Lab 1 Power analysis

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Power analysis

Power in a circuit without harmonics

Calculation of the active power in an arbitrary block with voltage and current according to Figure 1.

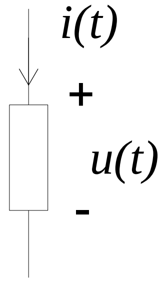

Figure 1

Considering the voltage and current in Figure 1 being pure fundamental frequency we can describe them as:

$$u(t) = \hat{U}\cos(\omega t + \phi_u)[V]$$
$$i(t) = \hat{I}\cos(\omega t + \phi_i)[A]$$

where $\omega = 2\pi f = \frac{2\pi}{T}$

The instantaneous power is the product of voltage and current as:

$$p(t) = u(t) \cdot i(t)$$

Average power is calculated over one fundamental frequency cycle.

$$P_{av} = \int_0^T p(t)dt = \frac{1}{T}\int_0^T u(t) \cdot i(t)\, dt = \frac{1}{T}\int_0^T \hat{U}\hat{I}\cos(\omega t + \phi_u) \cdot \cos(\omega t + \phi_i)dt$$

Regarding a product of two cosines we use the following trigonometric identity

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$$\cos a \cdot \cos b = \frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b)$$

This results in the voltage-current product being separated into one dc-term and one term with the double frequency.

$$P_{av} = \frac{1}{T}\int_0^T \frac{\hat{U}\hat{I}}{2}\big(\cos(\phi_u - \phi_i) + \cos(2\omega t + \phi_u + \phi_i)\big)dt =$$
$$= \frac{\hat{U}\hat{I}}{2}\cos(\phi_u - \phi_i) + \frac{1}{T}\int_0^T \cos(2\omega t + \phi_u + \phi_i)dt$$

If we consider the expression inside the integral, it corresponds to the instantaneous power. Consequently, the instantaneous power consists of a constant (dc-) term and a time varying term with frequency 2ω. Regarding the average power, only the dc-term shall be considered since the average of the second term, with double frequency, is zero due to symmetry between positive and negative half-cycles. Remaining, is the dc-term which is defined by the product of the rms-values of voltage and current ($\frac{\hat{U}}{\sqrt{2}} \cdot \frac{\hat{I}}{\sqrt{2}}$) and cosine of the angle difference ($\phi_u - \phi_i$).

$$P_{av} = \frac{\hat{U}}{\sqrt{2}} \cdot \frac{\hat{I}}{\sqrt{2}}\cos(\phi_u - \phi_i) = U \cdot I \cdot \cos\phi [W]$$
$$\phi = \phi_u - \phi_i$$

Power in a circuit with harmonics

Again, analyzing the simple circuit, where we now assume it to be a resistive load.

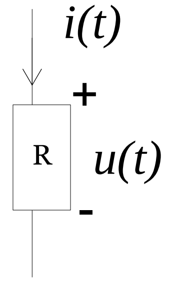

Figure 2

Considering the current in Figure 2 being a combination of direct current and 2nd harmonic frequency we can describe it as:

$$u(t) = \hat{U}\cos(\omega t + \phi_u)[V]$$
$$i(t) = I_d + \hat{I}_2\cos(2\omega t + \phi_i)[A]$$

The voltage can be calculated as

$$u(t) = \mathrm{R} \cdot i(t)$$

The instantaneous power is the product of voltage and current as:

$$p(t) = u(t) \cdot i(t) = R \cdot i^2(t)$$

Average power is calculated over one fundamental frequency cycle.

$$P_{av} = \int_0^T p(t)dt = \frac{1}{T}\int_0^T R \cdot i(t) \cdot i(t)\ dt$$

$$= \frac{R}{T}\int_0^T \left(I_d^2 + \hat{I}_2^2\cos(2\omega t + \phi_i) \cdot \cos(2\omega t + \phi_i) + 2I_d \cdot \hat{I}_2\cos(2\omega t + \phi_i)\right) dt$$

Using the trigonometric identity for the product of cosines yields:

$$P_{av} = \frac{R}{T}\int_0^T \left(I_d^2 + \frac{\hat{I}_2^2}{2} + \frac{\hat{I}_2^2}{2}\cos(4\omega t + 2\phi_i) + 2I_d \cdot \hat{I}_2\cos(2\omega t + \phi_i)\right) dt$$

$$= R \cdot I_d^2 + R \cdot \frac{\hat{I}_2^2}{2} + \frac{R}{T}\int_0^T \left(\frac{\hat{I}_2^2}{2}\cos(4\omega t + 2\phi_i) + 2I_d \cdot \hat{I}_2\cos(2\omega t + \phi_i)\right) dt$$

The average value of the oscillating components is zero which gives:

$$P_{av} = R \cdot \left(I_d^2 + \frac{\hat{I}_2^2}{2}\right)$$

The average power can also be calculated using the total rms value for the current. Total rms is calculated as the sum of squares of the rms of individual frequency components (including dc).

$$I_{rms} = \sqrt{I_d^2 + \left(\frac{\hat{I}_2}{\sqrt{2}}\right)^2} = \sqrt{I_d^2 + \frac{\hat{I}_2^2}{2}}$$

$$P_{av} = R \cdot I_{rms}^2 = R \cdot \left(I_d^2 + \frac{\hat{I}_2^2}{2}\right)$$

The power related to individual frequency components are calculated as:

$$P_{dc} = R \cdot I_d^2$$

$$P_2 = R \cdot \frac{\hat{I}_2^2}{2}$$

where the total average power is

$$P_{av} = P_{dc} + P_2$$

In the figure below an example is illustrated based on the following data:
$I_d = 2\,A$, $\hat{I}_2 = 2\sqrt{2}\,A$, f=50 Hz
Total rms is calculated as $I_{rms} = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ A.
Total average power created by this current through a 40 ohm resistor is then

$$P_{av} = R \cdot I_{rms}^2 = 40 \cdot 8 = 320\,W$$

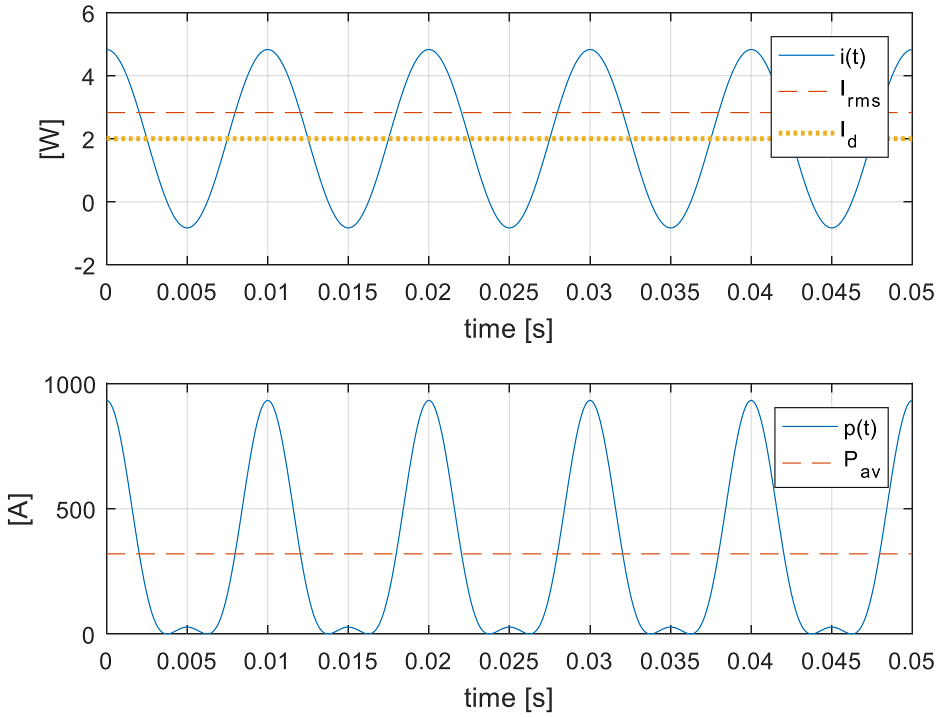

Figure 3

From the power graph above, it can also be understood that when making a Fourier analysis of the instantaneous power, the average value corresponds to the total average while the 2nd and 4th harmonics corresponds to the oscillating power components which have zero average value.

Reference

Section 3-2-2, 3-2-4 in Mohan Power electronics