms are given in Figure 2.4.

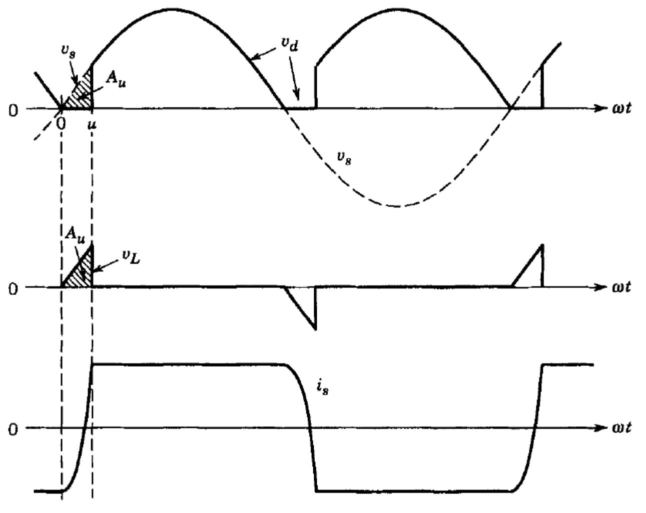

Figure 2.4: Full-wave rectifier waveforms.

(b) Commutation is the periodic reversal of the current direction.

To determine the commutation angle, we first apply Kirchhoff's voltage law to the i_s loop in Figure 2.3(b),

$$v_s(t) = v_L(t) + v_d(t), \qquad \Longrightarrow v_s(t) = v_L(t),$$

since $v_d = 0\,\text{V}$. The voltage drop across the inductor is

$$v_L(t) = L_s \frac{di_s(t)}{dt}. \tag{2.3}$$

Assuming a pure sinusoidal input voltage,

$$v_L(t) = \hat{v}_s \sin(\omega t). \tag{2.4}$$

From (2.3) and (2.4), we get

$$\hat{v}_s \sin(\omega t) = L_s \frac{di_s(t)}{dt}, \qquad \Longrightarrow \hat{v}_s \sin(\omega t)\, dt = L_s di_s(t),$$

$$\Longrightarrow \hat{v}_s \sin(\omega t)\, d(\omega t) = \omega L_s di_s(t), \qquad \Longrightarrow \int_0^u \hat{v}_s \sin(\omega t)\, d(\omega t) = \omega L_s \int_{-I_d}^{I_d} di_s,$$

$$\Longrightarrow -\hat{v}_s \left[\cos(\omega t)\right]_0^u = 2\omega L_s I_d, \qquad \Longrightarrow \hat{v}_s\left(1 - \cos(u)\right) = 2\omega L_s I_d$$

$$\text{i.e.,}\ \cos(u) = 1 - \frac{2\omega L_s I_d}{\hat{v}_s} \qquad (\text{or})\ \cos(u) = 1 - \frac{2\omega L_s I_d}{\sqrt{2}V_s},$$

where $\hat{v}_s$ and V_s are the peak and RMS values of the input voltage (v_s).

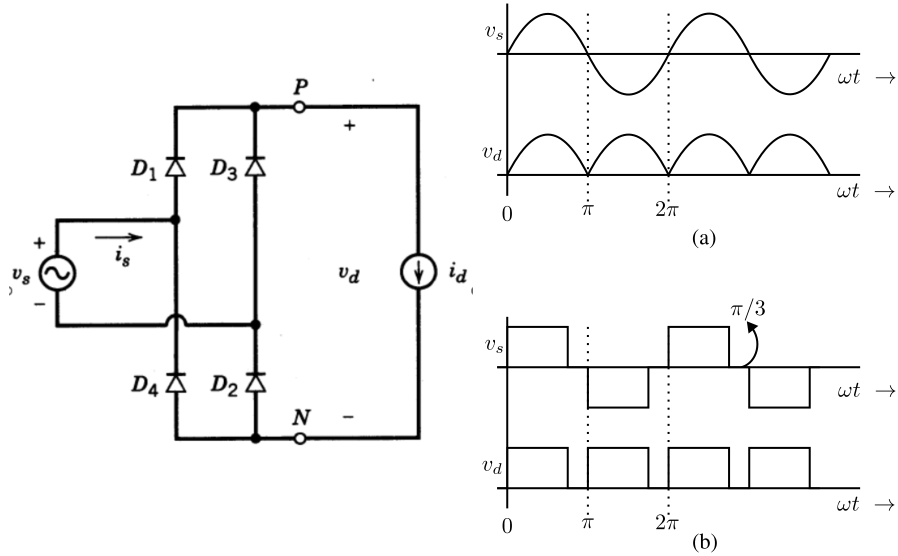

Figure 2.5: Exercise 2.3

Exercise 2.3 (5-4 in textbook)

(a) If v_s is sinusoidal and $V_s = 120\,\text{V}$, i.e.,

$$v_s(\omega t) = \sqrt{2}V_s \sin(\omega t).$$

The average output voltage, V_d (assuming a half-wave symmetry as shown in Fig- 2.5(a)) is

$$V_d = \frac{1}{\pi}\int_0^{\pi} \sqrt{2}V_s \sin(\omega t)\; d\omega t = 0.9V_s = 108\,\text{V}.$$

(b) If v_s is as shown in Fig- 2.5(b), and peak input voltage $\hat{V}_s = 200\,\text{V}$, then

$$V_d = \frac{1}{\pi}\int_0^{\frac{2\pi}{3}} \hat{V}_s\; d\omega t = \hat{V}_s\,\frac{2}{3} = 133.33\,\text{V}.$$

Exercise 2.4 (5-5 in textbook)

(a) The average output voltage, V_d ($v_d(t)$ as shown in Fig- 2.6) is

$$V_d = \frac{1}{\pi}\int_0^{\pi} \sqrt{2}V_s \sin(\omega t)\; d\omega t = 0.9\frac{V_s}{2} = 54\,\text{V}.$$

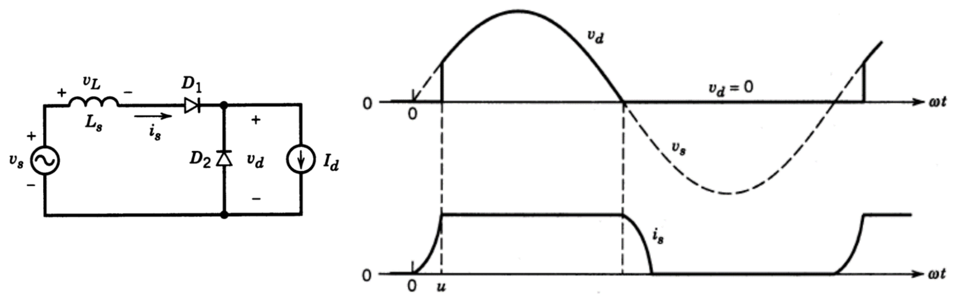

Figure 2.6: Exercise 2.4

The average output power P_d is

$$P_d = V_d I_d = 540\,\mathrm{W}.$$

(b) The inductor L_s of 5 mH forces v_d to 0 V for a period of u, called as the commutation period. During this period both diodes are turned on (or in forward mode). The derivation of the formula for the commutation angle is presented in section 5-3-2 in the course book and using (5-22) in the course book, the commutation angle is

$$\cos u = 1 - \frac{2\pi f L_s I_d}{\sqrt{2} V_s} \implies u = 27.26°.$$

V_d is calculated using (5-26) in the course book (section 5-3-2),

$$V_d = \frac{0.9}{2} V_s - \frac{2\pi f L_s}{2\pi} I_d = 51\,\mathrm{V}.$$

The average power is

$$P_d = V_d I_d = 510\,\mathrm{W}.$$

(c) Fig. 2.7 shows the output voltage $v_d(t)$ considering $v_s(t)$ in problem 2.1.

The average output voltage with $L_s = 0\,\mathrm{mH}$ ($\hat{V}_s = 200\,\mathrm{V}$ see Problem 2.2(b)) is

$$V_{d0} = \frac{1}{2\pi} \int_0^{2\pi} v_{d0}(\omega t)\ d\omega t = \frac{1}{2\pi} \int_0^{\pi} \hat{V}_s\ d\omega t = \hat{V}_s \frac{1}{3} = 66.67\,\mathrm{V}.$$

The region A_u (in Fig. 2.7) is the product of the voltage drop across the inductor ($L_s = 5\,\mathrm{mH}$) and u, i.e.,

$$A_u = \int_0^u \hat{V}_s\ d\omega t = 2\pi f L_s I_d = 18.85\,\mathrm{V}.$$

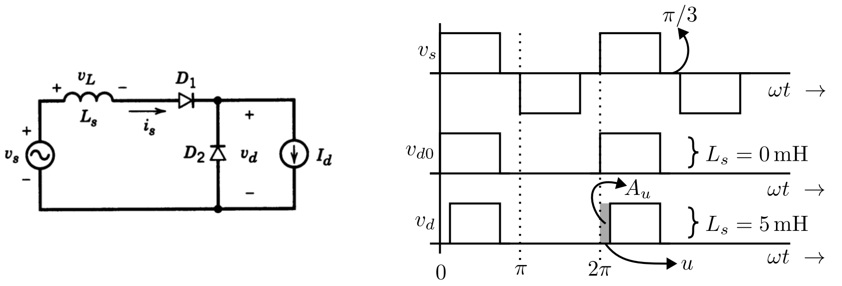

Figure 2.7: Problem 2.2(c)

The average output voltage with $L_s = 5\,\mathrm{mH}$, is

$$V_d = V_{d0} - \frac{A_u}{2\pi} = 63.67\,\mathrm{V}.$$

The average power P_d is

$$P_d = V_d I_d = 636.7\,\mathrm{W}.$$

Exercise 2.5 (5-8 in textbook)

(a) Using (5-32) in the course book (Section 5-3-2), the commutation angle u is

$$\cos u = 1 - \frac{2\omega L_s}{\sqrt{2}V_s} I_d \qquad \Longrightarrow \quad u = 17.14°.$$

The average output voltage V_d, using (5-33) in the course book (Section 5-3-2) is

$$V_d = 0.9V_s - \frac{2\omega L_s}{\pi} I_d = 105.6\,\mathrm{V}.$$

The average power P_d is

$$P_d = V_d I_d = 1056\,\mathrm{W}.$$

(b) The average output voltage with $L_s = 0\,\mathrm{mH}$ is

$$V_{d0} = 0.9V_s = 108\,\mathrm{V}.$$

The percentage voltage drop in V_d (ΔV_d) due to the inductor $L_s = 5\,\mathrm{mH}$ is

$$\Delta V_d = \frac{V_{d0} - V_d}{V_{d0}} \times 100 = 2.22\%.$$

Answers for Chapter 3

DC-DC Step-down (Buck) and Step-up (Boost) Converters

Exercise 3.1 (7-1 in textbook)

For a given load and output voltage, the likelihood that the inductor current will fall to zero is increased by lowering the duty ratio and increasing the off time. The duty ratio is lowest when $V_d = 40\,\text{V}$.

The output current is

$$I_o = \frac{P_o}{V_o} = \frac{5}{5} = 1\,textA.$$

The duty ratio D is

$$D = \frac{5}{40} = 0.125.$$

For continuous conduction, from (7-5) in the course book (in Section 7-3-2),

$$I_o \geq \frac{D}{2 f_s L}\,(V_d - V_o) \quad \Longrightarrow \; L \geq \frac{D}{2 f_s I_o}\,(V_d - V_o)$$

$$L = \frac{0.125}{2 \cdot 50000 \cdot 1}\,(40 - 5) = 43.75\,\mu\text{H}.$$

Exercise 3.2 (7-2 in textbook)

The duty ratio is

$$D = \frac{V_o}{V_d} = \frac{5}{12.6} = 0.397.$$

Is the circuit in the continuous conduction mode?

Using (7-5) in the course book, the boundary current (I_{oB}) is

$$I_{oB} = \frac{D}{2f_sL}\left(V_d - V_o\right) = 75.4\,\text{mA}.$$

Yes, it is in continuous conduction mode because $I_{oB} \leq I_o$.

In continuous conduction mode, the output voltage peak-peak ripple (using 7-24 in Section 7-3-4 in the course book), ΔV_o is

$$\Delta V_o = \frac{(1-D)\,V_o}{8f_s^2LC} = \frac{(1-0.397)\cdot 5}{8\cdot(2000)^2\cdot 0.001\cdot 470\cdot 10^{-6}} = 2.01\,mV.$$

General Comment

When calculating the ripple voltage across the filter capacitor, in practice, the equivalent series resistance (ESR) of the capacitor causes a significant portion of the overall ripple voltage. Therefore, The ESR must be included in the ripple voltage calculations.

Exercise 3.3 **(7-7 in textbook)**

the minimum output current (I_o) is

$$I_o = \frac{P_o}{V_o} = \frac{5}{24} = 0.21\,\text{A}.$$

Case 1: Let $V_d = 8\,V$, the the duty ratio, D, is given by

$$\frac{V_o}{V_d} = \frac{1}{1-D} \qquad \Longrightarrow D = 1 - \frac{V_d}{V_o} = 1 - \frac{8}{24} = 0.67.$$

The minimum inductance L_{min} required to keep the converter in continuous conduction mode (using (7-29) from Section 7-4-2 in course book) is

$$L_{min} = \frac{V_o}{2f_sI_o}D\,(1-D)^2 = \frac{24}{2\cdot 20000\cdot 0.21}\cdot 0.67\cdot(1-0.67)^2 = 0.213\,\text{mH}.$$

Case 2: Let $V_d = 16\,V$, the the duty ratio, D, is given by

$$\frac{V_o}{V_d} = \frac{1}{1-D} \qquad \Longrightarrow D = 1 - \frac{V_d}{V_o} = 1 - \frac{16}{24} = 0.33.$$

The minimum inductance L_{min} required to keep the converter in continuous conduction mode (using (7-29) from Section 7-4-2 in course book) is

$$L_{min} = \frac{V_o}{2f_sI_o}D\,(1-D)^2 = \frac{24}{2\cdot 20000\cdot 0.21}\cdot 0.33\cdot(1-0.33)^2 = 0.427\,\text{mH}.$$

The minimum inductance required to ensure that the step-up converter is in continuous conduction mode for input voltages between 8 to 16 V is

$\max\left[L_{min} : \text{Case 1},\ L_{min} : \text{Case 2}\right]$, i.e., $L_{min} = 0.427\,\text{mH}$.

Exercise 3.4 (7-8 in textbook)

Assuming that the converter is in continuous conduction mode, then the duty ratio D is

$$D = 1 - \frac{V_d}{V_o} = 1 - \frac{12}{24} = 0.5.$$

The boundary output current I_{oB}, using (7-29) in the course book from Section 7-4-2, is

$$I_{oB} = \frac{V_o}{2 f_s L} D \left(1 - D\right)^2 = \frac{24}{2 \cdot \left(20000\right)^2 \cdot 150 \cdot 10^{-6}} \cdot 0.5 \cdot (1 - 0.5) = 0.5\,\mathrm{A}.$$

The converter is operating at the boundary of continuous conduction mode since $I_o B = I_o = 0.5\,\mathrm{A}$.

The current through the diode (i_d) at the boundary of continuous conduction mode is shown in Fig. 3.1.

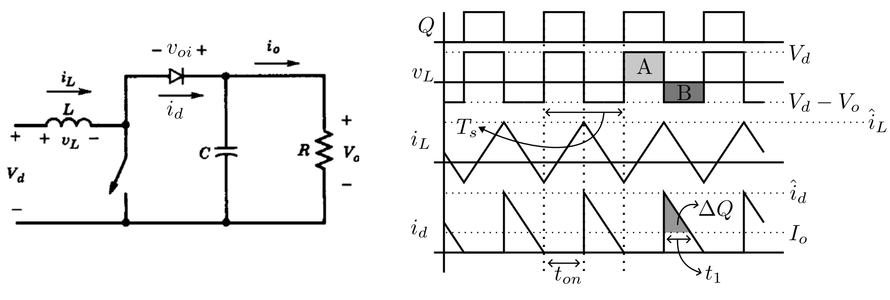

Figure 3.1: Step-down (buck) converter operation and schematic at the boundary of continuous conduction mode.

The peak current through the diode ($\hat{i}_d$) is the same as the peak current through the inductor ($\hat{i}_L$), i.e.,

$$\hat{i}_d = \hat{i}_L = \frac{V_d}{L} t_{on} = \frac{V_d}{L} D T_s = \frac{12}{150 \cdot 10^{-6}} \cdot \frac{0.5}{20000} = 2\,\mathrm{A}.$$

during the off period, i_d follows i_L, and

$$\frac{di_d}{dt} = \frac{V_d - V_o}{L} = \frac{12 - 24}{150 \cdot 10^{-6}} = -80000\,\mathrm{A/s}.$$

We know that,

$$-\frac{di_d}{dt} = \frac{\hat{i}_d - I_o}{t_1}. \quad \therefore, t_1 = \frac{\hat{i}_d - I_o}{-\frac{di_d}{dt}} = \frac{2 - 0.5}{80000} = 18.75\,\mu\mathrm{s}.$$

The peak-to-peak output voltage ripple is

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{\frac{1}{2}\left(\hat{i}_d - I_o\right) t_1}{C} = \frac{\frac{1}{2}\left(2 - 0.5\right) \cdot 18.75 \cdot 10^{-6}}{470 \cdot 10^{-6}} = 29.92\,\mathrm{mV}.$$

Note

The expression for ΔV_o given by (7-39) and (7-40) in the course book are valid only if the minimum value of the inductor current is greater than or equal to the average output current (i.e., $i_{L(min)} \geq I_o$) in the continuous condition mode of operation (as shown in Fig. 7.17a in the course book.)

Answers for Chapter 4

DC-AC Inverters

Exercise 4.1 (7-100 in textbook)

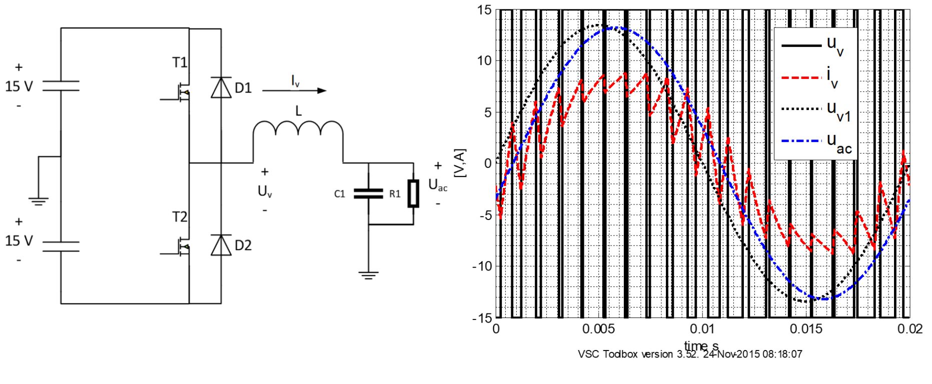

Figure 4.1: Half-bridge inverter example.

The half-bridge converter is shown in Fig. 4.1.

(a) What switching frequency is used

From the figure, it is clear that the fundamental period, $T_1 = 0.02\,\mathrm{s}$ and there are 19 switching pulses in the fundamental period, i.e.,

$$m_f = \frac{f_s}{f_1} = 19 \qquad \Longrightarrow \quad f_s = m_f f_1 = 19 \cdot 50 = 950\,\mathrm{Hz}.$$

(b) Estimate the inductance value

Around $t = 0.005\,\mathrm{s}$, the duty cycle (or duty ratio) of the pulses is almost 100%, and during this time, the voltage across the inductor is

$$v_L = L\frac{di}{dt}, \quad i.e., \; u_v - \hat{u}_{v1} = L\frac{\Delta i_v}{T_s}$$
$$\Longrightarrow L = \frac{u_v - \hat{u}_{v1}}{\Delta i_v f_s} = \frac{15 - 13.5}{(9-7)\cdot 950} = 0.8\,\mathrm{mH} \approx 1\,\mathrm{mH}.$$

(c) Estimate the peak fundamental current

Around $t = 0.005\,\mathrm{s}$, the output current reaches its peak and the average current during the switching period is $(9+7)/2$, which is $8\,\mathrm{A}$.

(d) Modulation index m_a

The peak fundamental voltage $\hat{u}_{v1}$ is $13.5\,\mathrm{V}$, and the pole-to-ground DC voltage (V_d) is $15\,\mathrm{V}$. Therefore, m_a is

$$m_a = \frac{\hat{u}_{v1}}{V_d} = \frac{13.5}{15} = 0.9.$$

(e) Estimate the active and reactive power on the load side. Consider the current i_v to be in phase with u_{ac}

From the figure, it is clear that i_v and u_{ac} are in phase. Therefore, the power factor $\cos\phi = 1$, and the active (P_{ac}) and reactive (Q_{ac}) powers are

$$P_{ac} = \frac{\hat{u}_{ac}\hat{i}_v}{2}\cos\phi = \frac{13.2\cdot 8}{2}\cdot 1 = 52\,\mathrm{W}.$$
$$Q_{ac} = \frac{\hat{u}_{ac}\hat{i}_v}{2}\sin\phi = \frac{13.2\cdot 8}{2}\cdot 0 = 0\,\mathrm{var}.$$

Estimate the phase angle of the fundamental current with respect to the fundamental component of the switched converter side voltage, u_{v1}

From the figure, the time delay between i_v, and u_v is about $0.004\,\mathrm{s}$. The phase angle $\phi_v$is

$$\phi_v = 0.004\cdot 2\pi/0.02 \approx 15^\circ.$$

Calculate P and Q on the converter.

The active (P_v) and reactive (Q_v) on the converter is

$$P_v = \frac{\hat{u}_{ac}\hat{i}_v}{2}\cos\phi_v = \frac{13.2\cdot 8}{2}\cdot 0.97 = 52\,\mathrm{W}.$$
$$Q_v = \frac{\hat{u}_{ac}\hat{i}_v}{2}\sin\phi_v = \frac{13.2\cdot 8}{2}\cdot 0.26 = 14\,\mathrm{var}.$$

Exercise 4.2 (7-101 in textbook)

(a) Using (8-7) from the course book, the output voltage rating is defined by the minimum value of the dc pole-to-pole voltage (V_d^{min}) and the maximum modulation index m_a^{max}, i.e.,

$$V_{o1,rms} = \frac{V_d^{min}}{2\sqrt{2}}m_a^{max} = \frac{295}{2\cdot\sqrt{2}}\cdot 1 = 104\,\mathrm{V}.$$

(b) When V_d is the maximum value, m_a must be reeduced to maintain constant output RMS voltage ($V_{o1,rms}$) of 104 V, i.e.,

$$m_a^{min} = 2\sqrt{2}\frac{V_{o1,rms}}{V_d^{max}} = 2\cdot\sqrt{2}\cdot\frac{104}{325} = 0.91.$$

(c) The RMS output current ($I_{o1,rms}$) is

$$I_{o1,rms} = \frac{S}{V_{o1,rms}} = \frac{2000}{104} = 19.2\,\mathrm{A}.$$

The peak output current ($\hat{I}_{o1} = \sqrt{2}I_{o1,rms}$) is 27.2 A.

Peak voltage of the switch, $V_T = V_d^{max} = 325\,\mathrm{V}$.

Peak current of the switch, $I_T = \hat{I}_{o1} = 27.2\,\mathrm{A}$.

Exercise 4.3 (8-1 in textbook)

(a) Using (8-19) from the course book, the output voltage rating is defined by the minimum value of the dc pole-to-pole voltage (V_d^{min}) and the maximum modulation index m_a^{max}, i.e.,

$$V_{o1,rms} = \frac{V_d^{min}}{\sqrt{2}}m_a^{max} = \frac{295}{2\cdot\sqrt{2}}\cdot 1 = 208.6\,\mathrm{V}.$$

(b) The RMS output current ($I_{o1,rms}$) is

$$I_{o1,rms} = \frac{S}{V_{o1,rms}} = \frac{2000}{208.6} = 9.6\,\mathrm{A}.$$

The peak output current ($\hat{I}_{o1} = \sqrt{2} I_{o1,rms}$) is 13.6 A.

Peak voltage of the switch, $V_T = V_d^{max} = 325$ V.

Peak current of the switch, $I_T = \hat{I}_{o1} = 13.6$ A.

(c) The output voltage of the full-bridge inverter is twice that of the half-bridge. However, the current rating of the switches and the output current is half.

Answers for Chapter 5

MOSFET Switching, Losses and Thermal Modeling

Exercise 5.1 (5-100 in textbook)

To determine the gate resistor which gives the turn-on dV/dt according to the given data, the dynamics of the MOSFET during turn-on are modeled as shown in Fig. 5.1(a). In the figure, the drain current (I_d) is a function of gate-source voltage (V_{gs}), and since V_{gs} is constant, I_d is also constant. During turn-on, the

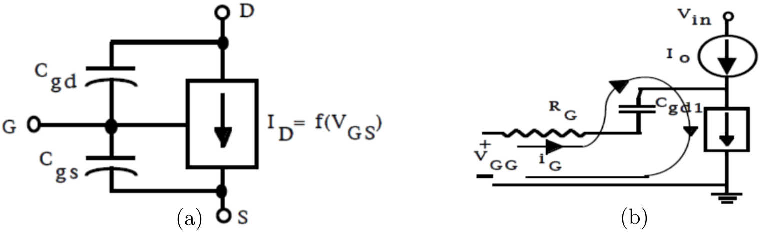

Figure 5.1: Step-down converter.

dV/dt between of the drain-source voltage will discharge the gate-drain capacitance C_{gd} giving a current defined as

$$I_{gd} = C_{gd}\frac{dV_d}{dt} = 120 \cdot 10^{-12} \cdot \frac{100}{200 \cdot 10^{-9}} = 60\,\text{mA}.$$

Specifically related to the Miller plateau where the gate-source voltage is constant during the collapse of the drain voltage, all current from the gate of the MOSFET will go through the gate-drain capacitance according to Fig. 5.1(b).

Consequently, the gate current is defined by the gate-drain capacitance and the dV/dt, i.e.,

$$I_g = I_{gd}$$

Related to the gate drive the following equation applies to the gate current when the gate-source voltage is defined by the Miller plateau voltage, V_{gp},

$$V_{gg} = I_g R_g + V_{gp}, \quad \Longrightarrow R_g = \frac{V_{gg} - V_{gp}}{I_g} = \frac{10 - 4}{60 \cdot 10^{-3}} = 100\,\Omega.$$

Exercise 5.2 **(29-6 in textbook)**

The maximum MOSFET losses (P^l_{mos}) for a given junction (T_j) and case temperature (T_j), and junction-to-case thermal resistance ($R_{\theta jc}$) is

$$T_j - T_c = P^l_{mos} R_{\theta jc} \quad \Longrightarrow P^l_{mos} = \frac{T_j - T_c}{R_{\theta jc}} = \frac{150 - 50}{1} = 100\,\mathrm{W}.$$

P^l_{mos} can also be written as the sum of on-state (conduction) losses ($P^l_{mos(c)}$) and switching losses ($P^l_{mos(s)}$), i.e.,

$$\begin{aligned} P^l_{mos} &= P^l_{mos(c)} + P^l_{mos(s)} = 50 + 10^{-3} f_s \\ \Longrightarrow f_s &= \left(P^l_{mos} - 50\right) \cdot 10^3 = (100 - 50) \cdot 10^3 = 50\,\mathrm{kHz} \end{aligned}$$

Exercise 5.3 **(29-7 in textbook)**

The maximum MOSFET losses (P^l_{mos}) is the sum of on-state (conduction) losses ($P^l_{mos(c)}$) and switching losses ($P^l_{mos(s)}$), i.e.,

$$P^l_{mos} = P^l_{mos(c)} + P^l_{mos(s)} = 50 + 10^{-3} f_s = 50 + 10^{-3} \cdot 25 \cdot 10^3 = 75\,\mathrm{W}.$$

The total thermal resistance between the junction-to-ambient, or junction-to-ambient thermal resistance ($R_{\theta ja}$) is the sum of the junction-to-case ($R_{\theta jc}$) and case-to-ambient ($R_{\theta ca}$) thermal resistances, i.e.,

$$\begin{aligned} R_{\theta ja} = R_{\theta jc} + R_{\theta ca} &= \frac{T_{j,max} - T_a}{P^l_{mos}} \quad \Longrightarrow R_{\theta ca} = \frac{T_{j,max} - T_a}{P^l_{mos}} - R_{\theta jc} \\ \Longrightarrow R_{\theta ca} &= \frac{150 - 35}{75} - 1 = 0.53\,\mathrm{K/W}. \end{aligned}$$

Exercise 5.4 **(12-101 in textbook)**

(a) The on-state loss of the MOSFET IRF540 is given by its drain current and the drain-source on-state resistance, $R_{ds(on)}$, of $0.077\,\Omega$, given in the datasheet. The full-bridge inverter has an RMS load current of 17 A. Considering an average duty cycle of the MOSFET as 50%, i.e., $D = 0.5$, given an RMS MOSFET current I_d,

$$I_d = \sqrt{\frac{1}{T}\int_0^{DT_s} I_o^2 dt} = I_o\sqrt{D} = 17 \cdot \sqrt{0.5} = 12\,\text{A}.$$

The on-state (conduction) loss ($P^l_{mos(c)}$) is

$$P^l_{mos(c)} = I_d^2 R_{ds(on)} = 12^2 \cdot 0.077 = 11.1\,\text{W}.$$

(b) The thermal resistance data from the datasheet are,

$$R_{\theta ja} = 62\,\text{K/W} \qquad\qquad R_{\theta jc} = 1\,\text{K/W}.$$

This means the total thermal resistance from junction to ambient is 62 K/W. The thermal resistance between the junction and the case is specified separately but will be part of the total thermal resistance from junction to ambient.

$$\begin{aligned} &T_j - T_a = R_{\theta ja} P^l_{mos(c)} \\ &\implies T_j = R_{\theta ja} P^l_{mos(c)} + T_a = 25 + 62 \cdot 11.1 = 715^\circ\text{C}. \end{aligned}$$

The device will be destroyed immediately since the maximum junction temperature is 175°C. The case temperature is calculated by subtracting the temperature difference, junction-to-case.

$$\begin{aligned} &T_j - T_c = R_{\theta ja} P^l_{mos(c)} \\ &\implies T_c = T_j - R_{\theta ja} P^l_{mos(c)} = 715 - 1 \cdot 11.1 = 704^\circ\text{C}. \end{aligned}$$

Practically, the junction and case have the same temperature under these conditions.

(c) The MOSFET will be mounted on a heatsink that has a thermal resistance to ambient $R_{\theta sa}$. The interface between the MOSFET and the heatsink gives a thermal resistance $R_{\theta cs} = 0.50\,\text{K/W}$ as defined in the datasheet. Assuming the same power dissipation is calculated in (a), the case-to-ambient temperature rise, $T_c - T_a$, is

$$\begin{aligned} &T_c - T_a = (R_{\theta cs} + R_{\theta sa})\, P^l_{mos(c)} \\ &\implies R_{\theta sa} = \frac{T_c - T_a}{P^l_{mos(c)}} - R_{\theta cs} = \frac{80 - 25}{11.1} - 0.5 = 4.5\,\text{K/W}. \end{aligned}$$

To ensure the temperature is below 80°C, a heatsink with thermal resistance 4.5 K/W is required.

(d) The junction temperature is defined by the temperature rise related to the junction-to-case interface with $R_{\theta jc}$

$$T_j - T_c = R_{\theta jc} P^l_{mos(c)} \qquad \implies T_j = T_c + R_{\theta jc} P^l_{mos(c)} = 80 + 1 \cdot 11.1 = 91^\circ\text{C}.$$

Exercise 5.5 (12-102 in textbook)

(a) The on-state loss of the MOSFET IRF540 is given by its drain current and the drain-source on-state resistance, $R_{ds(on)}$, of $0.077\,\Omega$, given in the datasheet. The full-bridge inverter has an RMS load current of 17 A. Considering an average duty cycle of the MOSFET as 50%, i.e., $D = 0.5$, given an RMS MOSFET current I_d,

$$I_d = \sqrt{\frac{1}{T}\int_0^{DT_s} I_o^2 dt} = I_o\sqrt{D} = 17 \cdot \sqrt{0.5} = 12\,\text{A}.$$

The on-state (conduction) loss ($P^l_{mos(c)}$) is

$$P^l_{mos(c)} = I_d^2 R_{ds(on)} = 12^2 \cdot 0.077 = 11.1\,\text{W}.$$

With $U_d = 15\,\text{V}$, and RMS load current (I_o) of 17 A the average current ($I_{o,av}$) is over half-fundamental period is

$$I_{o,av} = \frac{1}{\pi}\int_0^{\pi} i_o(\omega t)\; d\omega t = \frac{2\sqrt{2}}{\pi} I_o = \frac{2\cdot\sqrt{2}}{\pi}\cdot 17 = 15.3\,\text{A}.$$

The MOSFET switching loss ($P^l_{mos(s)}$) considering $f_s = 50\,\text{kHz}$, and the current and voltage rise and fall times, $t_{ri} = 38\,\text{ns}$, $t_{fv} = 690\,\text{ns}$, $t_{rv} = 24\,\text{ns}$, $t_{fi} = 32\,\text{ns}$., is

$$\begin{aligned} P^l_{mos(s)} &= \frac{1}{2} V_d I_{o,av} f_s \left(t_{ri} + t_{fv} + t_{rv} + t_{fi}\right) \\ &= \frac{1}{2}\cdot 15 \cdot 15.3 \cdot 50 \cdot 10^3 \cdot (38 + 690 + 24 + 32)\cdot 10^{-9} = 4.5\,\text{W}. \end{aligned}$$

The total MOSFET loss, P^l_{mos}, is

$$P^l_{mos} = P^l_{mos(c)} + P^l_{mos(s)} = 11.1 + 4.5 = 15.6\,\text{W}.$$

(b) The thermal resistance data from the datasheet are,

$$R_{\theta ja} = 62\,\text{K/W} \qquad\qquad R_{\theta jc} = 1\,\text{K/W}.$$

This means the total thermal resistance from junction to ambient is 62 K/W. The thermal resistance between the junction and the case is specified separately but will be part of the total thermal resistance from junction to ambient.

$$\begin{aligned} &T_j - T_a = R_{\theta ja} P^l_{mos} \\ &\implies T_j = R_{\theta ja} P^l_{mos} + T_a = 25 + 62 \cdot 15.6 = 994^\circ\text{C}. \end{aligned}$$

The device will be destroyed immediately since the maximum junction temperature is 175°C. The case temperature is calculated by subtracting the temperature difference, junction-to-case.

$$\begin{aligned} &T_j - T_c = R_{\theta ja} P^l_{mos} \\ &\implies T_c = T_j - R_{\theta ja} P^l_{mos} = 994 - 1 \cdot 15.6 = 978^\circ\text{C}. \end{aligned}$$

Practically, the junction and case have the same temperature under these conditions.

(c) The MOSFET will be mounted on a heatsink that has a thermal resistance to ambient $R_{\theta sa}$. The interface between the MOSFET and the heatsink gives a thermal resistance $R_{\theta cs} = 0.50\,\mathrm{K/W}$ as defined in the datasheet. Assuming the same power dissipation is calculated in (a), the case-to-ambient temperature rise, $T_c - T_a$, is

$$\begin{aligned} T_c - T_a &= (R_{\theta cs} + R_{\theta sa})\, P^l_{mos} \\ \implies R_{\theta sa} &= \frac{T_c - T_a}{P^l_{mos}} - R_{\theta cs} = \frac{80 - 25}{15.6} - 0.5 = 3\,\mathrm{K/W}. \end{aligned}$$

To ensure the temperature is below 80°C, a heatsink with thermal resistance 3 K/W is required.

(d) The junction temperature is defined by the temperature rise related to the junction-to-case interface with $R_{\theta jc}$

$$T_j - T_c = R_{\theta jc} P^l_{mos} \qquad \implies T_j = T_c + R_{\theta jc} P^l_{mos} = 80 + 1 \cdot 15.6 = 103.4°\mathrm{C}.$$

Exercise 5.6 (22-13 in textbook)

Two overstress possibilities are overvoltage across drain-source terminals because of stray inductance and excessive power dissipation.

Overvoltage

During the turn-off transient, specifically, during the fall of drain current I_d there is a voltage drop across the inductor (see Fig. 5.2), i.e.,

$$V_d = L_p \frac{di_d}{dt} + V_{di} + V_{dst\text{-off}} \qquad \implies V_{dst\text{-off}} = V_d - V_{di} - L_p \frac{di_d}{dt}.$$

$$\begin{aligned} V_{dst\text{-off}} &= V_d - V_{di} - L_p \frac{0 - I_o}{t_{fi}} = 100 - 0 - 100 \cdot 10^{-9} \cdot \frac{0 - 100}{50 \cdot 10^{-9}} \\ &= 300\,\mathrm{V} > 150\,\mathrm{V}\ (V^{max}_{ds}). \end{aligned}$$

Excessive power dissipation

The maximum power losses to ensure that the junction temperature, $T^{max}_j = 150°\mathrm{C}$ is given by the relation

$$P^{l(max)}_{mos} R_{\theta ja} = T_j - T_a. \qquad \implies P^{l(max)}_{mos} = \frac{T_j - T_a}{R_{\theta ja}} = \frac{150 - 50}{1} = 100\,\mathrm{W}.$$

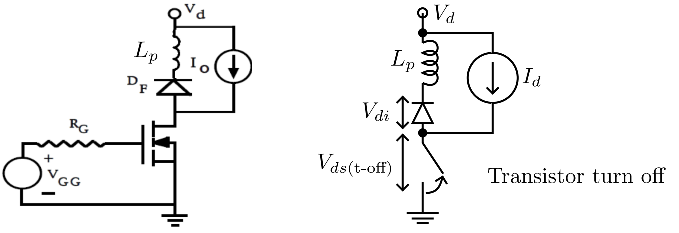

Figure 5.2: Step-down converter with stray inductance and the transistor turn-off transient.

The on-state (conduction) loss ($P^l_{mos(c)}$) for the MOSFET with duty cycle (D), is

$$P^l_{mos(c)} = DI_o^2 R_{ds(on)} = \frac{1}{2} 100^2 \cdot 0.01 = 50\,\mathrm{W}.$$

The MOSFET switching loss ($P^l_{mos(s)}$) considering a switching frequency, f_s, and the current and voltage rise and fall times, t_{ri}, t_{fv}, t_{rv}, and t_{fi}, is

$$\begin{aligned} P^l_{mos(s)} &= \frac{1}{2} V_d I_{o,av} f_s \left(t_{ri} + t_{fv} + t_{rv} + t_{fi}\right) \\ &= \frac{1}{2} \cdot 100 \cdot 100 \cdot 30 \cdot 10^3 \cdot (50 + 200 + 50 + 200) \cdot 10^{-9} = 75\,\mathrm{W}. \end{aligned}$$

The total MOSFET loss, P^l_{mos}, is

$$P^l_{mos} = P^l_{mos(c)} + P^l_{mos(s)} = 50 + 75 = 125\,\mathrm{W} > 100\,\mathrm{W}\ (P^{l(max)}_{mos}).$$

MOSFET is overstressed by both overvoltages and excessive power dissipation.

Answers for Chapter 6

DC-AC Inverter Harmonic Calculations

Exercise 6.1 (8-100 in textbook)

The half-bridge converter is shown in Fig. 6.1.

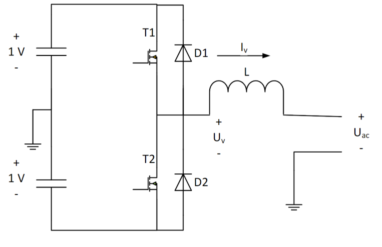

Figure 6.1: Half-bridge inverter.

(a) The waveforms are shown in Fig. 6.2, where the PWM voltage reference is shown as a signal that is sampled at the peaks of the triangular wave. The sampled voltage reference is then compared with the triangular wave to define the switchings.

(b) To determine the largest current harmonic, we start from the harmonics in U_v. From Table 8-1 (or Table 6.1 in the compendium) in the course book we find at $m_a = 0.8$,

$$\frac{\left(\hat{U}_v\right)_h}{\frac{U_d}{2}} = 0.818,$$

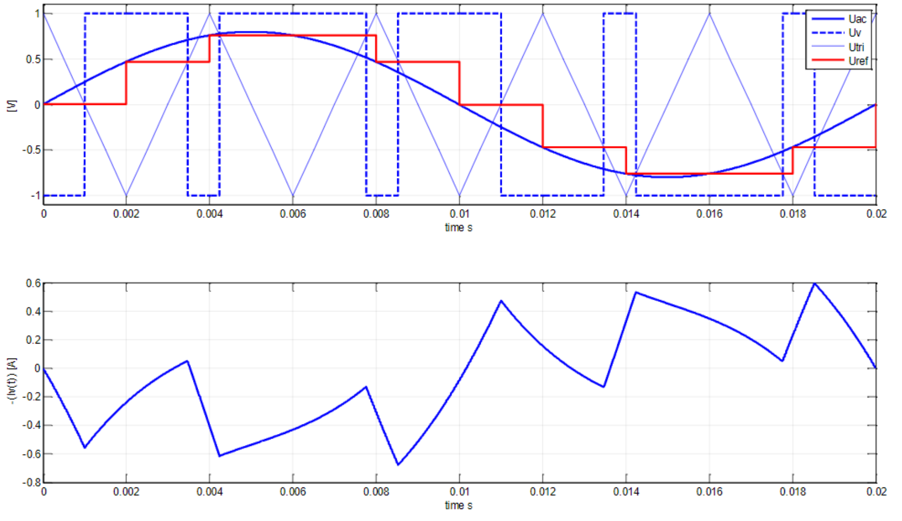

Figure 6.2: Half-bridge inverter waveforms.

at $h = m_f$. Consecutively,

$$\left(\hat{U}_v\right)_{m_f} = 1 \cdot 0.818 = 0.818\,\mathrm{V}.$$

(c) The harmonic current is calculated based on the harmonic voltage across the inductor. The harmonic voltage on the U_{ac} side is zero, giving

$$\left(\hat{I}_v\right)_{m_f} = \frac{\left(\hat{U}_v\right)_{m_f} - \left(\hat{U}_{ac}\right)_{m_f}}{\omega L m_f} = \frac{0.818 - 0}{2 \cdot \pi \cdot 50 \cdot 2 \cdot 10^{-3} \cdot 5} = 0.26\,\mathrm{A},$$

at 250 Hz.

Exercise 6.2 (8-101 in textbook)

In a full-bridge converter with U_d=2 V and $L = 2\,\mathrm{mH}$, PWM is done with a 50 Hz reference having $m_a = 0.8$ and zero phase angle

(a) The output voltage u_v will switch between $+U_d$ and $-U_d$ ($\pm 2\,\mathrm{V}$). The fundamental frequency component (50 Hz) will have a magnitude defined by the PWM reference.

$$\hat{U}_{v1} = m_a U_d = 1.6\,\mathrm{V},$$

giving the RMS value,

$$U_{v1} = \frac{1.6}{\sqrt{2}} = 1.13\,\mathrm{V}.$$

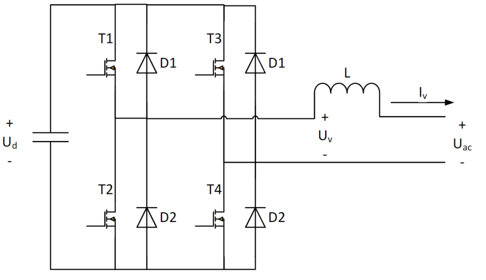

Figure 6.3: Full-bridge inverter with filter internal resistance.

we know that,

$$\begin{aligned} u_{v1}(t) &= \hat{U}_{v1} \sin(\omega t) \\ U_{v1} &= 1.13\,\text{V}. \\ U_{ac1} &= \frac{1}{\sqrt{2}} = 0.707\,\text{V}. \end{aligned}$$

The current, i_v, is defined by the voltage across L, where the fundamental component is calculated using complex RMS quantities:

$$I_{v1} = \frac{U_{v1} - U_{ac1}}{2\pi f L} = \frac{1.13 - 0.707}{2\pi \cdot 50 \cdot 2 \cdot 10^{-3}} = 0.67\,\text{A}.$$

(b) The largest harmonic components of the current is calculated based on the harmonic voltage in u_v.

For **bipolar Switching**, consider the same harmonics as for the half-bridge but twice the amplitude. From Table 8-1 in the course book, at $m_a = 0.8$,

$$\begin{aligned} \frac{\left(\hat{U}_v\right)_h}{\frac{U_d}{2}} &= 0.818|_{\text{at } h=m_f}. \\ \text{i.e., } \left(\hat{U}_v\right)_{m_f} &= 2 \cdot 0.818 = 1.64\,\text{V}. \end{aligned}$$

The harmonic current is calculated based on the harmonic voltages across the inductor. The harmonic voltage on the U_{ac} side is zero, giving:

$$\left(\hat{I}_v\right)_{m_f} = \frac{\left(\hat{U}_v\right)_{m_f} - \left(\hat{U}_{ac}\right)_{m_f}}{2\pi m_f f L} = \frac{1.64 - 0}{2 \cdot \pi \cdot 5 \cdot 50 \cdot 2 \cdot 10^{-3}} = 0.52\,\text{A}.$$

For **unipolar switching** the m_f component will be canceled and the next large component is $2m_f \pm 1$. From Table 8-1 in the course book, at $m_a = 0.8$,

$$\frac{\left(\hat{U}_v\right)_h}{\frac{U_d}{2}} = 0.314|_{\text{at } h=2m_f \pm 1}.$$

Remark, the double voltage magnitude is considered because a full-bridge converter is analyzed.

$$\left(\hat{U}_v\right)_{2m_f \pm 1} = 2 \cdot 0.314 = 0.628\,\text{V}.$$

The harmonic current is calculated based on the harmonic voltages across the inductor. The frequency considered here is $(2m_f - 1)\,f$ because the inductive reactance is low at low frequencies and thereby resulting higher current at $(2m_f - 1)\,f$ than $(2m_f + 1)\,f$. The harmonic voltage on the U_{ac} side is zero, giving:

$$\begin{aligned}\left(\hat{I}_v\right)_{2m_f-1} &= \frac{\left(\hat{U}_v\right)_{2m_f-1} - \left(\hat{U}_{ac}\right)_{2m_f-1}}{2\pi\,(2m_f - 1)\,fL} = \frac{0.628 - 0}{2 \cdot \pi \cdot (2 \cdot 5 - 1) \cdot 50 \cdot 2 \cdot 10^{-3}} \\ &= 0.11\,\text{A}.\end{aligned}$$

Exercise 6.3 (8-102 in textbook)

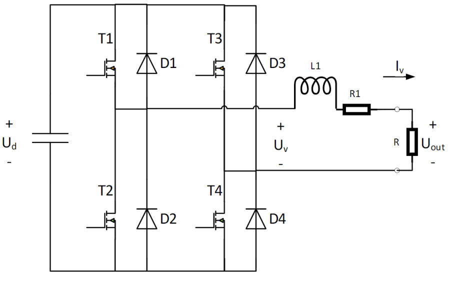

Figure 6.4: Full-bridge inverter with filter internal resistance.

(a) The fundamental RMS voltage of U_v:

$$U_{v1} = m_a \frac{U_d}{\sqrt{2}} = 0.8 \cdot \frac{15}{\sqrt{2}} = 8.5\,\text{V}.$$

The fundamental peak voltage of U_{out} is defined by the voltage drop across the load resistor R.

$$\begin{aligned}U_{out1} &= U_{v1} \frac{R}{|R + R_1 + j2\pi f_1 L|} = 8.5 \cdot \frac{10}{|10 + 0.35 + j2\pi \cdot 50 \cdot 2 \cdot 10^{-3}|} \\ &= 8.1\,\text{V}.\end{aligned}$$

(b) To determine the two largest output voltage harmonics, we start from the harmonics in U_v. For **unipolar switching** the m_f component will be canceled and the next large component is $2m_f \pm 1$. From Table 8-1 in the course book (or Table 6.1 in the compendium), at $m_a = 0.8$,

$$\frac{\left(\hat{U}_v\right)_h}{\frac{U_d}{2}} = 0.314|_{\text{at } h=2m_f\pm1}.$$

$$\text{i.e.,} \quad \left(\hat{U}_v\right)_{2m_f\pm1} = 15 \cdot 0.314 = 4.71\,\text{V}.$$

The harmonics in the output voltage are defined by the same impedance relation as in (a), i.e.,

$$\begin{aligned}
(U_{out})_h &= (U_v)_h \frac{R}{|R + R_1 + j2\pi h f_1 L|} \\
(U_{out})_{2m_f-1} &= (U_v)_{2m_f-1} \frac{R}{|R + R_1 + j2\pi\,(2m_f - 1)\, f_1 L|} \\
&= 4.71 \cdot \frac{10}{|10 + 0.35 + j2 \cdot \pi \cdot (2 \cdot 19 - 1) \cdot 150 \cdot 2 \cdot 10^{-3}|} = 0.47\,\text{V}. \\
(U_{out})_{2m_f+1} &= (U_v)_{2m_f-1} \frac{R}{|R + R_1 + j2\pi\,(2m_f - 1)\, f_1 L|} \\
&= 4.71 \cdot \frac{10}{|10 + 0.35 + j2 \cdot \pi \cdot (2 \cdot 19 + 1) \cdot 150 \cdot 2 \cdot 10^{-3}|} = 0.45\,\text{V}.
\end{aligned}$$

(c) Based on the two largest harmonics, the total harmonic distortion (THD) is

$$THD = \frac{\sqrt{\sum_{h=2}^{\infty} (U_{out})_h^2}}{(U_{out})_1} = \frac{\sqrt{0.47^2 + 0.45^2}}{8.1\sqrt{2}} = 5.7\%.$$